RUB

Evgeny Epelbaum, RUB

ECT* workshop Three-Body Forces: From Matter to Nuclei, Mai 5-9 2014

Improved chiral nuclear potentials

Motivation

Construction of the potential

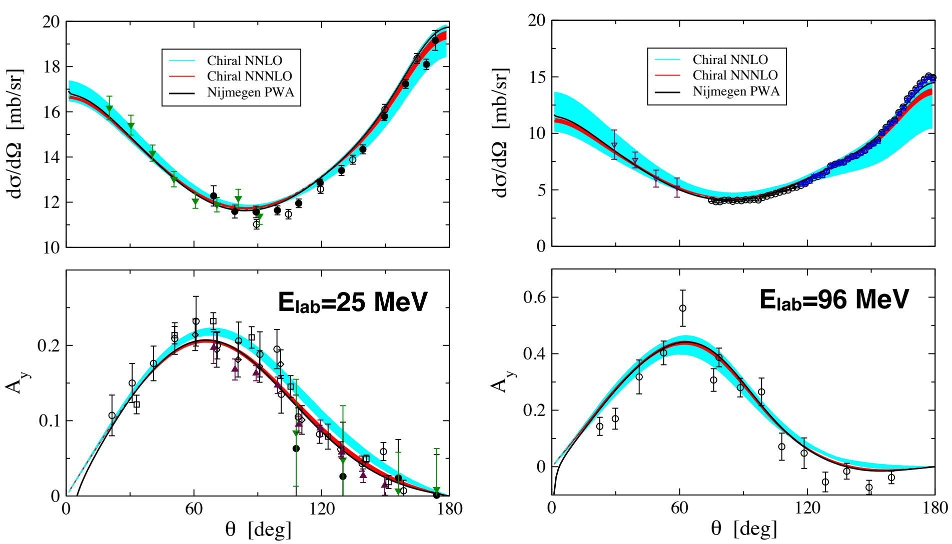
Some results

Summary & outlook

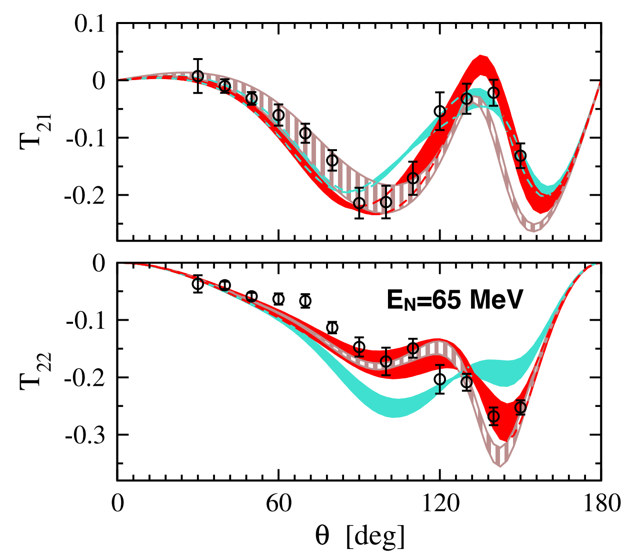
Motivation

- Chiral 3NF at N^{3}LO & N^{4}LO are derived using DR while the EGM 2NFs employ SFR…
- Would like to update πN LECs (and have a flexibility to use different sets of them)
- Need to think about relativistic corrections
- Nd scattering is the most natural testing ground for chiral 3NF. Large 3NF effects are expected/needed at intermediate & higher energies [Nasser's talk]
  → need to increase the accuracy of chiral 2NF:
    - go to higher orders in the chiral expansion
    - try to reduce finite-cutoff artefacts [similar philosophy for NLEFT, Dean's talk]

np differential cross section & analyzing power



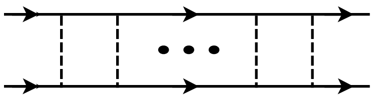
Nd tensor analyzing powers at 65 MeV



Finite-cutoff artifacts

Why cutoff?

$$T = \underbrace{V}_{\text{truncated at a given order in the expansion}} + VG_0T = \underbrace{V + VG_0V + VG_0VG_0V + \ldots}_{\text{increasingly UV divergent integrals are generated through iterations}}$$



Ideally, would like to calculate the integrals

$$T^{(n)} = V[G_0V]^{n-1} = m^{n-1}\int_0^\Lambda d^3l_1 \ldots d^3l_{n-1} \frac{V(\vec{p}\,', \vec{l}_1)\, V(\vec{l}_1, \vec{l}_2) \ldots V(\vec{l}_{n-1}, \vec{p})}{[p^2 - l_1^2 + i\epsilon] \ldots [p^2 - l_{n-1}^2 + i\epsilon]}$$

subtract the UV divergences ($\sim \log\Lambda;\ \Lambda;\ \Lambda^2; \ldots$) and take the limit $\Lambda \to \infty$. This is not possible in practice (except for pionless EFT) ⟶ let Λ finite and adjust bare C_i to exp data (= implicit renormalization). Λ should be not taken too high [Lepage, EE, Meißner, Gegelia], in practice $\max[\Lambda] \sim 600$ MeV (otherwise spurious BS...)

Price to pay: finite cutoff artifacts (i.e. terms $\sim 1/\Lambda;\ 1/\Lambda^2;\ 1/\Lambda^3; \ldots$), may become an issue at higher energies (e.g. $E_{lab} \sim 200$ MeV corresponds to $p \sim 310$ MeV/c)

Is it possible to eliminate or at least reduce finite cutoff artifacts?

1st option:
Renormalizable chiral EFT for NN (Λ=∞)

Refrain from doing non relativistic expansion prior to solving the integral equation EE, Gegelia '12

→ 3D equations which fulfill relativistic elastic unitarity, e.g.:

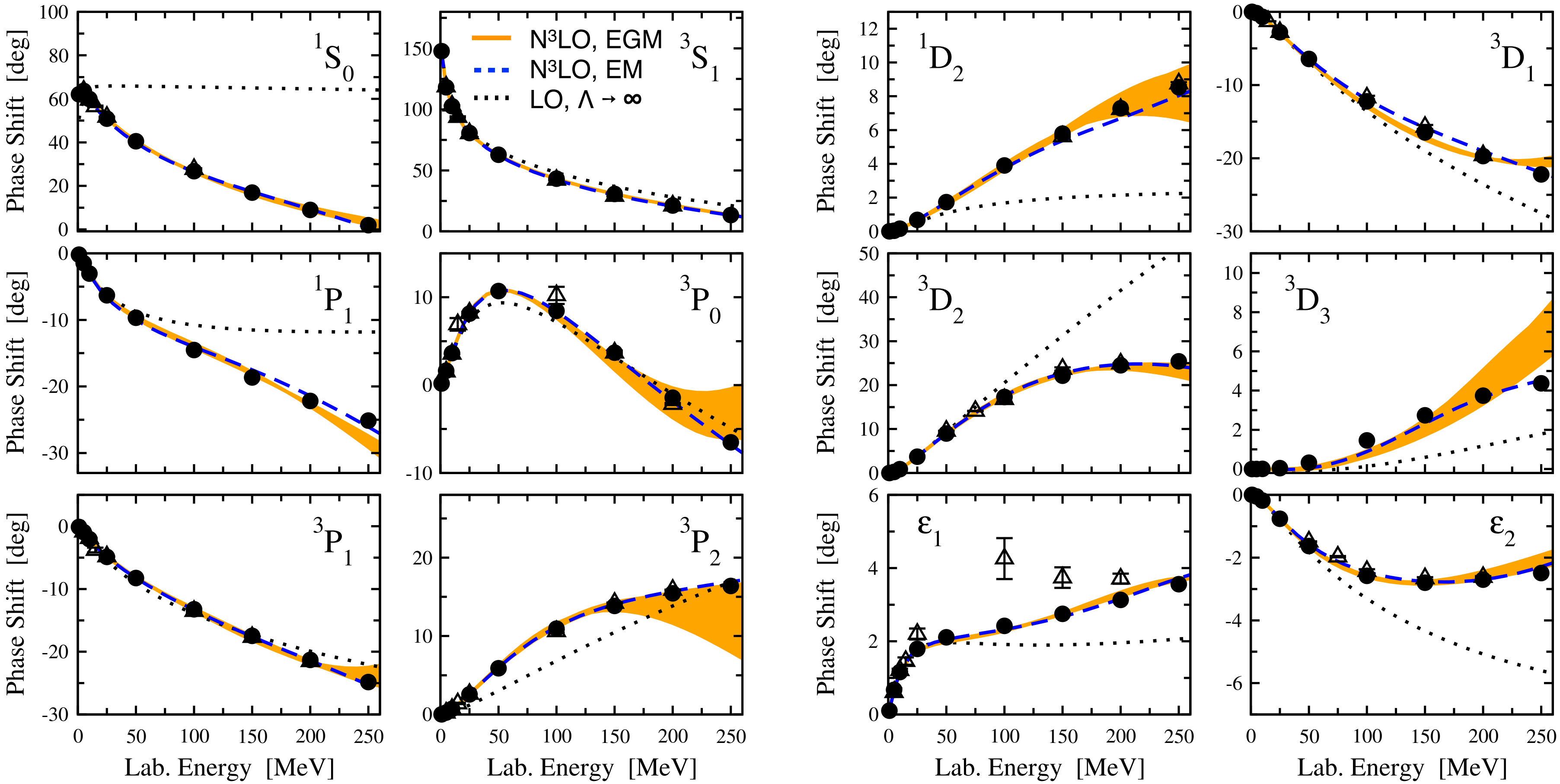
$$T(\vec{p}\,',\vec{p}) = V_{2N}^{(0)}(\vec{p}\,',\vec{p}) + \frac{m_N^2}{2}\int \frac{d^3k}{(2\,\pi)^3}\,\frac{V_{2N}^{(0)}(\vec{p}\,',\vec{k})\,T(\vec{k},\vec{p})}{(k^2+m_N^2)\,(E-\sqrt{k^2+m_N^2}+i\,\epsilon)}$$

Kadyshevsky '68

- LO equation is log-divergent (i.e. renormalizable) → can safely take $\Lambda \to \infty$!
- corrections beyond LO are to be included perturbatively
- paramater-free results for m_q dependence of NN observables [EE, Gegelia '13] and the deuteron form factors at LO [EE, Gasparyan, Gegelia, Schindler '14]

Neutron-proton phase shifts at LO

EE, Gegelia '12



2nd option:

Keep Λ finite but try to reduce finite-Λ artifacts

Regularization of the chiral NN potentials

order of the chiral expansion

$$\langle \vec{p}'|\, V^{\rm NNLO}\, |\vec{p}\rangle = \left[V_{1\pi}^{(0)} + V_{2\pi}^{(2)} + V_{2\pi}^{(3)} + V_{\rm cont}^{(0)} + V_{\rm cont}^{(2)} \right] e^{\frac{-p'^4 - p^4}{\Lambda^4}}$$

The cutoff Λ should not be chosen too large (spurious bound states, nonlinearities, nonrenormalizable theory) Lepage'97, EE., Meißner '06, EE, Gegelia '09. On the other hand, smaller values of Λ introduce unnecessary errors.

Typical choice: Λ = 450...600 MeV [N³LO potentials by EGM, EM]

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Typical choice: Λ = 450...600 MeV [N^{3}LO potentials by EGM, EM]

Claim: while the above nonlocal regulator simplifies the determination of the LECs, it cuts off some model-independent long-range physics one would like to keep and leaves some model-dependent short-range physics one would like to cut off...

Given that $V_{1\pi}^{(0)} + V_{2\pi}^{(2)} + V_{2\pi}^{(3)}$ is local, local regulator will do a better job!

Reminder:

$$V_{\rm local}(\vec{p}\,', \vec{p}\,) \equiv \langle \vec{p}\,'|V_{\rm local}|\vec{p}\,\rangle = V(\vec{p}\,' - \vec{p}\,) \quad \longrightarrow \quad V(\vec{r}\,', \vec{r}\,) \equiv \langle \vec{r}\,'|V|\vec{r}\,\rangle = \delta^3(\vec{r}\,' - \vec{r}\,)V(\vec{r}\,)$$

Regularization of the chiral NN potentials

Peripheral NN scattering as a long-range filter: insensitive to short-range physics and determined by the model-independent long-range interaction ($V_{1\pi}$). Can be computed using Born approximation: $T_{\alpha'\alpha}(p) \equiv \langle p, \alpha' | T | p, \alpha \rangle = \langle p, \alpha' | V_{1\pi} | p, \alpha \rangle$

where $V_{1\pi}(\vec{q}) = -\frac{g_A^2}{4F_\pi^2} \frac{(\vec{q} \cdot \vec{\sigma}_1)(\vec{q} \cdot \vec{\sigma}_2)}{\vec{q}^{\,2} + M_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$

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- **Standard, nonlocal regularization** $V_{1\pi}^{\rm reg}(\vec{q}) = V_{1\pi}(\vec{q}) \; F\left(\frac{p'}{\Lambda}, \frac{p}{\Lambda}\right)$

  Partial-wave decomposition: $\langle p', \alpha' | V_{1\pi}^{\rm reg} | p, \alpha \rangle = \langle p', \alpha' | V_{1\pi} | p, \alpha \rangle \; F\left(\frac{p'}{\Lambda}, \frac{p}{\Lambda}\right)$

  Regulator affects all partial waves at high momenta independently on $\boldsymbol{\alpha}, \; \boldsymbol{\alpha'}$

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  Regulator affects all partial waves at high momenta independently on $\boldsymbol{\alpha}, \ \boldsymbol{\alpha}'$

- **Local regularization**

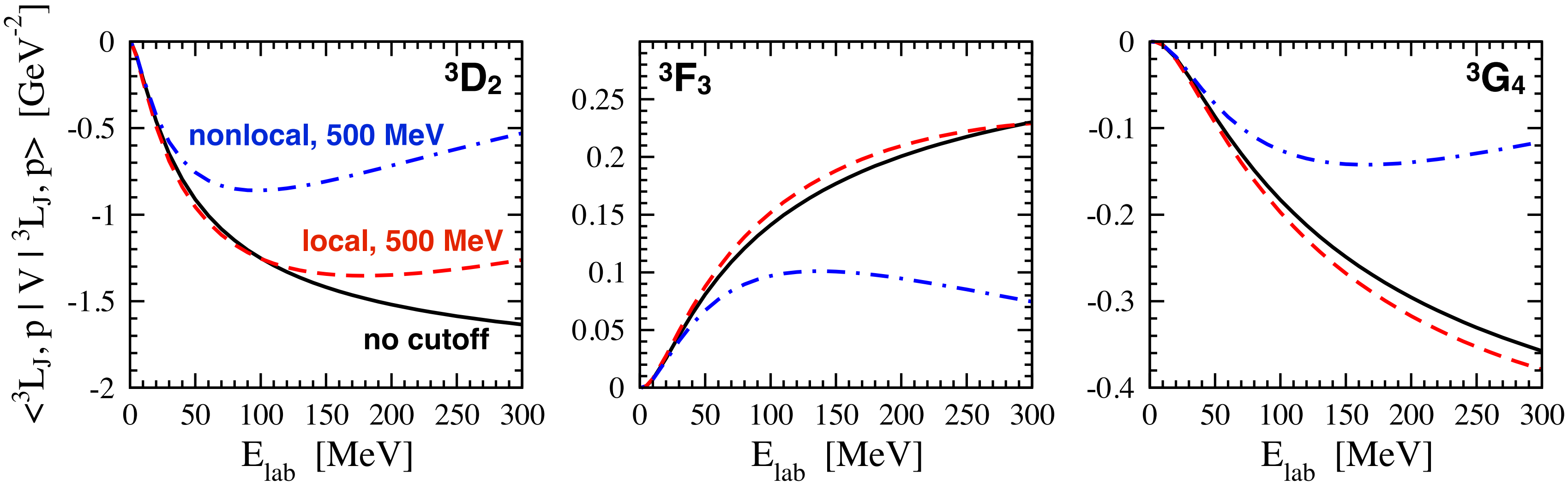
  $V_{1\pi}^{\rm reg}(\vec{q}) = V_{1\pi}(\vec{q})\, F\left(\frac{q}{\Lambda}\right)$ or, alternatively, $V_{1\pi}^{\rm reg}(\vec{r}) = V_{1\pi}(\vec{r})\, F(r/R_0)$

  Partial-wave matrix elements in momentum space:

$$\langle p', \alpha' | V_{1\pi}^{\rm reg} | p, \alpha \rangle \sim \underbrace{\int r^2 dr \, j_{l'}(p'r) \left[V_{1\pi}^{\alpha'\alpha}(r) F(r/R_0) \right] j_l(pr)}_{\text{becomes insensitive to } F \text{ for high } \boldsymbol{l}, \ \boldsymbol{l}'}$$

Regularization of the chiral NN potentials

PW projected MEs of the OPEP: $\exp[-(p'^2+p^2)/\Lambda^2]$ versus $\exp[-q^2/\Lambda^2]$ for Λ = 500 MeV



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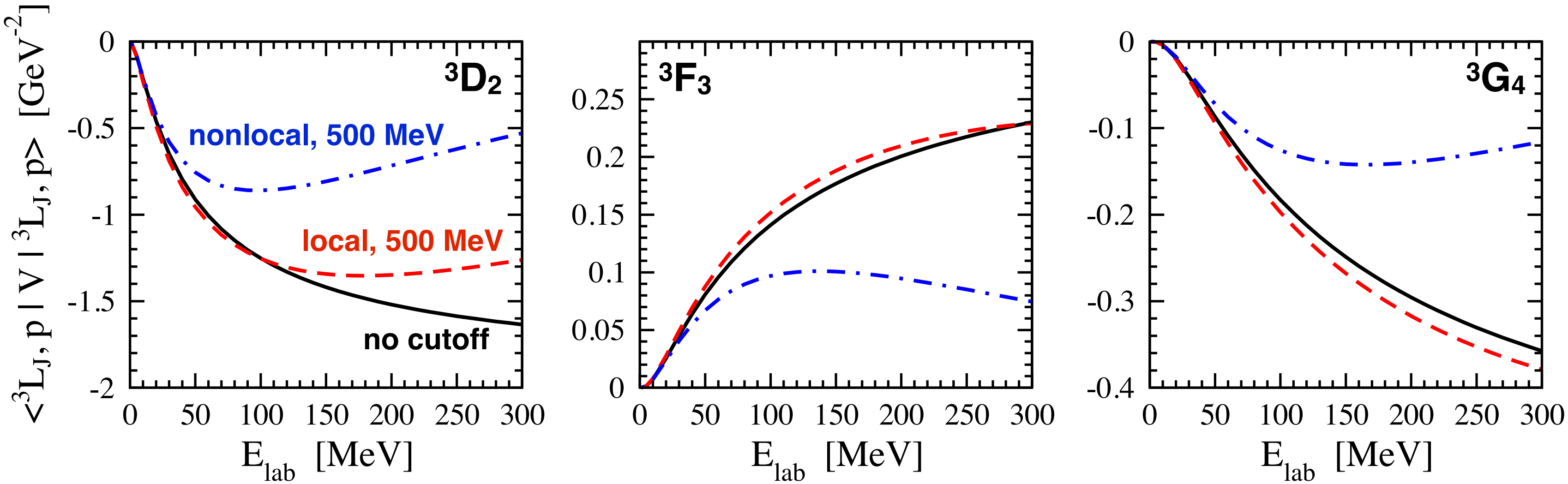
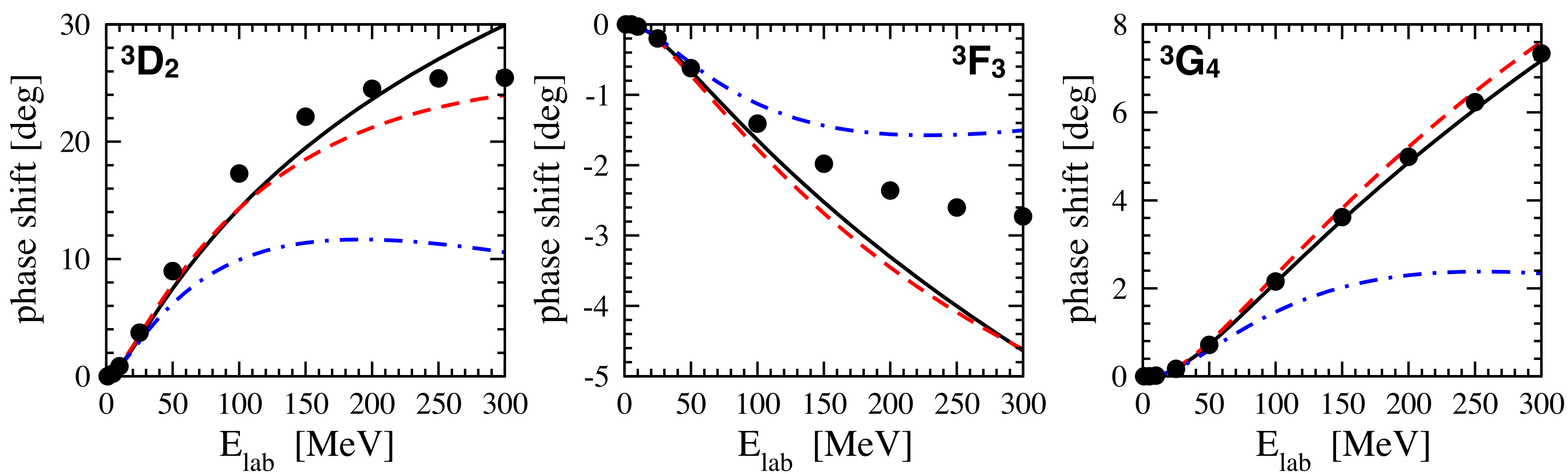


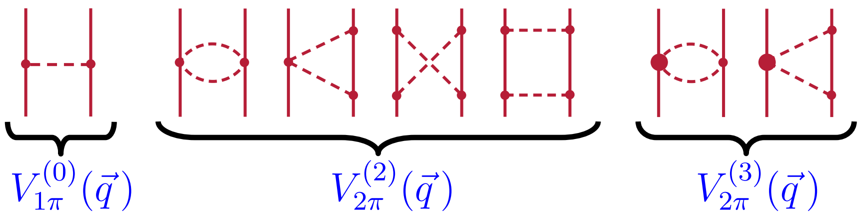
Peripheral partial waves based on the OPE potential (Born approx.)



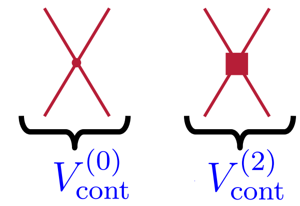
Construction of the potential (published local version)

Gezerlis, Tews, EE, Gandolfi, Hebeler, Nogga, Schwenk, PRL 111 (2013) 032501;   more details in the talk by Ingo

Long-range:



Short-range:



There are 9 isospin-concerving contact terms whose choice is not unique. Standard:

$$V_{\rm cont}^{(0)} = C_S + C_T(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$$

$$V_{\rm cont}^{(2)} = C_1 q^2 + C_2 k^2 + C_3 q^2 (\vec{\sigma}_1 \cdot \vec{\sigma}_2) + C_4 k^2 (\vec{\sigma}_1 \cdot \vec{\sigma}_2) + \frac{iC_5}{2}(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{q} \times \vec{k} + C_6 (\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q}) + C_7 (\vec{\sigma}_1 \cdot \vec{k})(\vec{\sigma}_2 \cdot \vec{k})$$

where $\vec{q} = \vec{p}\,' - \vec{p}, \quad \vec{k} = (\vec{p} + \vec{p}\,')/2$

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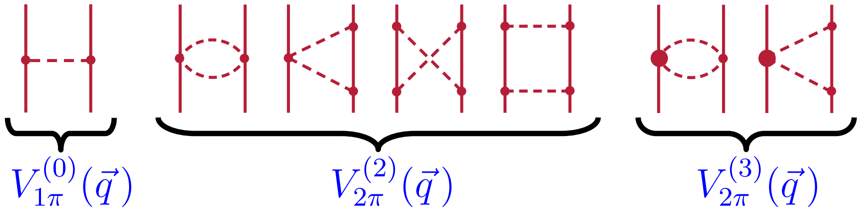
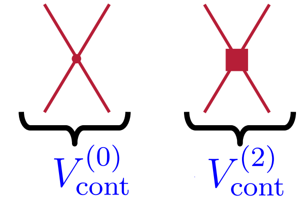
Long-range: 

Short-range: 

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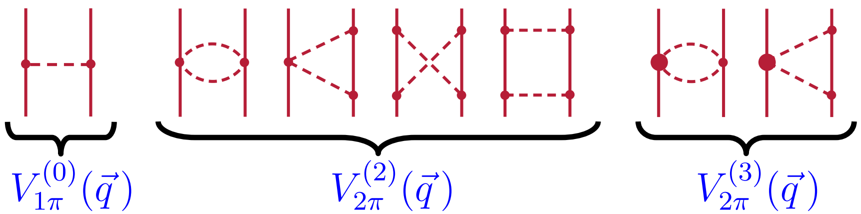

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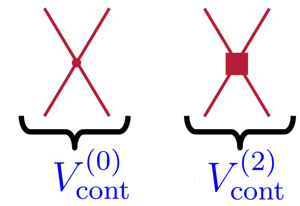

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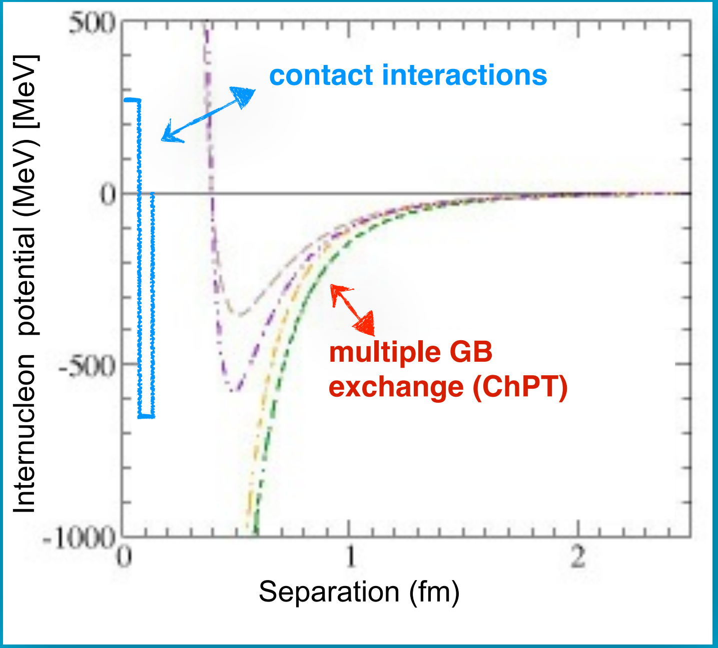
Make Fourier Transform and regularize in configuration space, e.g.:

$$V_{\rm long}(\vec{r}) \;\to\; V_{\rm long}(\vec{r})\left[1 - e^{-r^4/R_0^4}\right]$$ and $$\delta^3(\vec{r}) \;\to\; \alpha e^{-r^4/R_0^4}$$ where $$\alpha = \frac{1}{\pi \Gamma(3/4) R_0^3}$$

The LECs are determined from NN S-, P-waves and the mixing angle ε_1

Choice of the cutoff

What is the breakdown distance of the chiral expansion of the multiple-pion exchange ?



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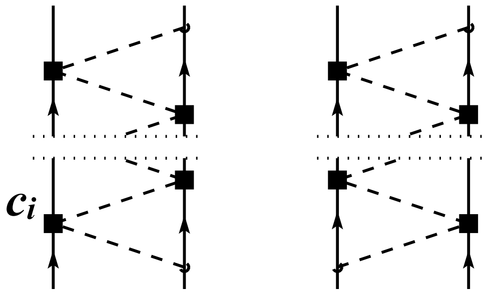
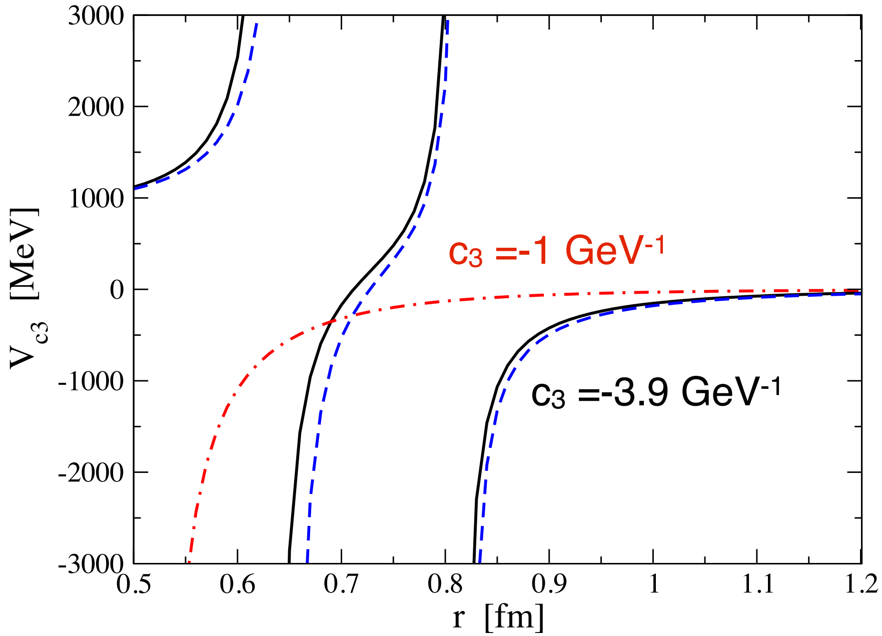
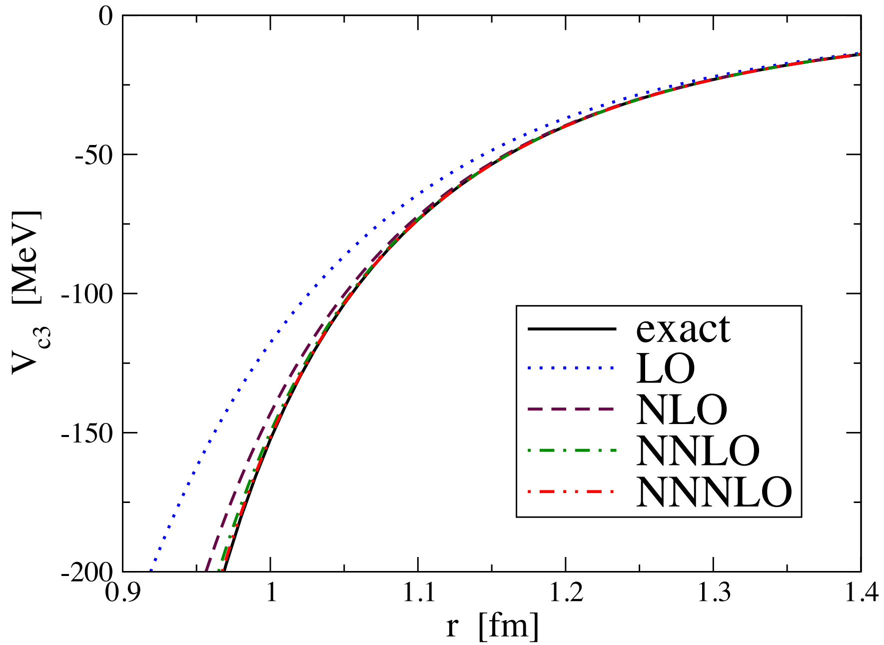


Certain classes of multiple-pion exchange diagrams (MSS) can be calculated analytically to an infinite order and resummed

Baru, EE, Hanhart, Hoferichter, Kudryavtsev, Phillips, EPJA 48 (12) 69

Resummed central potential generated by multi-pion exchange (c_3-part)





pole(!) ar r ~ 0.8 fm but good convergence of the chiral expansion for r > 1fm

New chiral NN interactions

Already available:

- Completely local (except for the short-range LS-term) potentials @ LO, NLO, N^{2}LO [R_0 = 1.0, 1.1 and 1.2 fm and Λ_{SFR} = 0.8…1.4 GeV]

  Freunek '08; Gezerlis, Tews, EE, Gandolfi, Hebeler, Nogga, Schwenk, PRL 111 (2013) 032501; in preparation

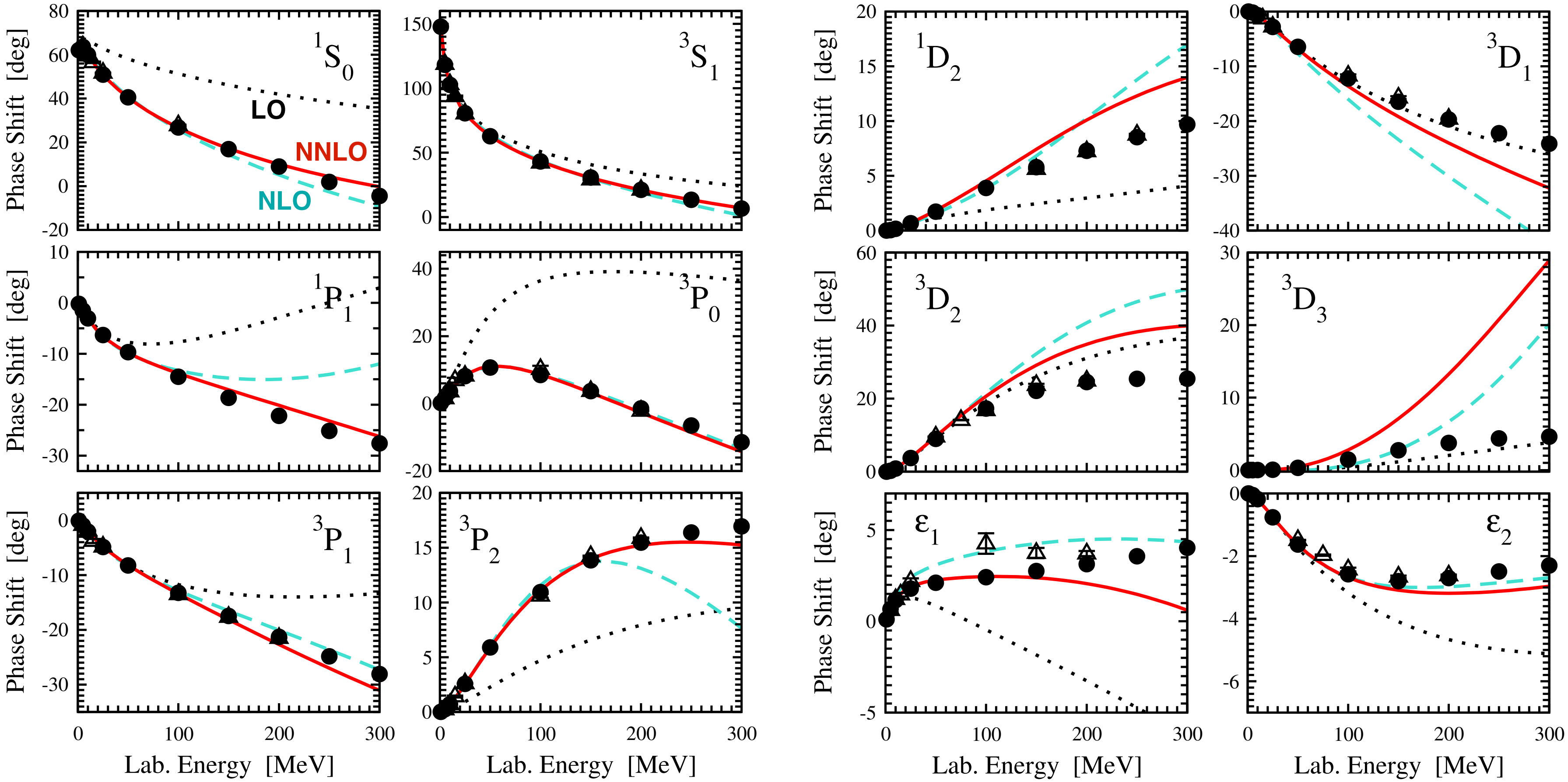
In development (testing) [in collaboration with: Krebs, Nogga, Meißner, Golak, Skibinski, Witala, Kamada]

- New version of l_{ocal}-**chiral** potentials @ LO, NLO, N^{2}LO [Λ_{SFR} up to Infinity, PWD MEs and operator form both in r-space and p-space]
- New $\text{i}_{\text{mproved}}$-**chiral** potentials up to N^{3}LO [Λ_{SFR} up to Infinity, PWD MEs and operator form in p-space]

Some results
(everything very preliminary)

I-chiral 2NF: Order-by-order improvement

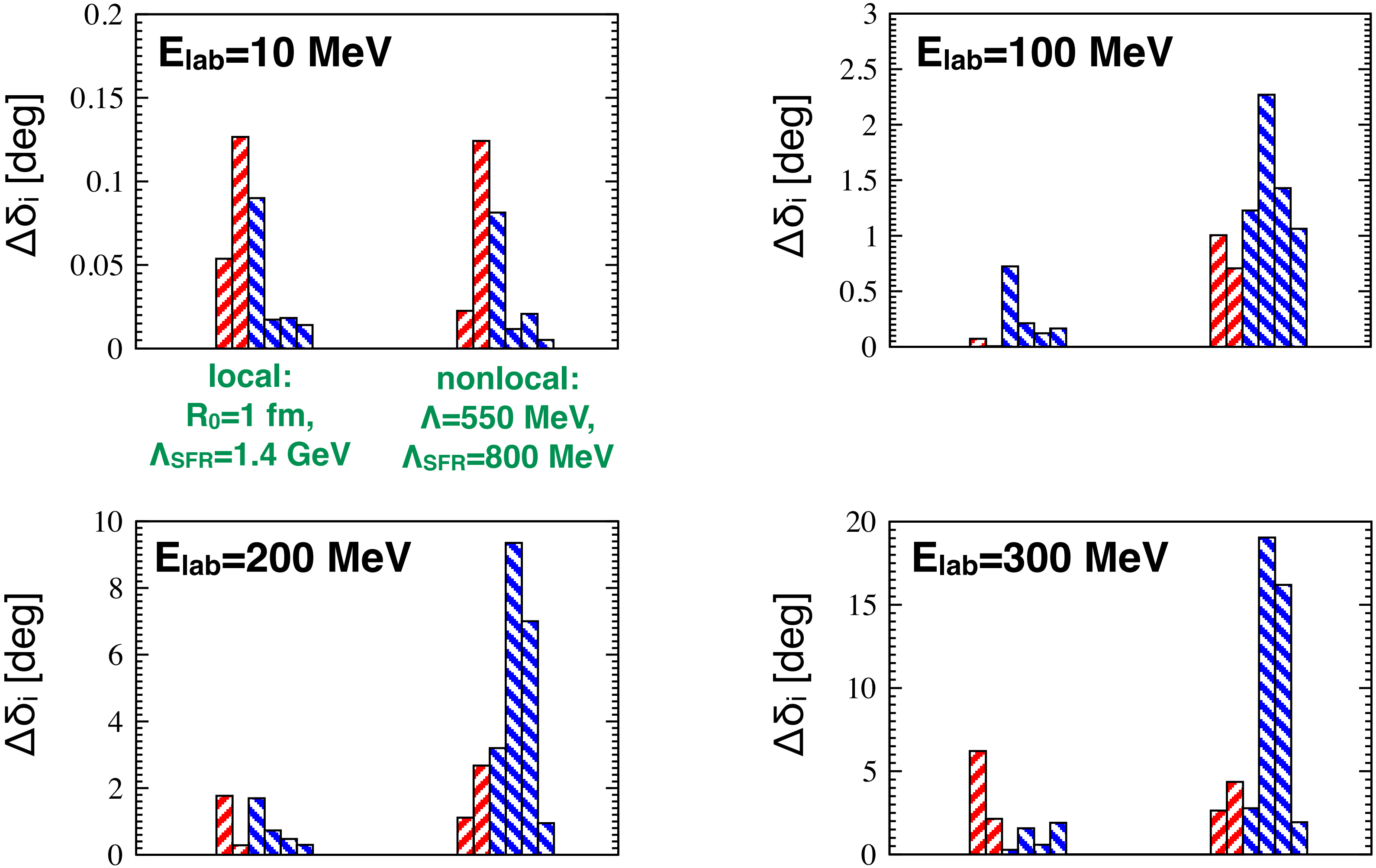
neutron-proton phase shifts on I-chiral 2NF at LO, NLO and N²LO



$R_0 = 1$ fm, $\Lambda_{SFR} = 2$ GeV

Error budget: local vs nonlocal regulators

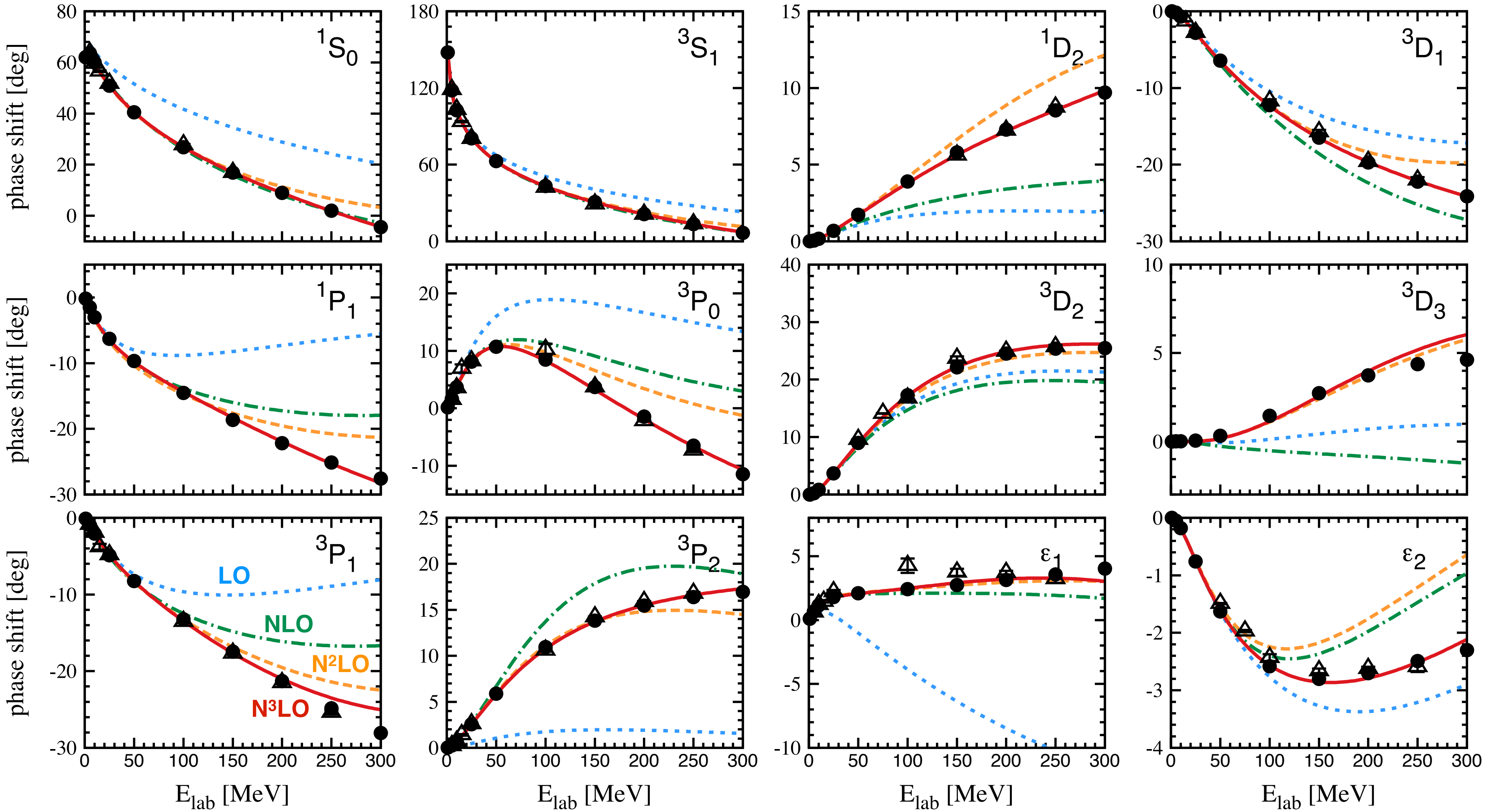
Absolute errors in S- and P-wave phase shifts at N^2LO



Ordering of partial waves: $^1S_0$, $^3S_1$, $^1P_1$, $^3P_0$, $^3P_1$, $^3P_2$

i-chiral 2NF: Order-by-order improvement

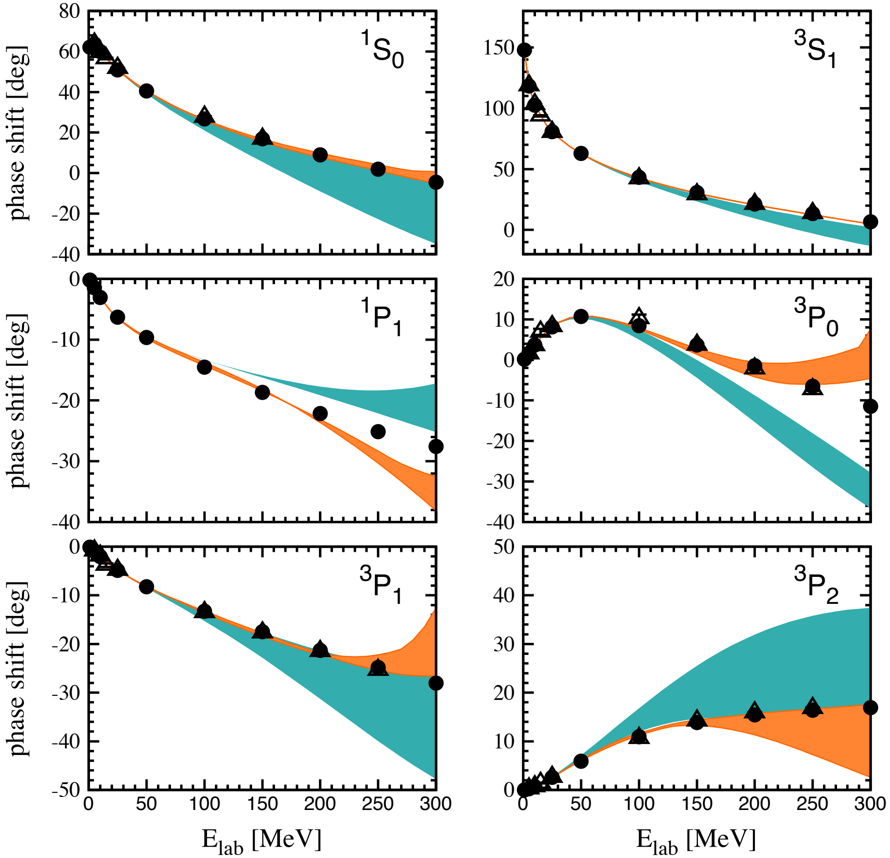
neutron-proton phase shifts on i-chiral 2NF at LO, NLO, N^2LO and N^3LO (w.o. 1/m)



R_0 = 0.9 fm, Λ_{SFR} = Infinity [i.e. DR]

Cutoff dependence: i-chiral vs old EGM’04

np phase shifts based on EGM’04 N²LO/N³LO 2NF



N²LO: Λ = 450…600 fm, Λ_{SFR} =500…700 MeV

N³LO: Λ = 450…600 fm, Λ_{SFR} =500…700 MeV

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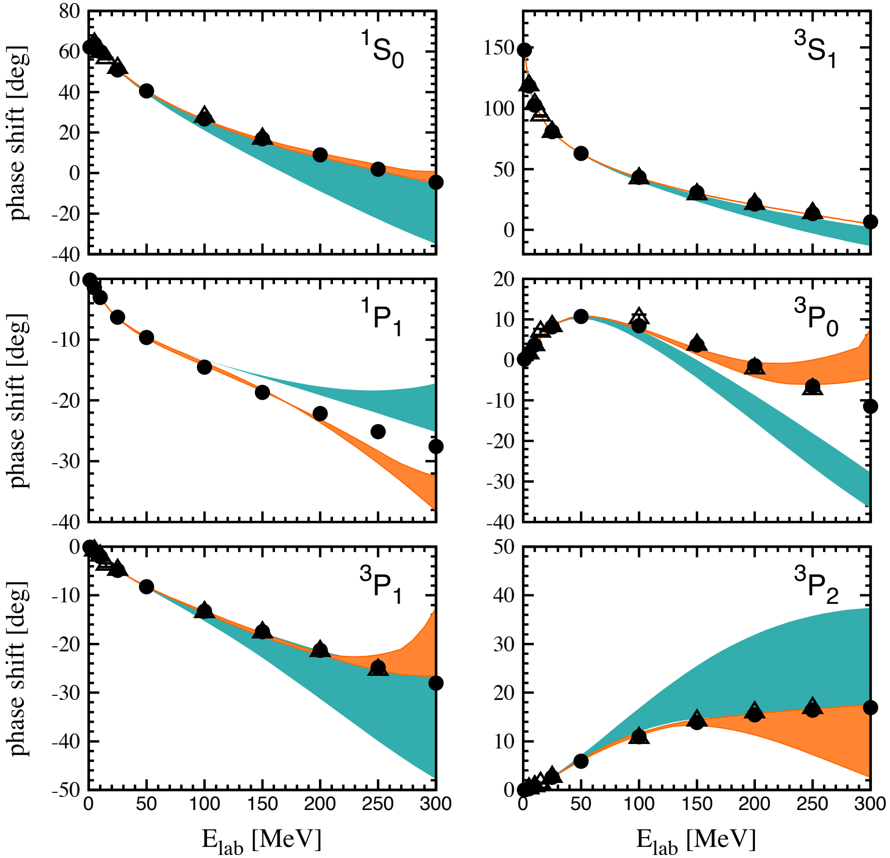
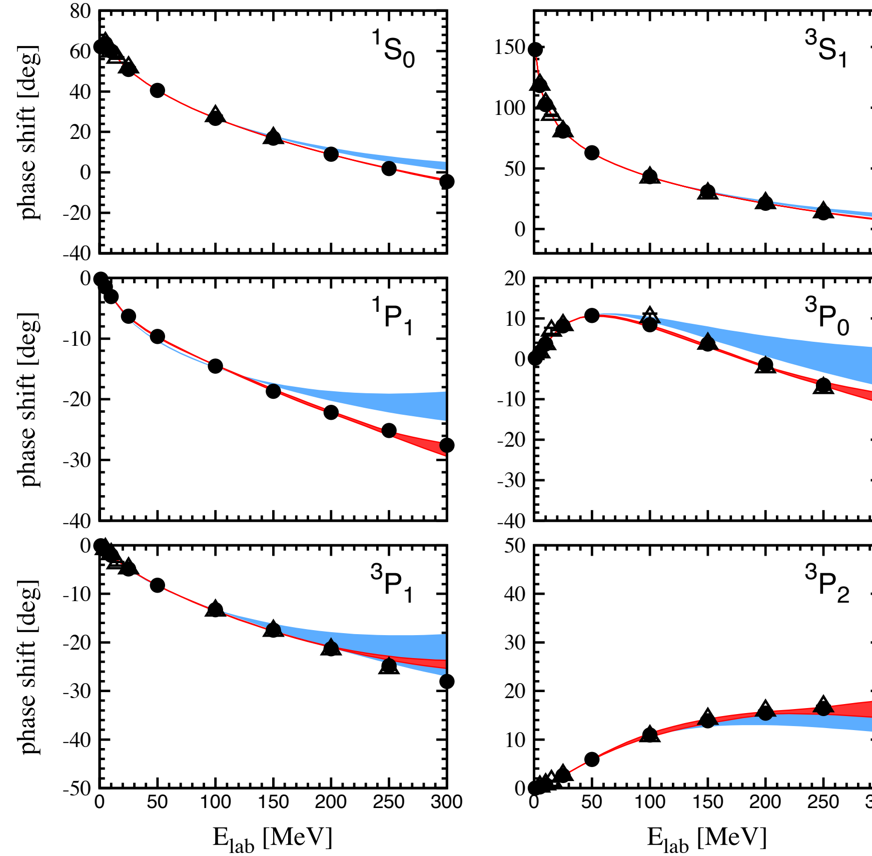


np phase shifts based on <u>i-chiral</u> N²LO/N³LO 2NF



N²LO: Λ = 450…600 fm, Λ_{SFR} =500…700 MeV

N³LO: Λ = 450…600 fm, Λ_{SFR} =500…700 MeV

N²LO: R_0 = 0.8…1.0 fm, Λ_{SFR} =1GeV…Infinity

N³LO: R_0 = 0.8…1.1 fm, Λ_{SFR} =1GeV…Infinity
LECs from Q⁴ KH πN

Cutoff dependence: i-chiral vs old EGM'04

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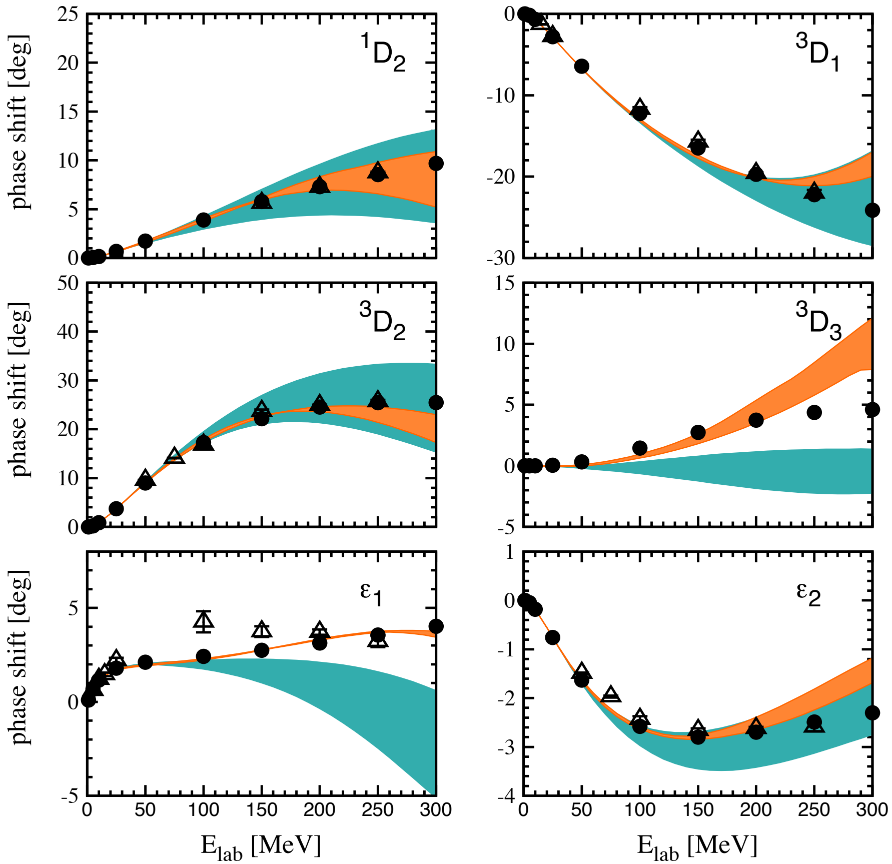

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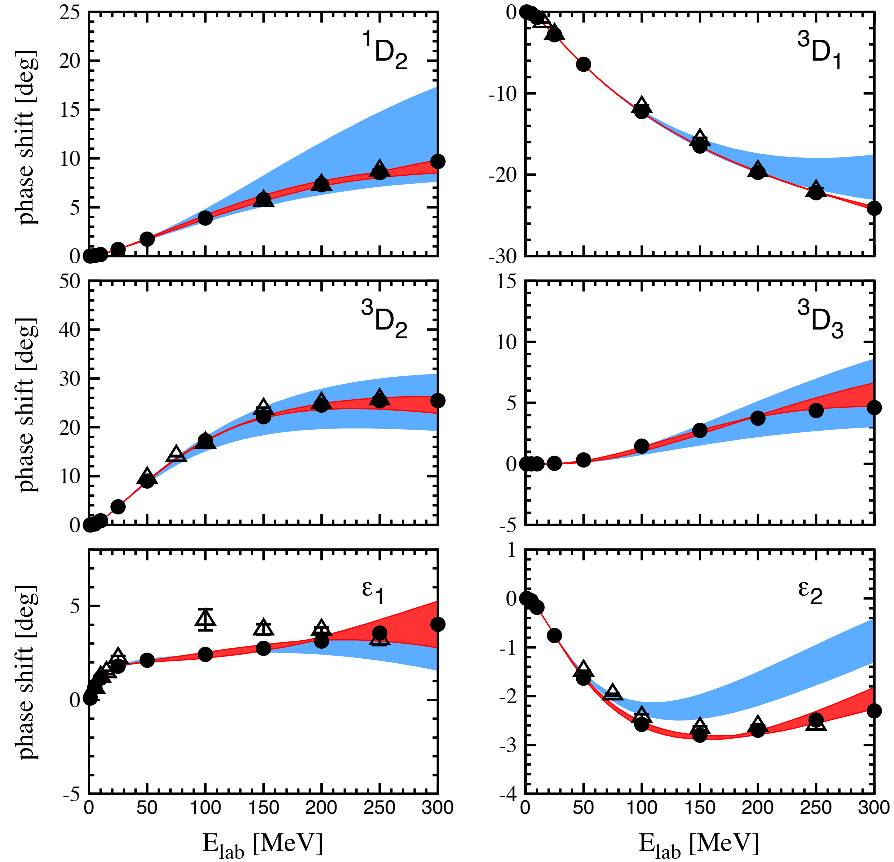


N^2LO: $\Lambda = 450\ldots600$ fm, $\Lambda_{SFR} = 500\ldots700$ MeV

N^3LO: $\Lambda = 450\ldots600$ fm, $\Lambda_{SFR} = 500\ldots700$ MeV

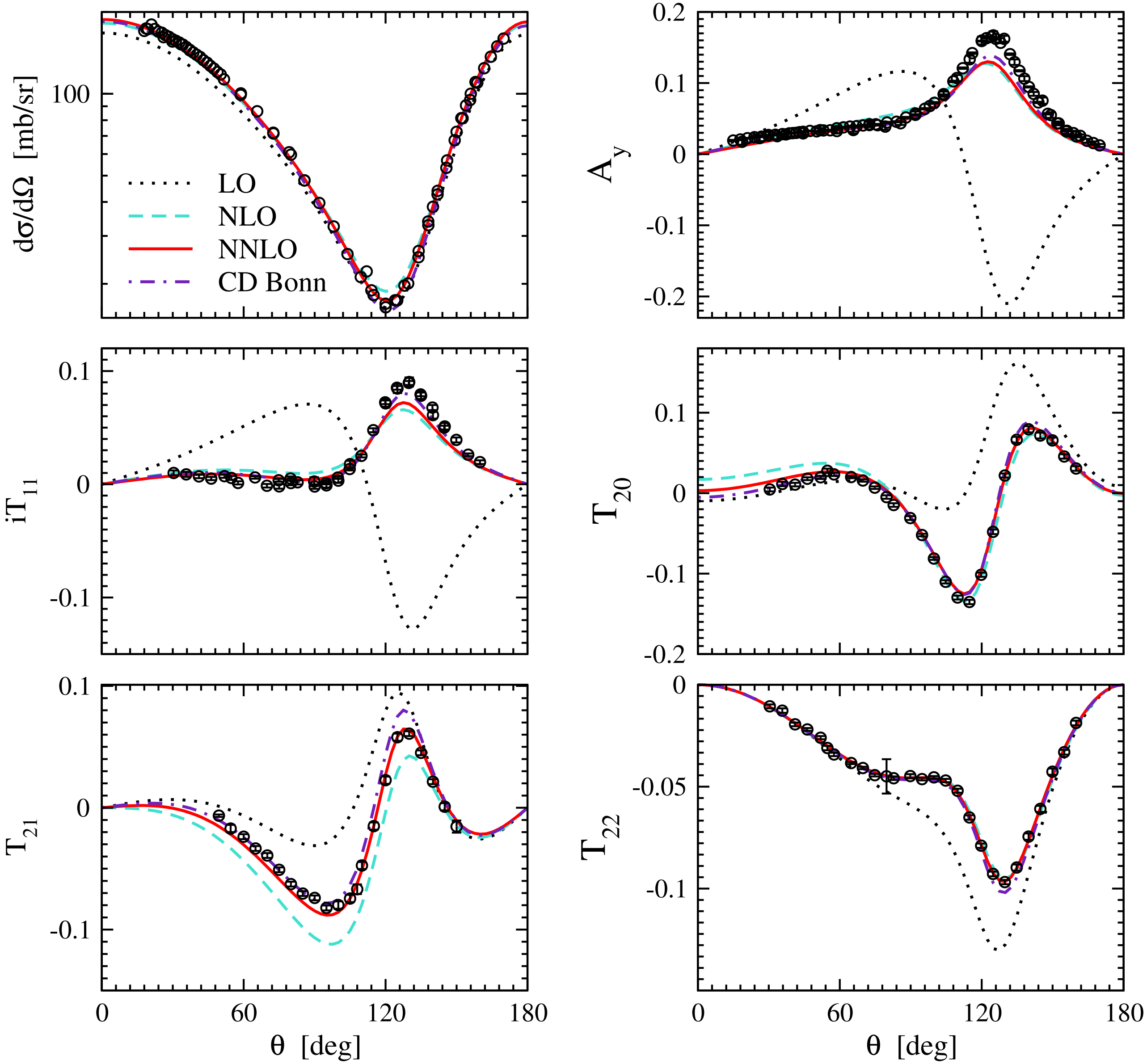
N^2LO: $R_0 = 0.8\ldots1.0$ fm, $\Lambda_{SFR} = 1$GeV…Infinity

N^3LO: $R_0 = 0.8\ldots1.1$ fm, $\Lambda_{SFR} = 1$GeV…Infinity
LECs from Q^4 KH πN

I-chiral 2NF: elastic nd scattering order-by-order

EE, Golak, Kamada, Krebs, Meißner, Nogga, Skibinski, Witala, in preparation

3 MeV:

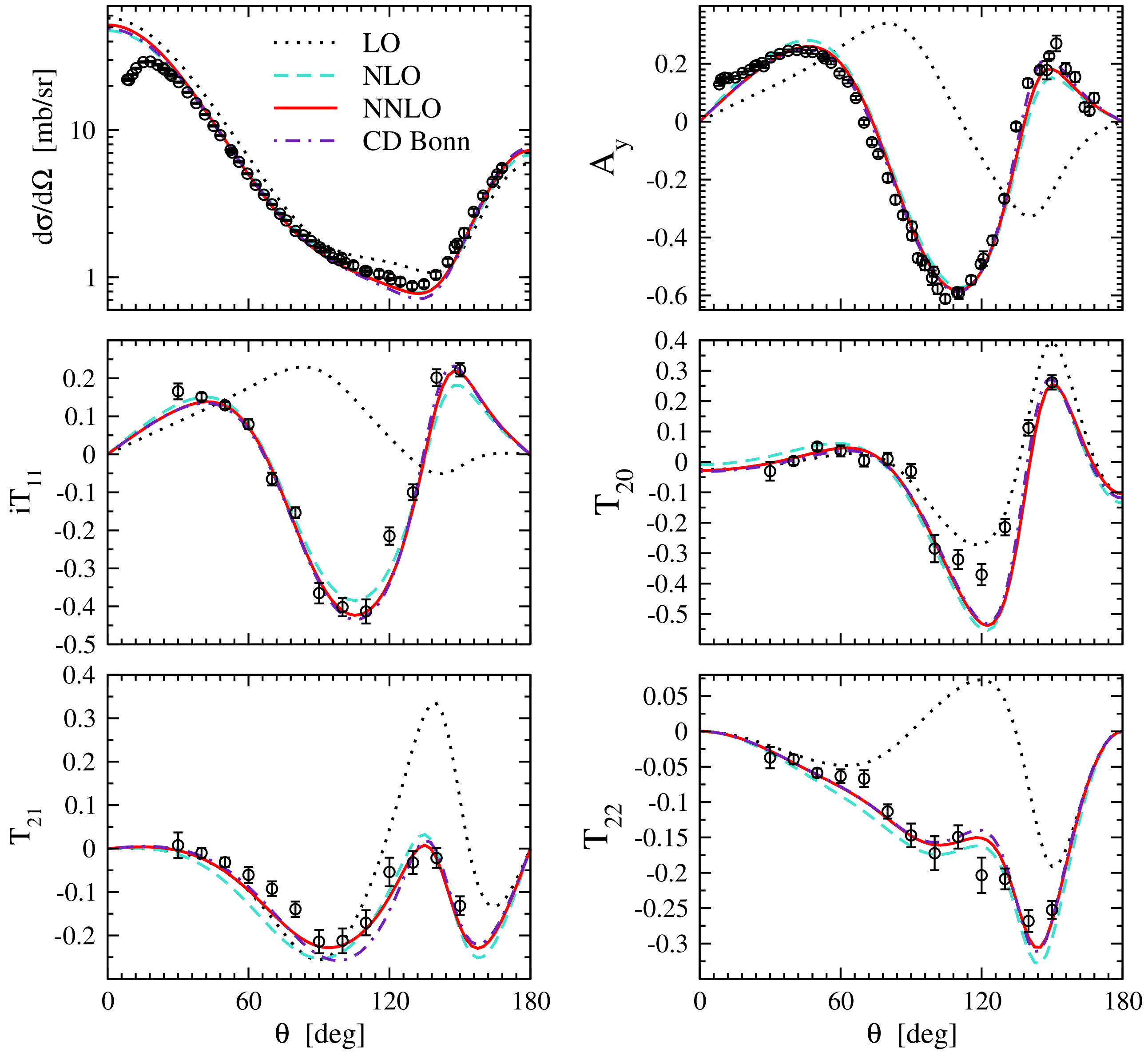


$R_0 = 1$ fm
$\Lambda_{SFR} = 2$ GeV

I-chiral 2NF: elastic nd scattering order-by-order

EE, Golak, Kamada, Krebs, Meißner, Nogga, Skibinski, Witala, in preparation

65 MeV:

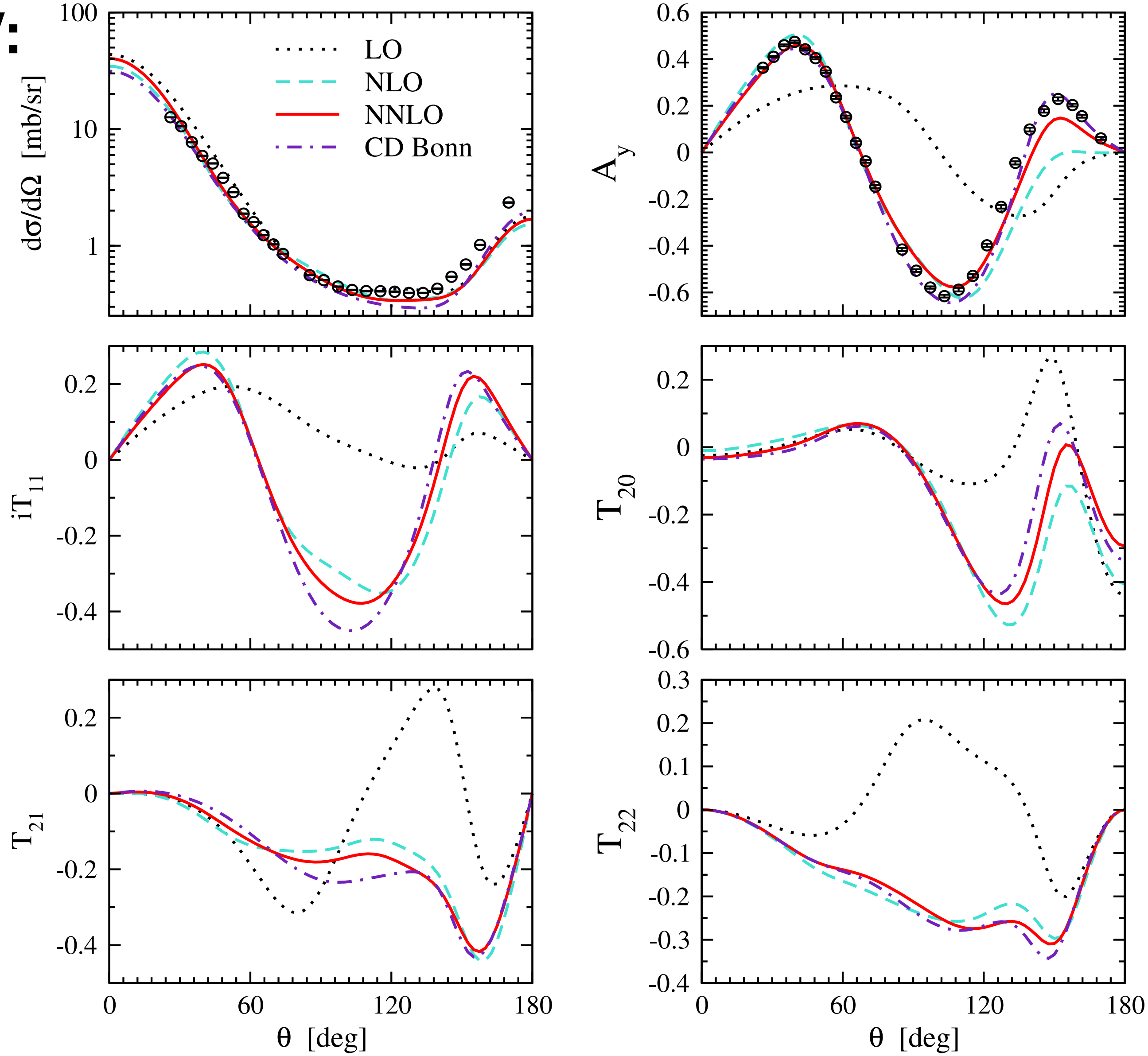


$R_0 = 1$ fm
$\Lambda_{SFR} = 2$ GeV

I-chiral 2NF: elastic nd scattering order-by-order

EE, Golak, Kamada, Krebs, Meißner, Nogga, Skibinski, Witala, in preparation

108 MeV:



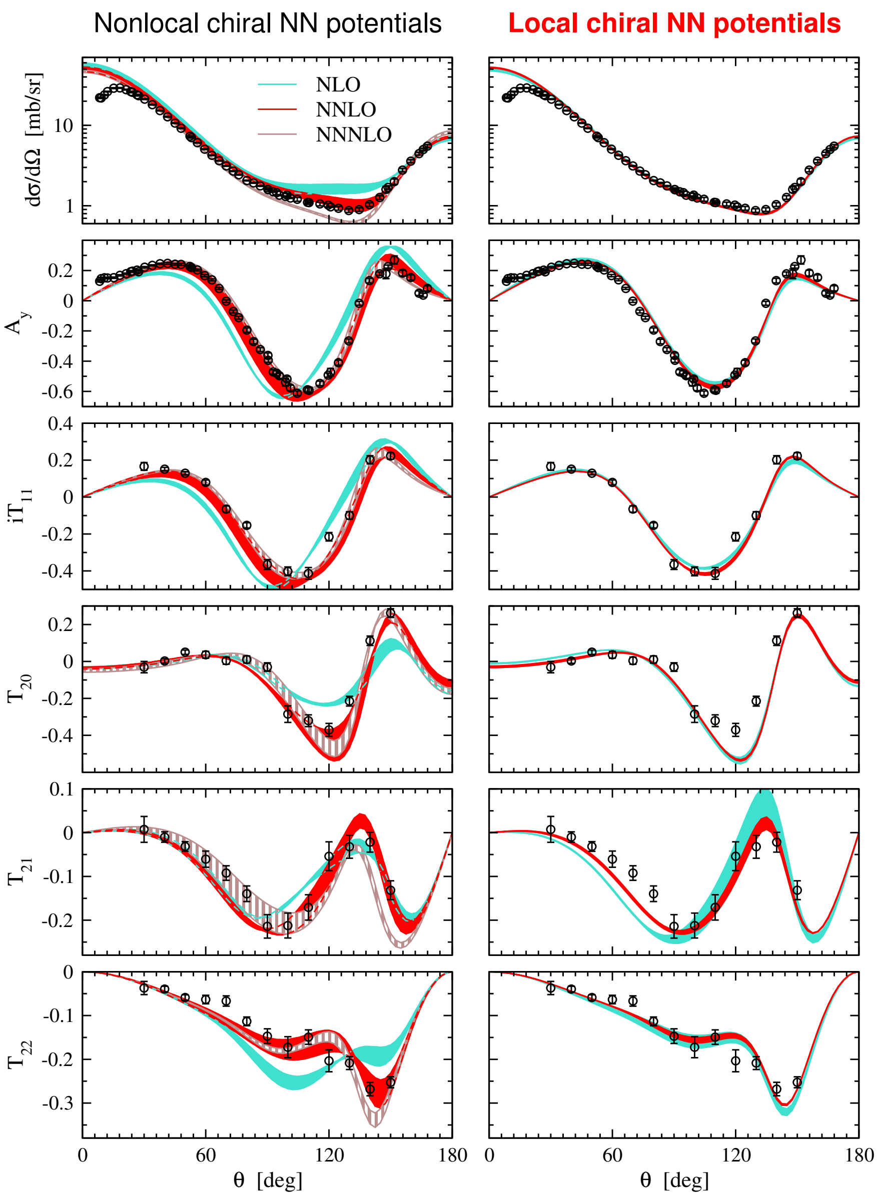
$R_0 = 1$ fm

$\Lambda_{SFR} = 2$ GeV

nd scattering with l-chiral 2NF: Cutoff dependence

EE, Golak, Kamada, Krebs, Meißner, Nogga, Skibinski, Witala, in preparation

65 MeV:



nonlocal NLO/N²LO/N³LO:
Λ = 450...600 MeV,
Λ_{SFR} = 500...700 MeV

local NLO/N²LO:
R_0 = 1...1.2 fm,
Λ_{SFR} = 1...2 GeV

Summary and outlook

A new generation of chiral NN potentials up to N^{3}LO is being developed:

l$_{ocal}$-chiral (up to N^{2}LO): local interactions, can be used in QMC

i$_{mproved}$-chiral (up to N^{3}LO): nonlocal potentials

Common features: better performance at higher energies, less sensitivity to cutoffs, no need for SFR, can use c_i's from πN.

First applications to Nd scattering (no 3NF yet) look very promising: given the increased accuracy at intermediate energies, can do interesting physics even at the N^{2}LO level. **Ongoing work: elastic Nd scattering and breakup, inclusion of the 3NF using the same regularization, sensitivity to c_i's, …**

Longer-term plans: Nd scattering and light nuclei at N^{3}LO, nuclear potentials from chiral EFT with explicit Δ's, four-body forces, N^{4}LO, …