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IOP Nuclear Physics Group Conference 2014, 7-9 April 2014, Surrey, UK

Chiral dynamics of light nuclei

Introduction

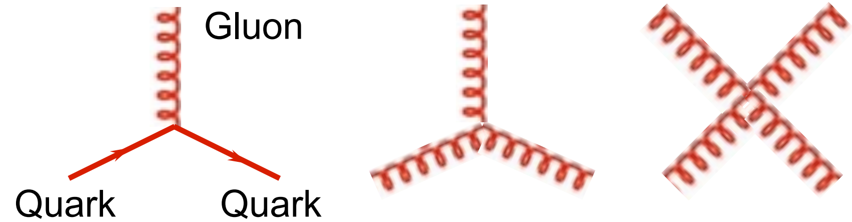
Chiral nuclear forces

Recent results from lattice simulations

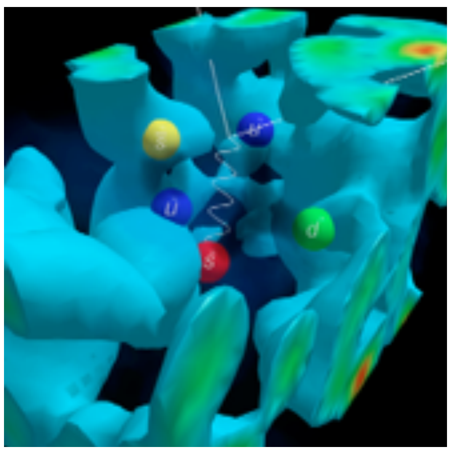
Summary & outlook

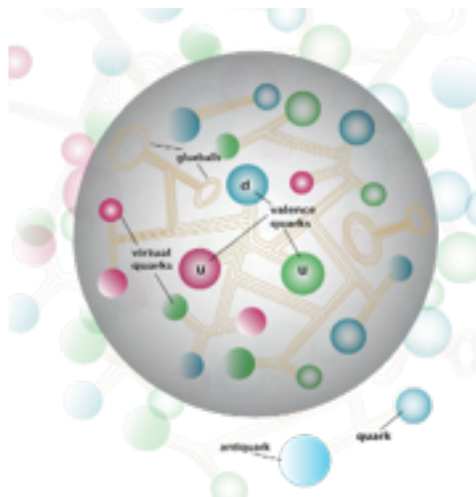
Facets of strong interactions

$$\mathcal{L}_{\mathrm{QCD}} = \bar{q}(i\not{D} - m)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu}$$



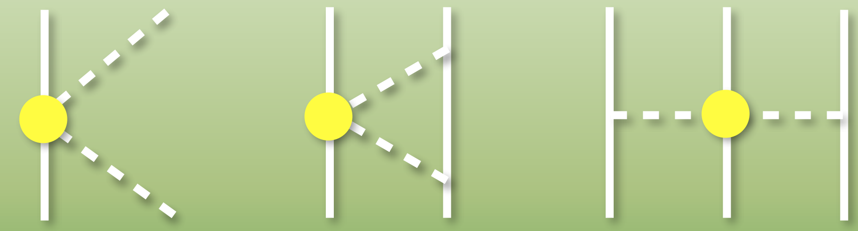
Seemingly very simple formulation is responsible for extremely complex phenomena!





From QCD to nuclear physics

- → **lattice** → **nuclear physics**
- → **effective chiral Lagrangian** → **(low-energy) nuclear physics**



Chiral perturbation theory

- **Ideal world [$m_u = m_d = 0$], zero-energy limit:** non-interacting massless GBs
  (+ strongly interacting massive hadrons)

- **Real world [$m_u,\ m_d \ll \Lambda_{QCD}$], low energy:** weakly interacting light GBs
  (+ strongly interacting massive hadrons)

→ expand about the ideal world (ChPT)

Chiral Perturbation Theory

Chiral Perturbation Theory: expansion of the scattering amplitude in powers of

Weinberg, Gasser, Leutwyler, Meißner, ...

$$Q = \frac{\text{momenta of pions and nucleons or } M_\pi \sim 140 \text{ MeV}}{\text{hard scales [at best } \Lambda_\chi = 4\pi F_\pi \sim 1 \text{ GeV]}}$$

Manohar, Georgi '84

Tool: Feynman calculus using the effective chiral Lagrangian

low-energy constants

$$\begin{aligned}
\mathcal{L}_\pi &= \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \ldots \\
\mathcal{L}_{\pi N} &= \underbrace{\bar{N}\left(i\gamma^\mu D_\mu[\pi] - m + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu[\pi]\right)N}_{\mathcal{L}_{\pi N}^{(1)}} + \underbrace{\sum_i \boldsymbol{c}_i \bar{N}\hat{O}_i^{(2)}[\pi]N}_{\mathcal{L}_{\pi N}^{(2)}} + \underbrace{\sum_i \boldsymbol{d}_i \bar{N}\hat{O}_i^{(3)}[\pi]N}_{\mathcal{L}_{\pi N}^{(3)}} + \ldots
\end{aligned}$$

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Pion-nucleon scattering up to Q^4 in heavy-baryon ChPT

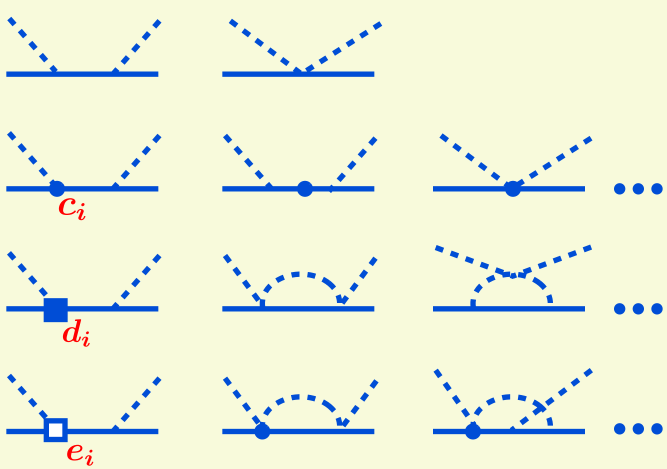
Fettes, Meißner '00; Krebs, Gasparyan, EE '12

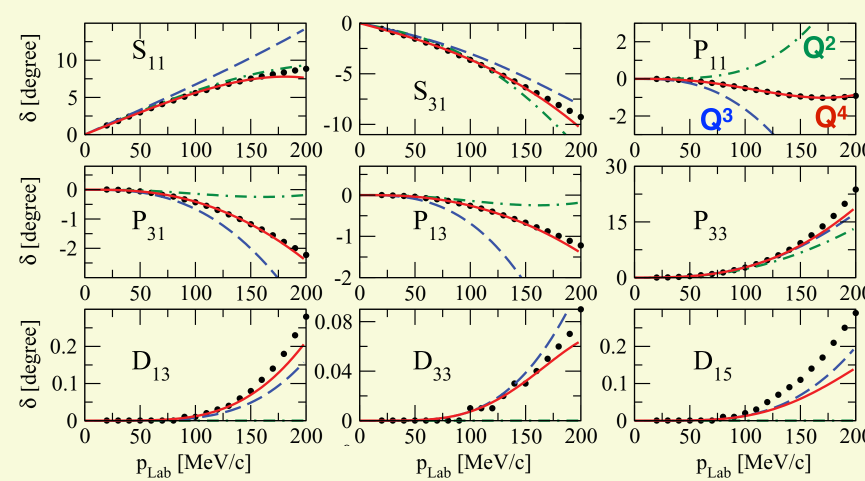
Order Q:

Order Q^2:

Order Q^3:

Order Q^4:





Nuclear chiral effective field theory

Nuclear chiral EFT

Weinberg, van Kolck, EE, Glöckle, Meißner, Machleidt, Entem...

- Schrödinger eq. for nucleons interacting via contact forces + long-range potentials (π-exchanges)

$$\left[\left(\sum_{i=1}^{A}\frac{-\vec{\nabla}_i^2}{2m_N}+\mathcal{O}(m_N^{-3})\right)+\underbrace{V_{2N}+V_{3N}+V_{4N}+\ldots}_{\text{derived in ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$

- access to heavier nuclei (ab initio few-/many-body methods)

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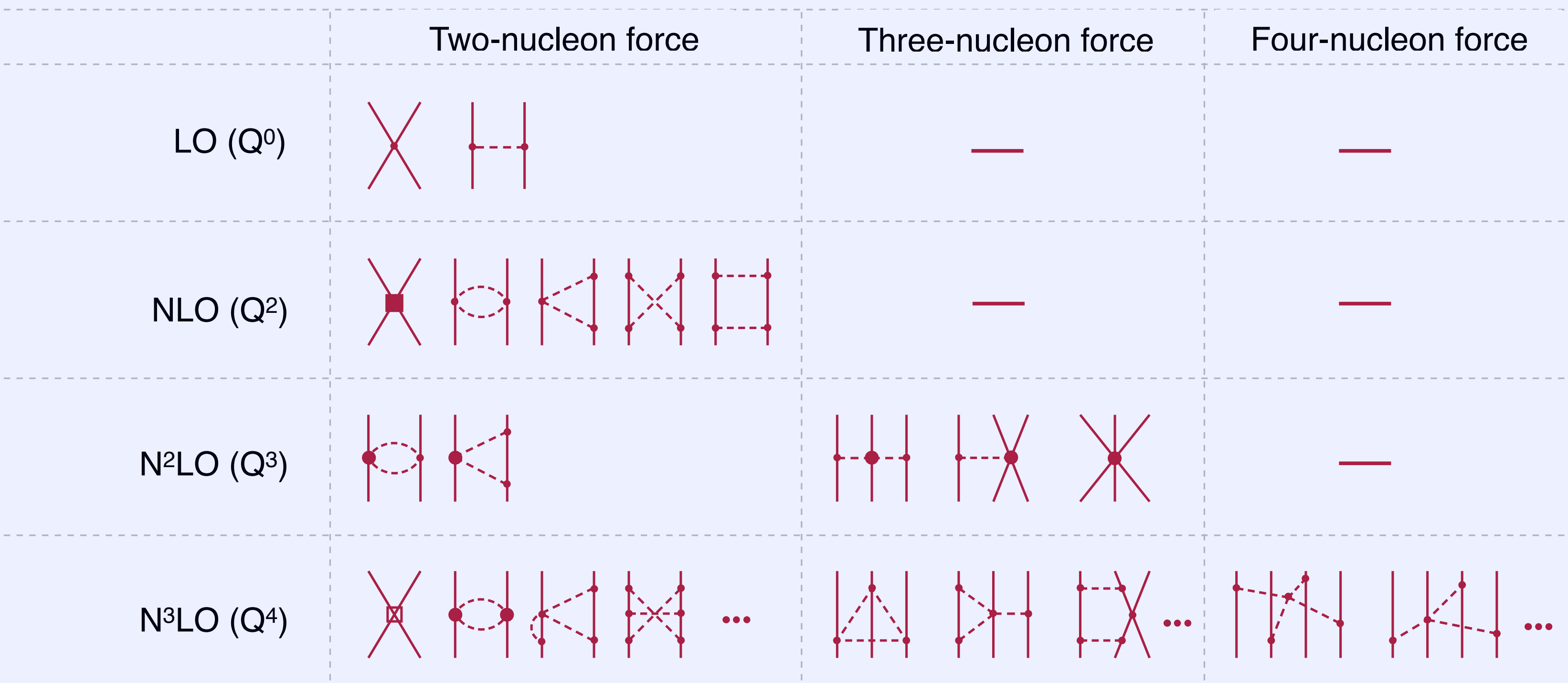
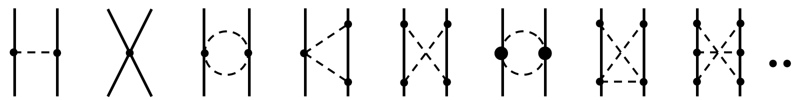


Nucleon-nucleon potential at N^{3}LO

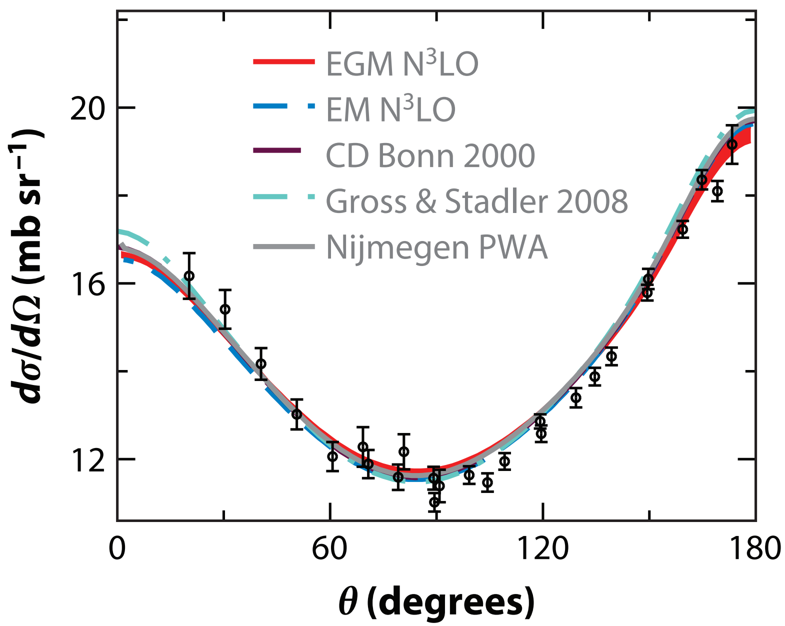
van Kolck et al.'94; Friar & Coon '94; Kaiser et al. '97; E.E. et al. '98,'03; Kaiser '99-'01; Higa, Robilotta '03; …

- Long-range: parameter-free (all LECs from πN)
- Short-range part: 24 LECs tuned to NN data
- Accurate description of NN data up to ~ 200 MeV

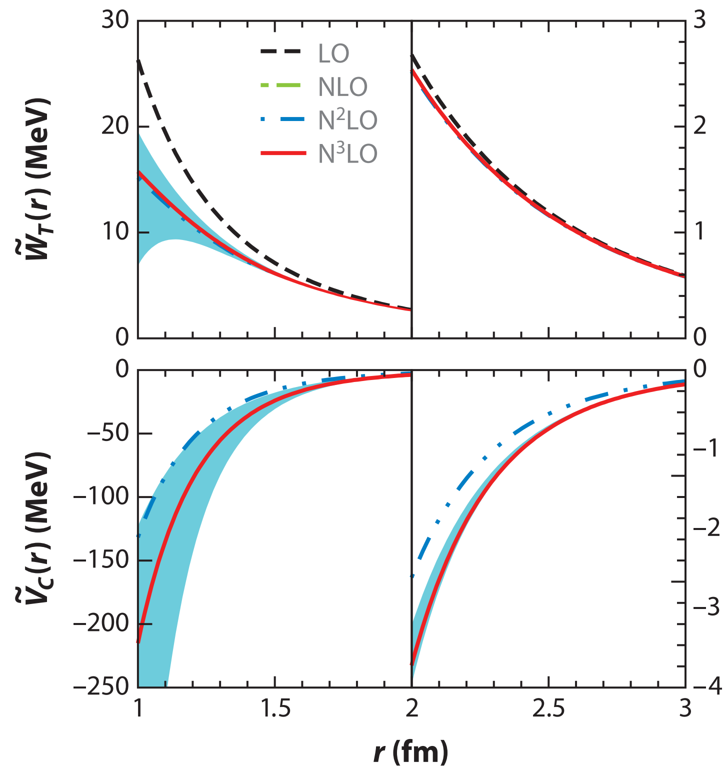


Entem-Machleidt, EE-Glöckle-Meißner

np cross section @ 50 MeV



χ expansion of the long-range force



Recent reviews:

EE, Prog. Part Nucl. Phys. 57 (06) 654;

EE, Hammer, Meißner, Rev. Mod. Phys. 81 (09) 1773;

Entem, Machleidt, Phys. Rept. 503 (11) 1;

EE, Meißner, Ann. Rev. Nucl. Part. Sci. 62 (12) 159.

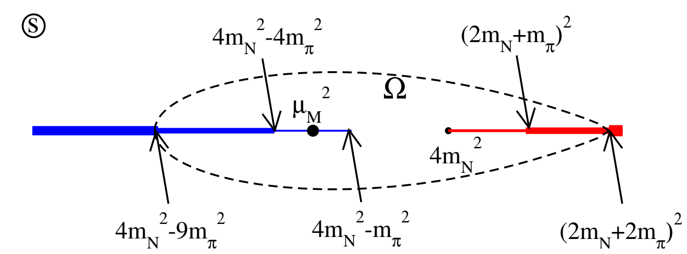
Some other topics & ongoing developments

Renormalitazion and power counting

van Kolck, Pavon Valderrama, Brise, Gegelia, EE, Machleidt, …

Merging chiral EFT with dispersion relations

Albaladejo, Oller '11,'12; Gasparyan, EE, Lutz '12; Guo, Oller, Rios '13



- Calculate the discontinuity of the amplitude along the left-hand cut using ChPT
- Reconstruct the amplitude in the physical region using dispersion relations + analytic cont. (conformal mapping)

Generalization to the SU(3) sector

Haidenbauer, Meißner, Kaiser, Petschauer, Nogga, …

New generation of chiral NN potentials

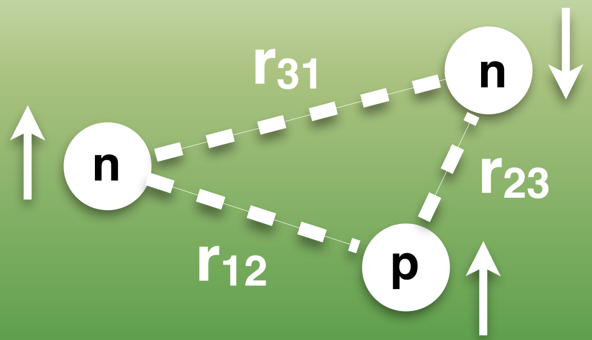
- Optimized N^{2}LO chiral nuclear force (tune LECs to reduce the impact of 3NF in the 3N & 4N systems) Ekström, Baardsen, et al. '13. Justified from an EFT point of view?
- Locally regularized chiral potentials Gezerlis, Tews, EE, Gandolfi, Hebeler, Nogga, Schwenk '13; EE et al. in progress

Nuclear parity violation

Schindler, Viviani, Kievski, Girlanda, de Vries, van Kolck, Kaiser, Meißner, EE, …

Partial wave analysis and the role of the chiral two-pion exchange potential

Rentmeester et al., Birse, McGovern, Navarro Perez, Ruiz Arriola et al.



The three-nucleon force

Inspite of decades of efforts, the structure of the 3NF is still poorely understood…

Kalantar-Nayestanaki, EE, Messchendorp, Nogga, Rev. Mod. Phys. 75 (2012) 016301

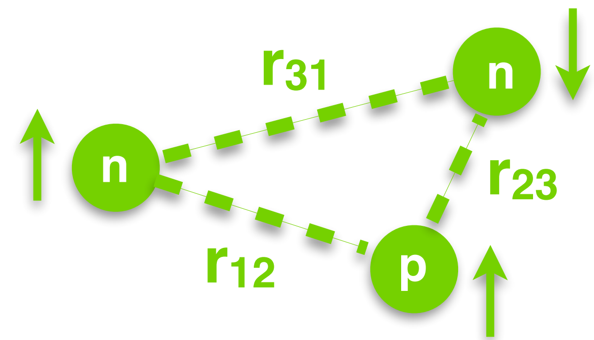
Kistryn, Stephan, J. Phys. G: Nucl. Part. Phys. 40 (2013) 063101

Most general structure of a local 3NF

Most general local isospin-conserving 3NF can be written via

$$V(q_1,q_2,q_3) = \sum_{i=1}^{20} \mathcal{G}_i \, F_i(q_1,q_2,q_3) + \text{permutations}$$

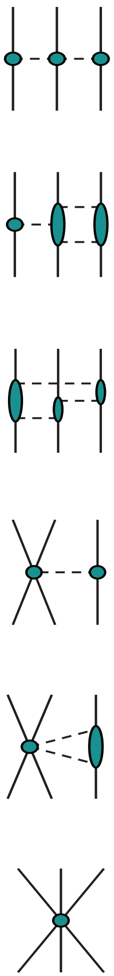
$$V(r_{12},r_{23},r_{31}) = \sum_{i=1}^{20} \tilde{\mathcal{G}}_i \, F_i(r_{12},r_{23},r_{31}) + \text{permutations}$$



(2 operators out of the 22 given in **Krebs, Gasparyan, EE, PRC87 (2013)** are redundant **EE, Gasparyan, Krebs, Schat, in preparation**)

Generators $\mathcal{G}$ in momentum space	Generators $\tilde{\mathcal{G}}$ in coordinate space
$\mathcal{G}_1 = 1$	$\tilde{\mathcal{G}}_1 = 1$
$\mathcal{G}_2 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3$	$\tilde{\mathcal{G}}_2 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3$
$\mathcal{G}_3 = \vec{\sigma}_1 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_3 = \vec{\sigma}_1 \cdot \vec{\sigma}_3$
$\mathcal{G}_4 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_4 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \vec{\sigma}_1 \cdot \vec{\sigma}_3$
$\mathcal{G}_5 = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_5 = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \vec{\sigma}_1 \cdot \vec{\sigma}_2$
$\mathcal{G}_6 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3)$	$\tilde{\mathcal{G}}_6 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \, \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3)$
$\mathcal{G}_7 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	$\tilde{\mathcal{G}}_7 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})$
$\mathcal{G}_8 = \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_8 = \hat{r}_{23} \cdot \vec{\sigma}_1 \, \hat{r}_{23} \cdot \vec{\sigma}_3$
$\mathcal{G}_9 = \vec{q}_1 \cdot \vec{\sigma}_3 \vec{q}_3 \cdot \vec{\sigma}_1$	$\tilde{\mathcal{G}}_9 = \hat{r}_{23} \cdot \vec{\sigma}_3 \, \hat{r}_{12} \cdot \vec{\sigma}_1$
$\mathcal{G}_{10} = \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{10} = \hat{r}_{23} \cdot \vec{\sigma}_1 \, \hat{r}_{12} \cdot \vec{\sigma}_3$
$\mathcal{G}_{11} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{11} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \hat{r}_{23} \cdot \vec{\sigma}_1 \, \hat{r}_{23} \cdot \vec{\sigma}_2$
$\mathcal{G}_{12} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{12} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \hat{r}_{23} \cdot \vec{\sigma}_1 \, \hat{r}_{12} \cdot \vec{\sigma}_2$
$\mathcal{G}_{13} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{13} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \hat{r}_{12} \cdot \vec{\sigma}_1 \, \hat{r}_{23} \cdot \vec{\sigma}_2$
$\mathcal{G}_{14} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{14} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \hat{r}_{12} \cdot \vec{\sigma}_1 \, \hat{r}_{12} \cdot \vec{\sigma}_2$
$\mathcal{G}_{15} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{q}_2 \cdot \vec{\sigma}_1 \vec{q}_2 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{15} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \hat{r}_{13} \cdot \vec{\sigma}_1 \, \hat{r}_{13} \cdot \vec{\sigma}_3$
$\mathcal{G}_{16} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_2 \vec{q}_3 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{16} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \hat{r}_{12} \cdot \vec{\sigma}_2 \, \hat{r}_{12} \cdot \vec{\sigma}_3$
$\mathcal{G}_{17} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{17} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \hat{r}_{23} \cdot \vec{\sigma}_1 \, \hat{r}_{12} \cdot \vec{\sigma}_3$
$\mathcal{G}_{18} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{\sigma}_3 \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	$\tilde{\mathcal{G}}_{18} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \, \vec{\sigma}_1 \cdot \vec{\sigma}_3 \, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})$
$\mathcal{G}_{19} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_3 \cdot \vec{q}_1 \vec{q}_1 \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)$	$\tilde{\mathcal{G}}_{19} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \, \vec{\sigma}_3 \cdot \hat{r}_{23} \, \hat{r}_{23} \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)$
$\mathcal{G}_{20} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot \vec{q}_3 \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	$\tilde{\mathcal{G}}_{20} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \, \vec{\sigma}_1 \cdot \hat{r}_{23} \, \vec{\sigma}_3 \cdot \hat{r}_{12} \, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})$

Chiral expansion of the 3NF (Δ-less EFT)



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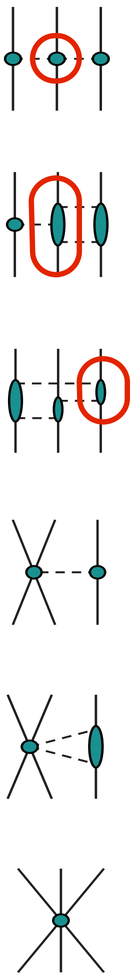
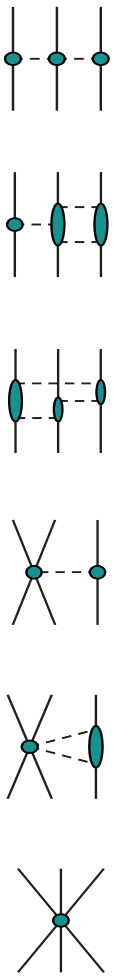


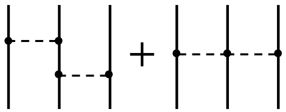
3NF structure functions at large distance are
model-independent and parameter-free predictions
based on χ symmetry of QCD + exp. information on πN system

Chiral expansion of the 3NF (Δ-less EFT)



Chiral expansion of the 3NF (Δ-less EFT)

NLO (Q^2)



Weinberg ’91, van Kolck ’94

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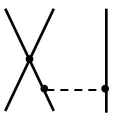

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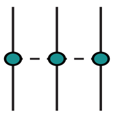
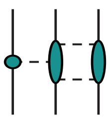
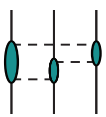
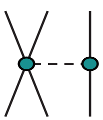
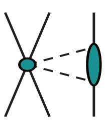
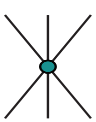
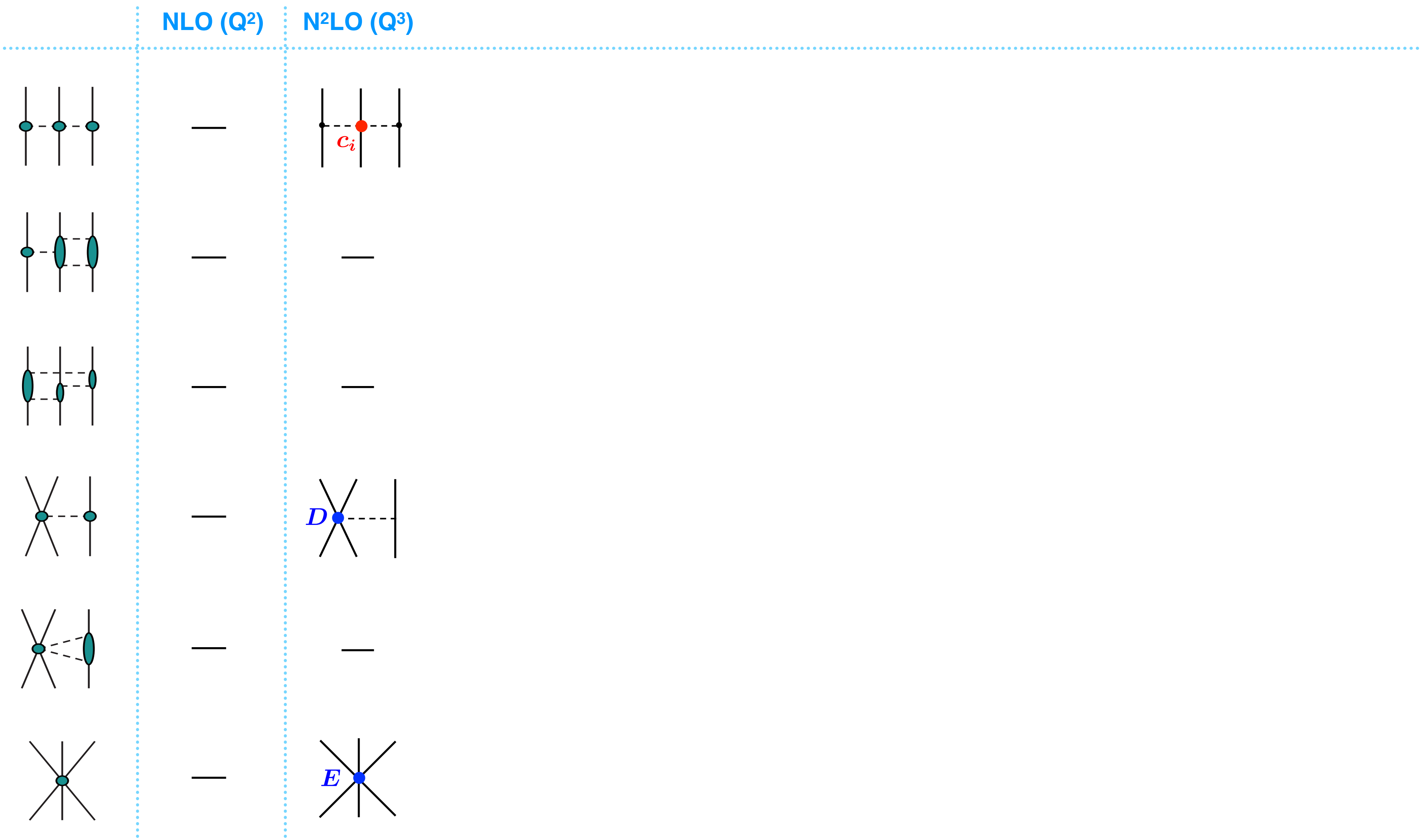
Chiral expansion of the 3NF (Δ-less EFT)

	NLO (Q^2)
	—
	—
	—
	—
	—
	—

Chiral expansion of the 3NF (Δ-less EFT)



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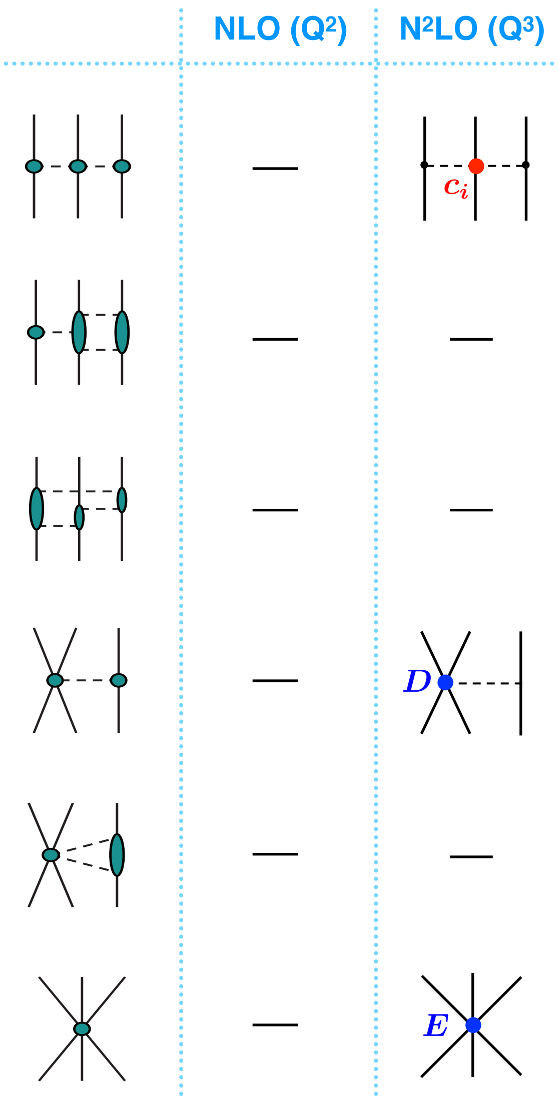
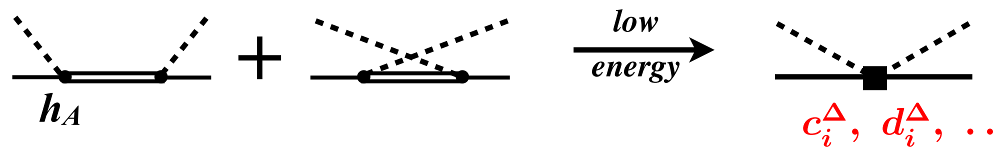


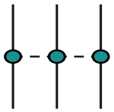
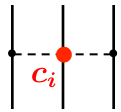
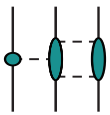
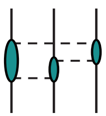
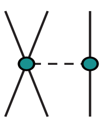
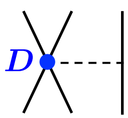
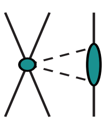
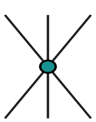
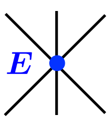
Notice: c_i receive large Δ(1232) contributions

Bernard, Kaiser, Meißner '97

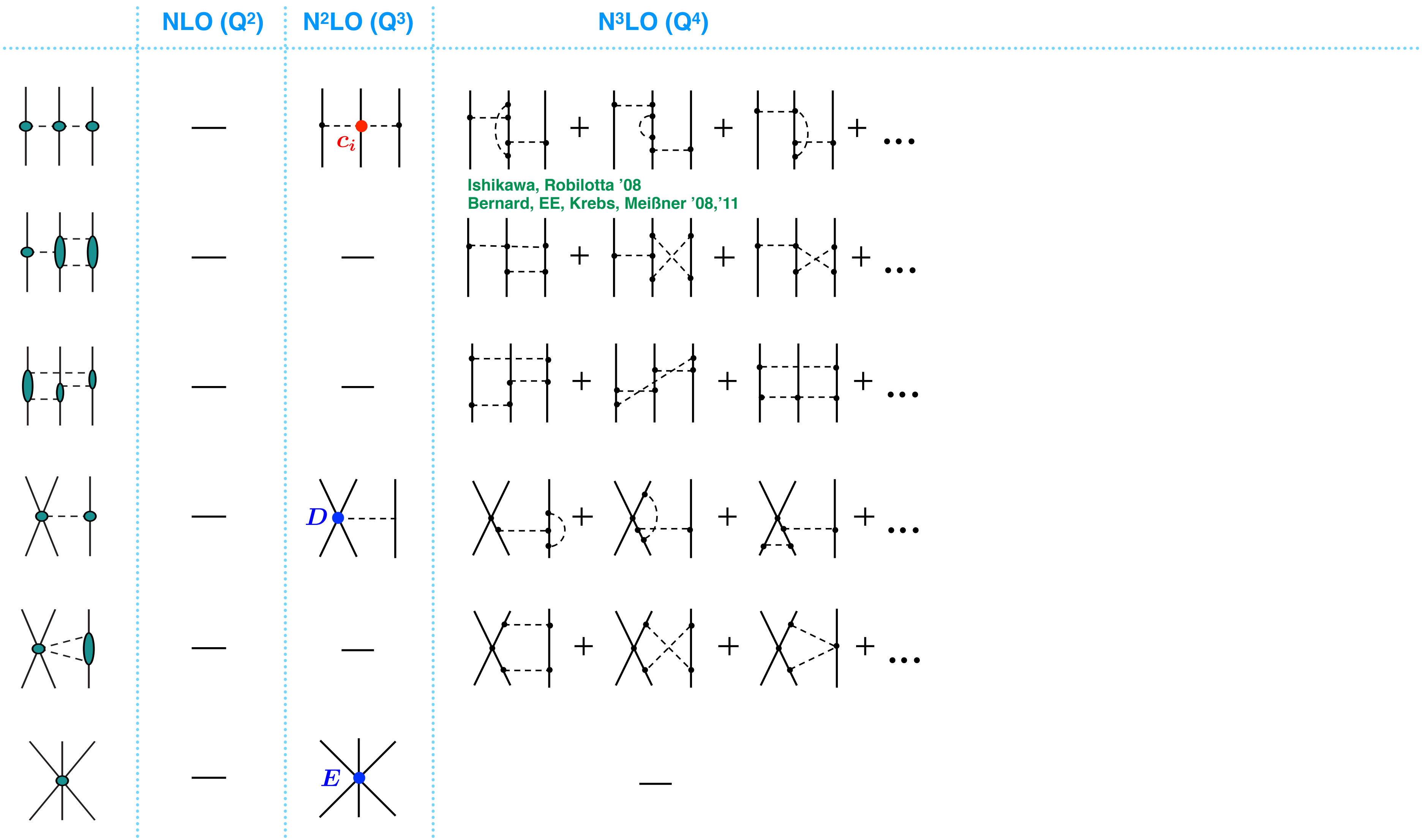


$$c_2^\Delta = -c_3^\Delta = 2c_4^\Delta = \frac{4h_A^2}{9(m_\Delta - m_N)} \simeq 2.8\,\mathrm{GeV}^{-1}$$

Chiral expansion of the 3NF (Δ-less EFT)

	NLO (Q^2)	N^2LO (Q^3)
	—	
	—	—
	—	—
	—	
	—	—
	—	

Chiral expansion of the 3NF (Δ-less EFT)



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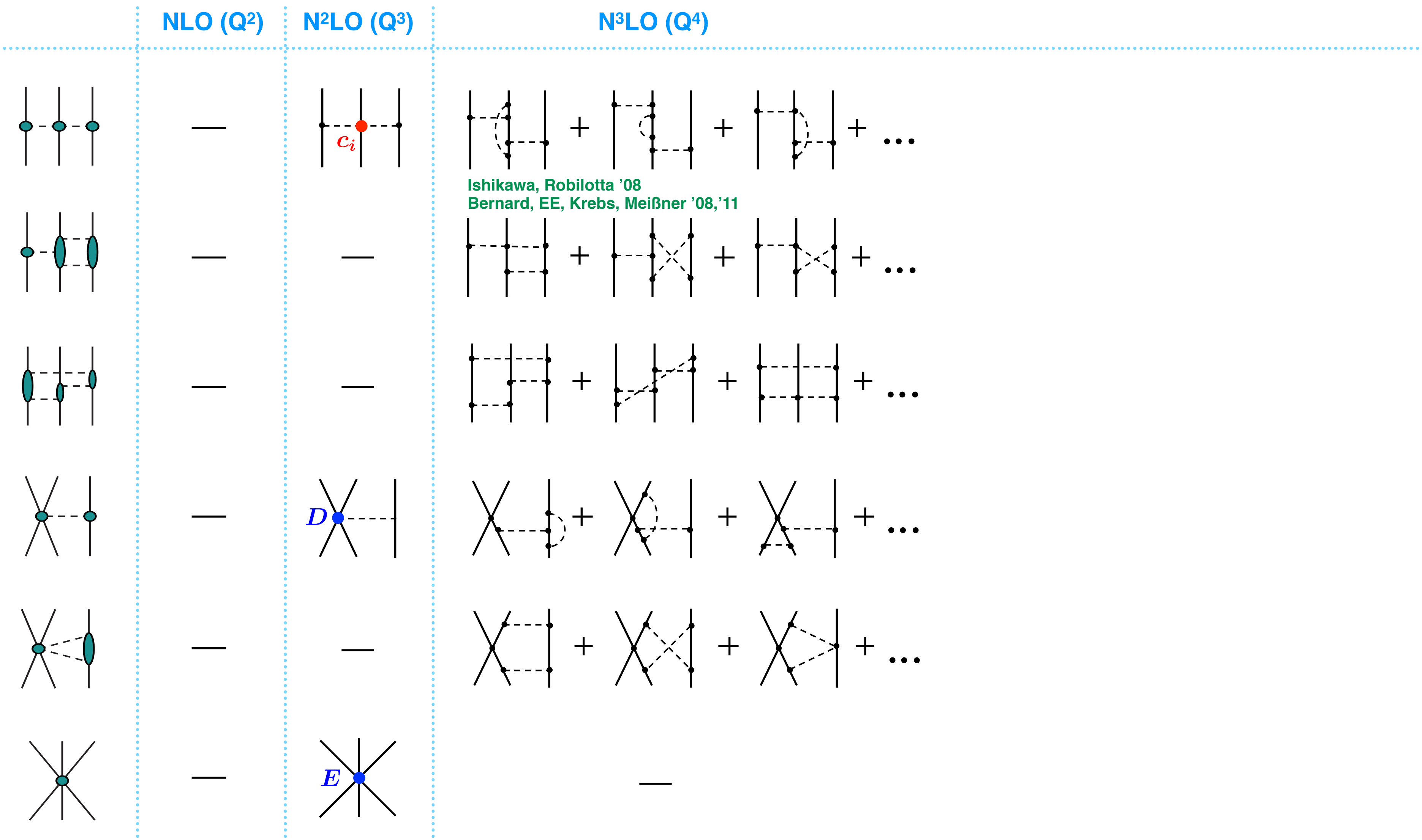

- parameter-free!
- Δ-effects are missing (except for the 2π 3NF) → expect large N^{4}LO corrections

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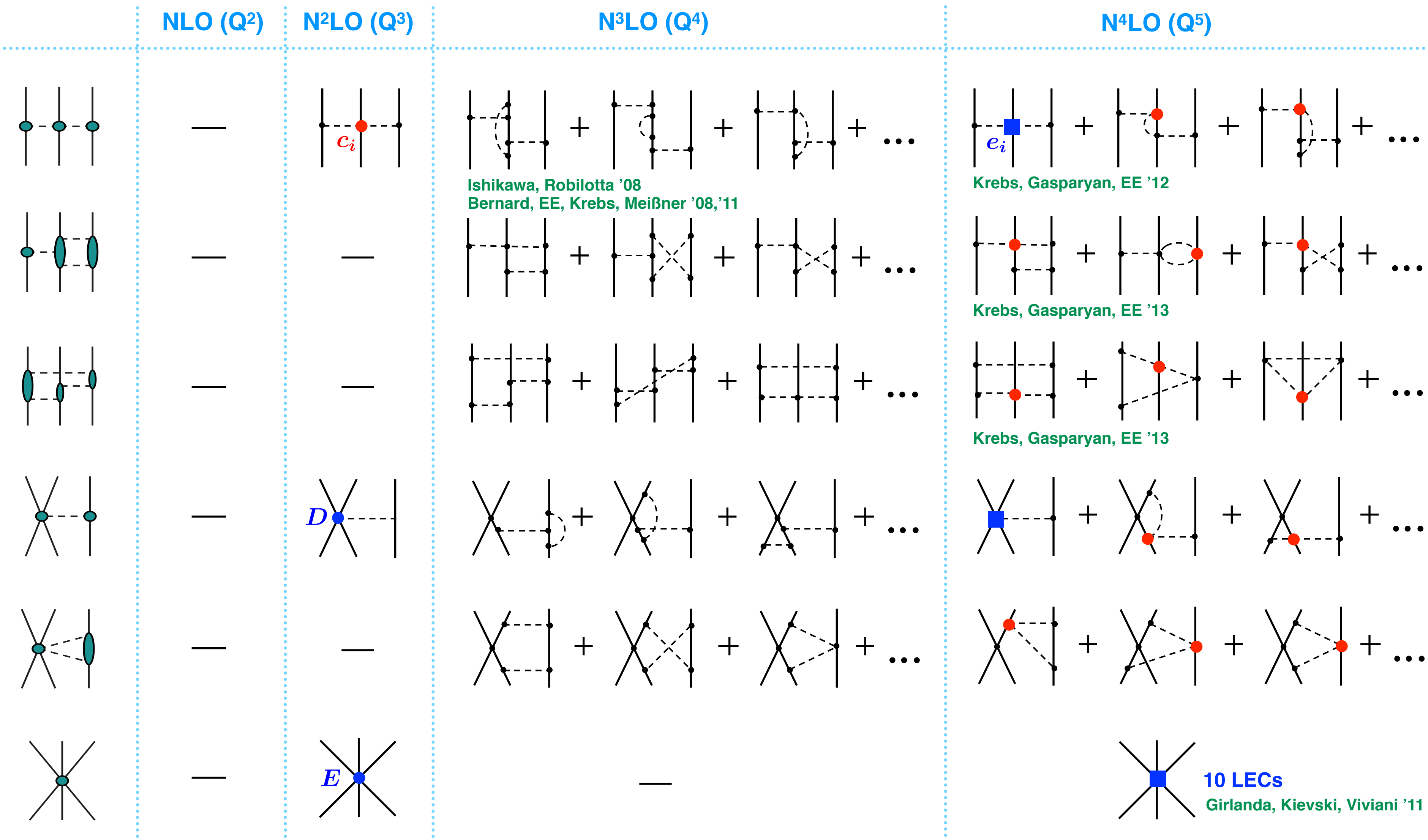

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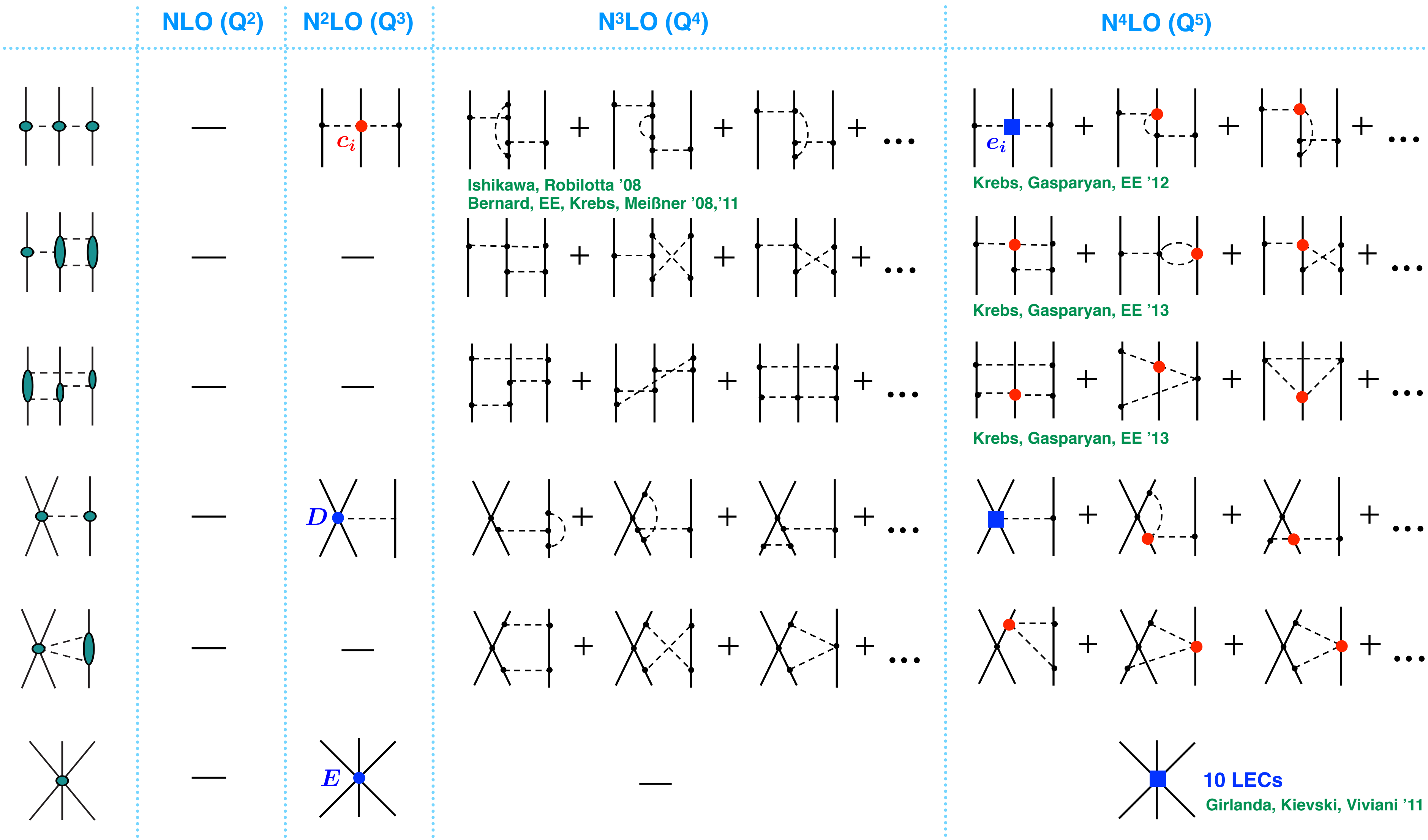

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- long range parameter-free (after determination of LECs in πN)
- converged?? (graphs ~ c_i^2, c_i^3…)

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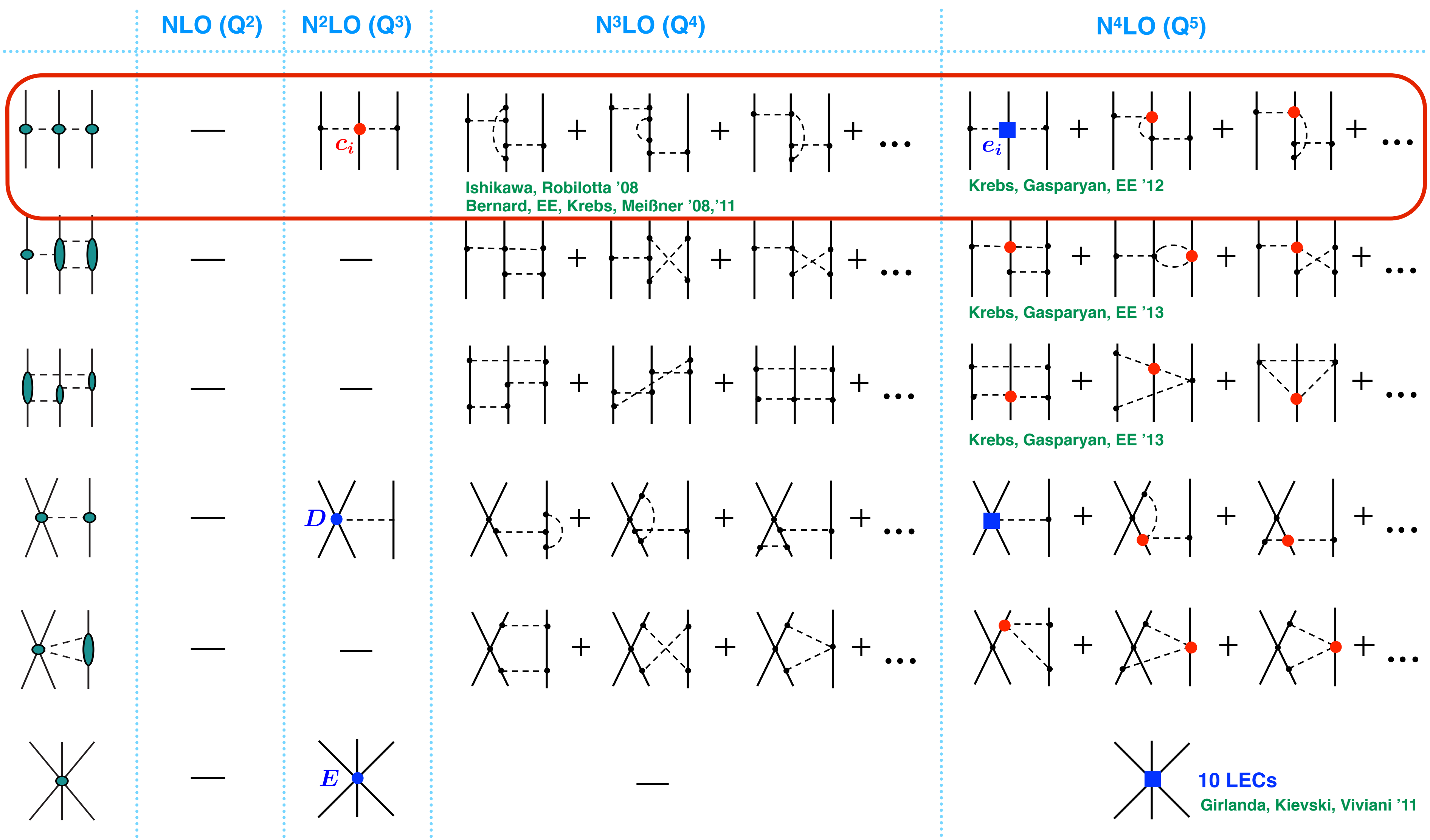

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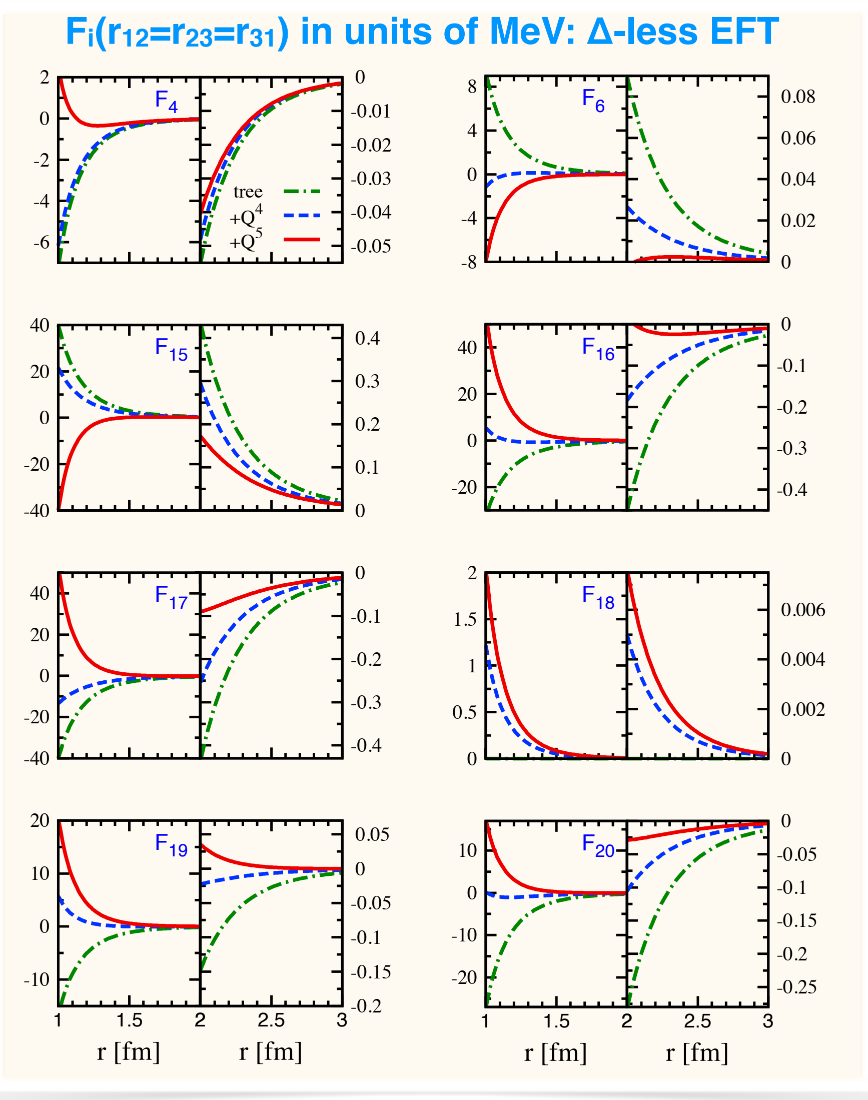
2π 3NF: Δ-full vs Δ-less (preliminary)

Krebs, Gasparyan, EE, to appear



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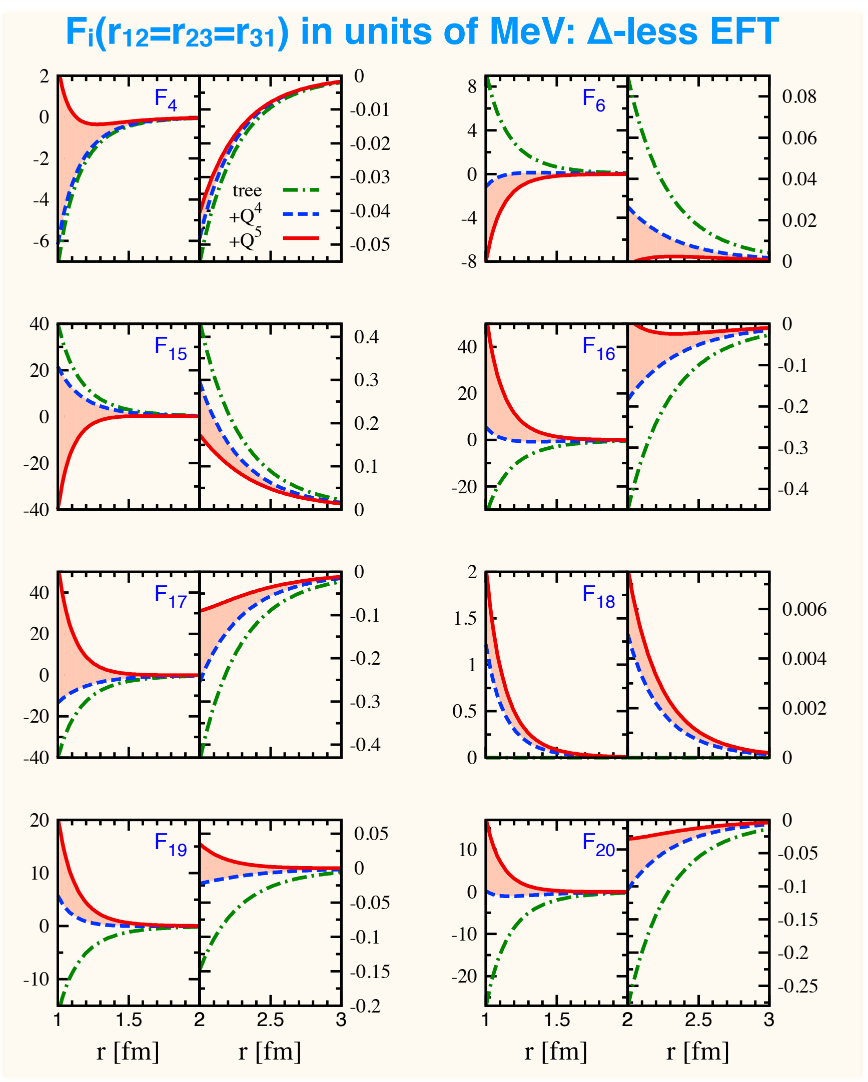
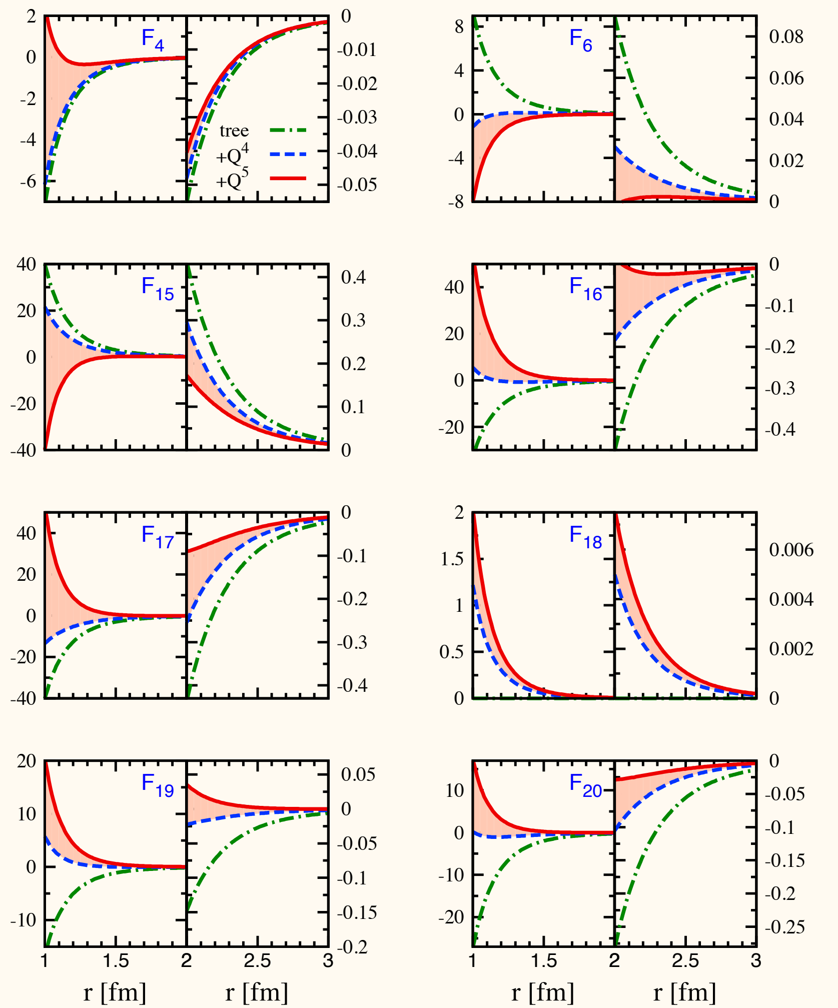


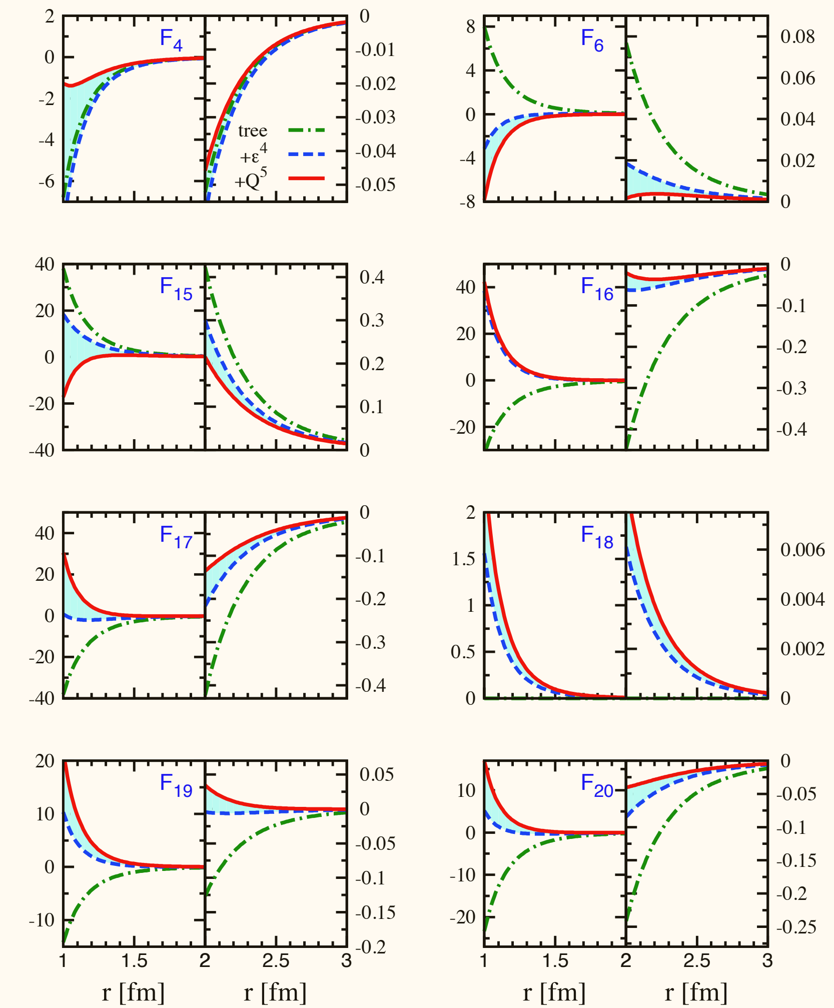
2π 3NF: Δ-full vs Δ-less (preliminary)

Krebs, Gasparyan, EE, to appear

$F_i(r_{12}=r_{23}=r_{31})$ in units of MeV: Δ-less EFT



$F_i(r_{12}=r_{23}=r_{31})$ in units of MeV: Δ-full EFT



2π 3NF: Δ-full vs Δ-less (preliminary)

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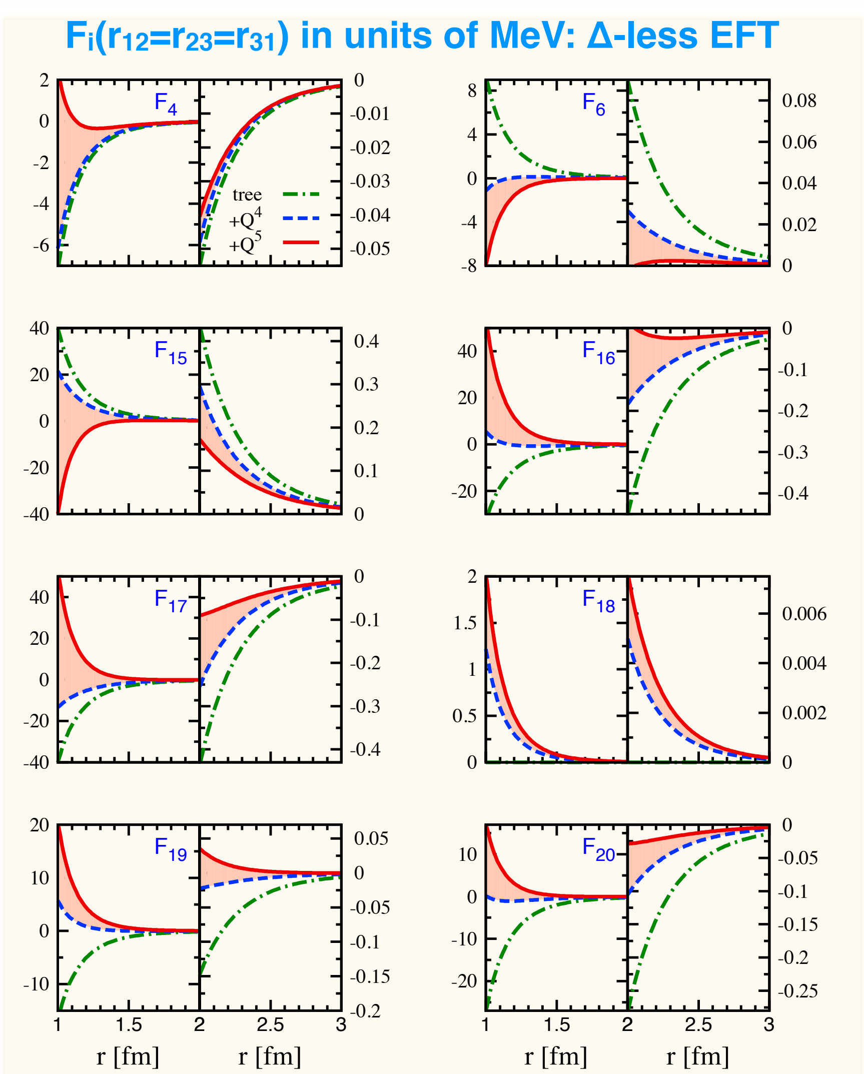
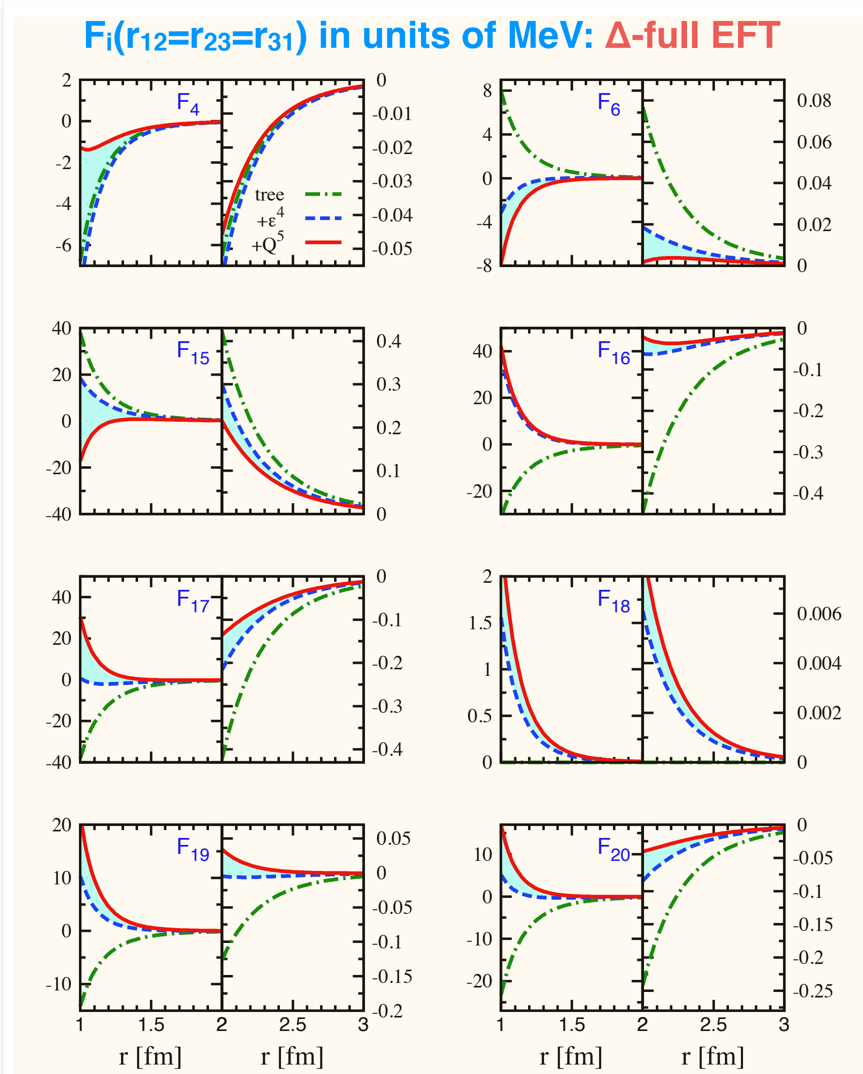




- Δ-full and Δ-less EFT predictions agree well with each other
- Δ-full approach shows clearly a superior convergence
- remarkably, the final 2π 3NF turns out to be rather weak at large distances…

2π 3NF: Δ-full vs Δ-less (preliminary)

Krebs, Gasparyan, EE, to appear

Numerical partial wave decomposition in progress
(currently on JUQUEEN@Jülich, INTREPID@Argonne)

Low **E**nergy **N**uclear **P**hysics **I**nternational **C**ollaboration
(LENPIC)

J. Golak, R. Skibinski, K. Topolnicki, H. Witala (Cracow)
EE, H.Krebs (Bochum)
S. Binder, A. Calci, K. Hebeler, J. Langhammer, R. Roth (Darmstadt)
P. Maris, H. Potter, J. Vary (Iowa State)
R. J. Furnstahl (Ohio State)
A. Nogga (Jülich)
H. Kamada (Kyushu)
U.-G. Meißner (Bonn)
V. Bernard (Orsay)

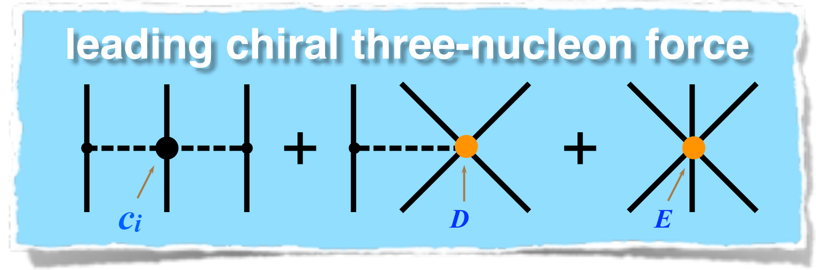
- Δ-full and Δ-less EFT predictions agree well with each other
- Δ-full approach shows clearly a superior convergence
- remarkably, the final 2π 3NF turns out to be rather weak at large distances…

Chiral 3NF@N²LO & nd elastic scattering

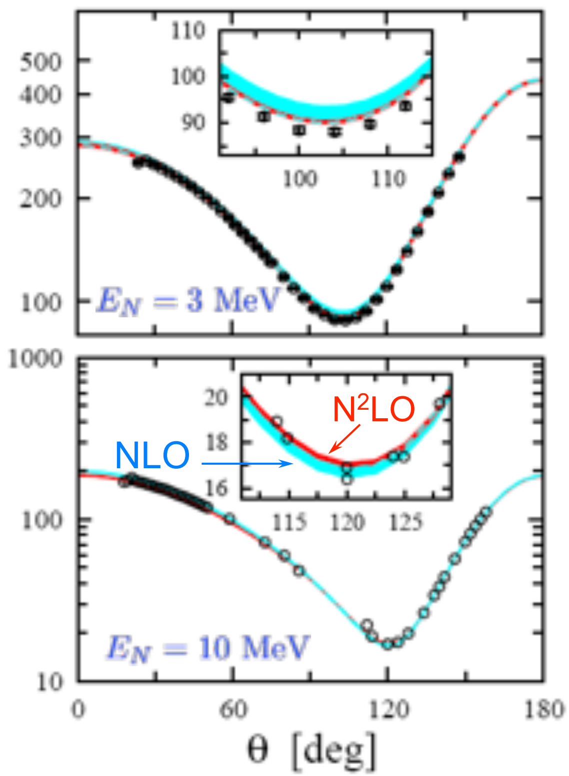
EE, Glöckle, Golak, Kamada, Nogga, Skibinski, Witala

The 3NF starts to contribute at N²LO

The LECs D,E can be fixed e.g. from ^{3}H BE and nd doublet scattering length

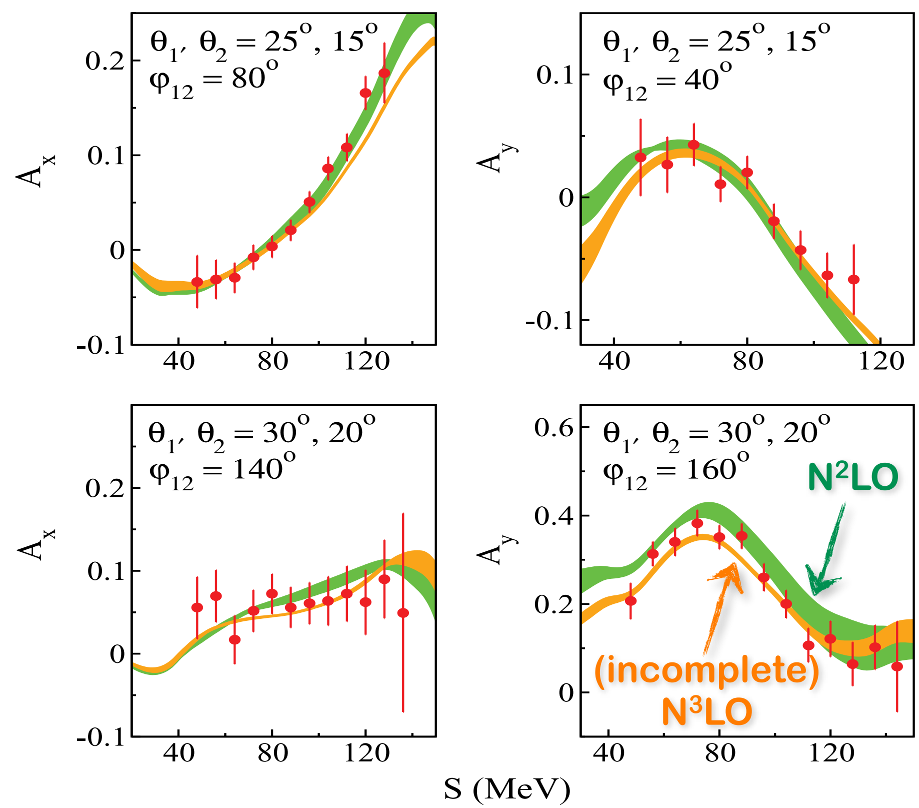


Nd elastic cross sections at low energies



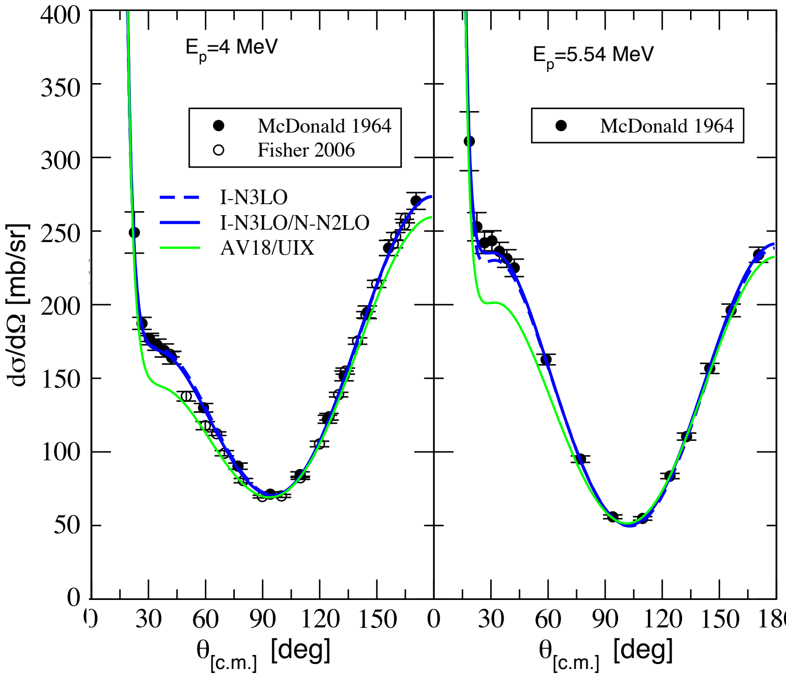
Nd breakup at E_d=130 MeV

Stephan et al., PRC 82 (2010) 014003

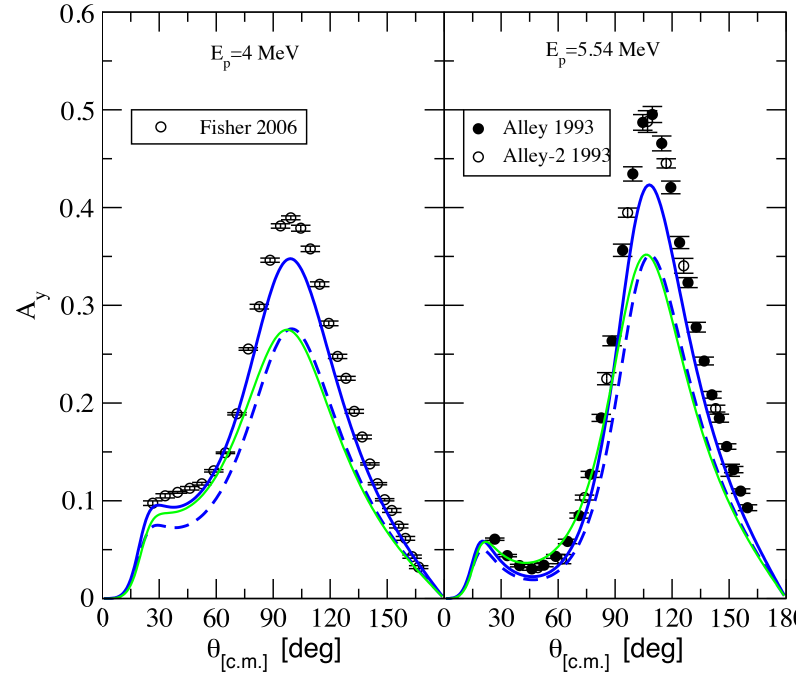


Chiral 3NF@N²LO and 4N scattering

p-³He differential cross section



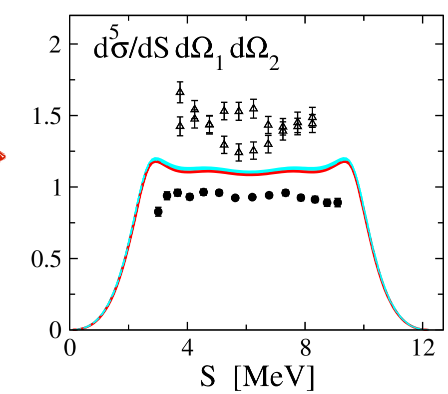
A_y-puzzle in p-³He elastic scattering



LECs D,E tuned to the ³H and ⁴He binding energies, figure from Viviani et al., arXiv:1004.1306

To summarize:

- Nd scattering: accurate description at low energy except for A_y (fine tuned) and Space Star breakup configuration
- Uncertainty increases with energy (higher-order 3NF?)
- 4N continuum: an emerging field...



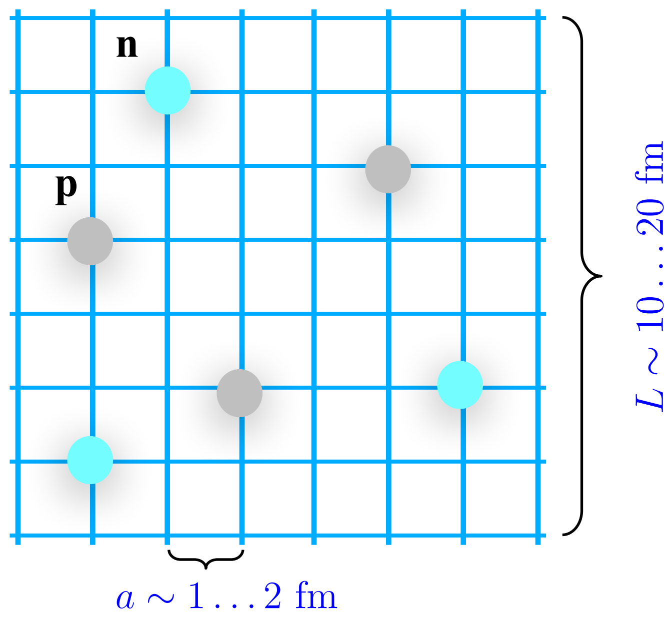
Impact of the 3NF on heavier systems & nuclear matter in the talks by Carlo Barbieri and Arnau Rios Huguet

Nuclear Lattice Effective Field Theory

The Collaboration:

EE, Hermann Krebs (Bochum), Timo Lähde (Jülich), Thomas Luu (Jülich/Bonn), Dean Lee (NC State), Ulf-G. Meißner (Bonn/Jülich), Gautam Rupak (MS State)

Borasoy, E.E., Krebs, Lee, Meißner, Eur. Phys. J. A31 (07) 105,
Eur. Phys. J. A34 (07) 185,
Eur. Phys. J. A35 (08) 343,
Eur. Phys. J. A35 (08) 357,
E.E., Krebs, Lee, Meißner, Eur. Phys. J A40 (09) 199,
Eur. Phys. J A41 (09) 125,
Phys. Rev. Lett 104 (10) 142501,
Eur. Phys. J. 45 (10) 335,
Phys. Rev. Lett. 106 (11) 192501,
E.E., Krebs, Lähde, Lee, Meißner, Phys. Rev. Lett. 109 (12) 252501,
Phys. Rev. Lett. 110 (13) 112502,
Eur. Phys. J. A49 (13) 82
Lähde, EE, Krebs, Lee, Meißner, Rupak, arXiv:1311.0477, to appear in PLB
EE, Krebs, Lähde, Lee, Meißner, Rupak, Phys. Rev. Lett. 112 (14) 102501



Deutsche Forschungsgemeinschaft DFG
HELMHOLTZ GEMEINSCHAFT
HadronPhysics2 Study of Strongly Interacting Matter
JÜLICH FORSCHUNGSZENTRUM
erc European Research Council

Calculation strategy

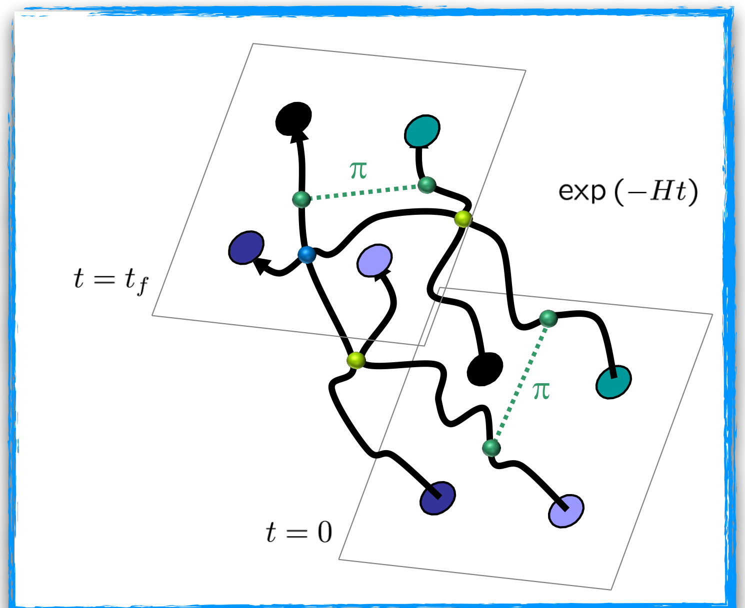
- Eucl.-time propagation of A nucleons → transition amplitude $Z_A(t) = \langle \Psi_A | \exp(-tH) | \Psi_A \rangle$

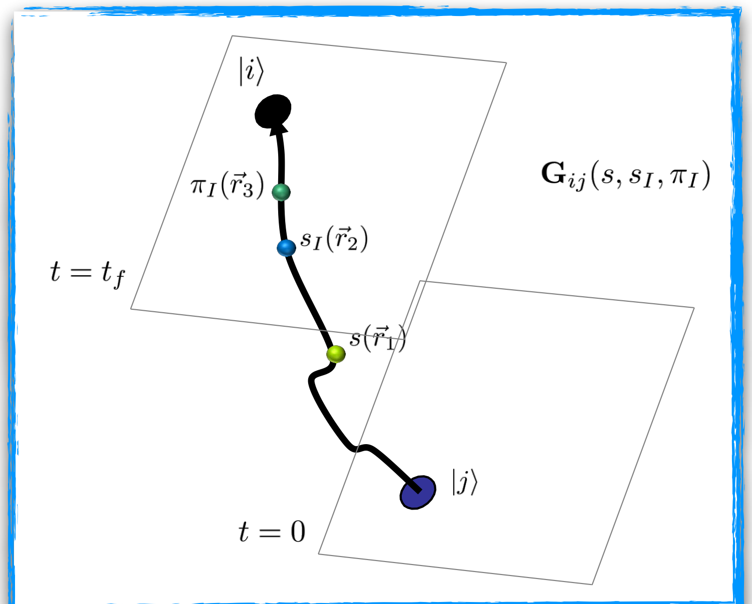
  → ground-state energies $E_A = - \lim_{t\to\infty} d(\ln Z_A)/dt$

- Excited state energies can be obtained from a large-t limit of a correlation matrix $Z_A^{ij}(t) = \langle \Psi_A^i | \exp(-tH) | \Psi_A^j \rangle$ between A-nucleon states Ψ_A^j with the proper quantum numb.

- Use $H_{\rm LO}$ to run the simulation, higher-order terms (incl. Coulomb, 3NF, ...) taken into account perturvatively via $Z_A^O(t) = \langle \Psi_A | \exp(-tH/2)\, O\, \exp(-tH/2) | \Psi_A \rangle$

- We use Auxiliary-Field QMC method

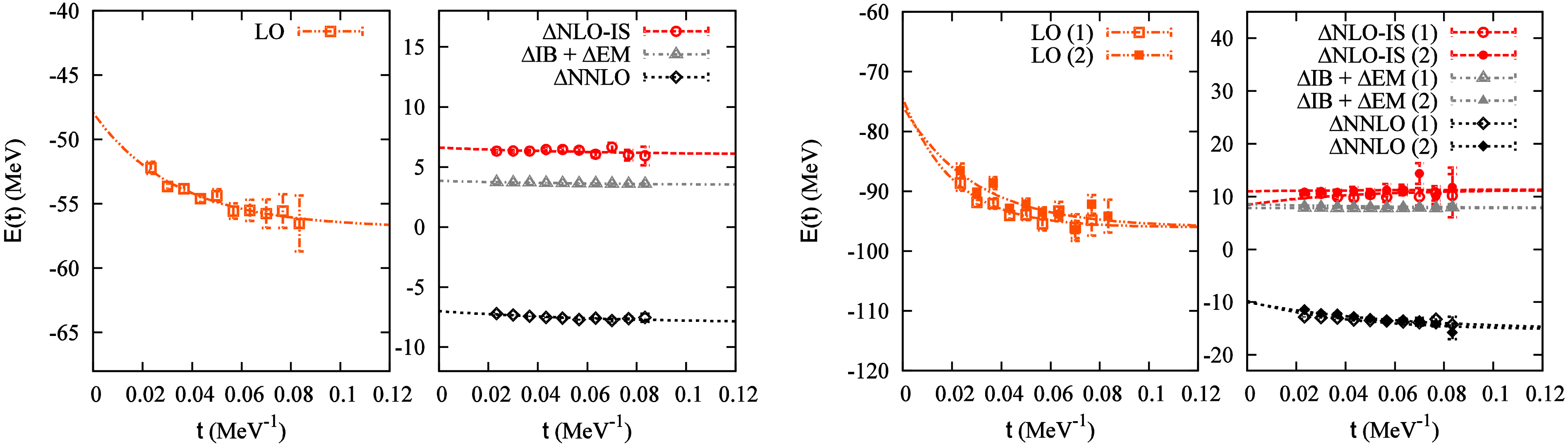




Ground states of ^{8}Be and ^{12}C

EE, Krebs, Lee, Meißner, PRL 106 (11) 192501

Simulations for ^{8}Be and ^{12}C, L=11.8 fm



Ground state energies (L=11.8 fm) of ^{4}He, ^{8}Be, ^{12}C & ^{16}O

	^{4}He	^{8}Be	^{12}C	^{16}O
LO $[Q^0]$, in MeV	$-28.0(3)$	$-57(2)$	$-96(2)$	$-144(4)$
NLO $[Q^2]$, in MeV	$-24.9(5)$	$-47(2)$	$-77(3)$	$-116(6)$
NNLO $[Q^3]$, in MeV	$-28.3(6)$	$-55(2)$	$-92(3)$	$-135(6)$
Experiment, in MeV	-28.30	-56.5	-92.2	-127.6

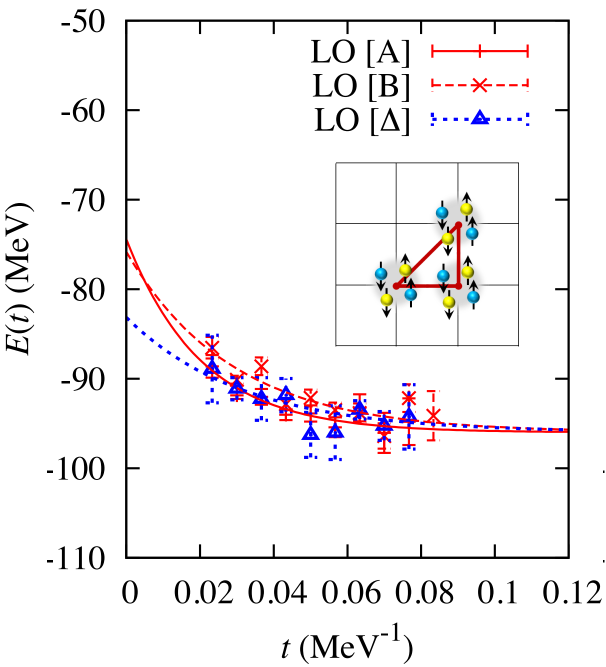
The Hoyle state

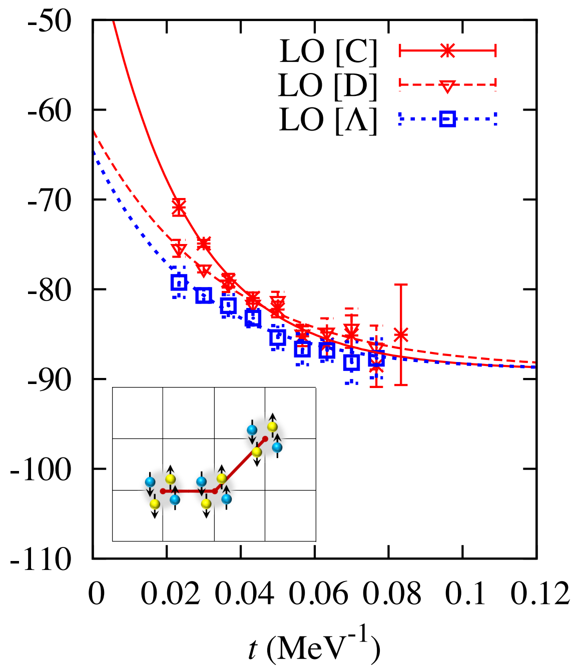
EE, Krebs, Lähde, Lee, Meißner, PRL 106 (2011) 192501; PRL 109 (2012) 252501

Lattice results for low-lying even-parity states of ^{12}C

	0_1^+	$2_1^+(E^+)$	0_2^+	$2_2^+(E^+)$
LO	$-96(2)$	$-94(2)$	$-89(2)$	$-88(2)$
NLO	$-77(3)$	$-74(3)$	$-72(3)$	$-70(3)$
NNLO	$-92(3)$	$-89(3)$	$-85(3)$	$-83(3)$
Exp	-92.16	-87.72	-84.51	$-82(1)$

Probing (α-cluster) structure of the 0_1^+, 0_2^+ states





RMS radii and quadrupole moments

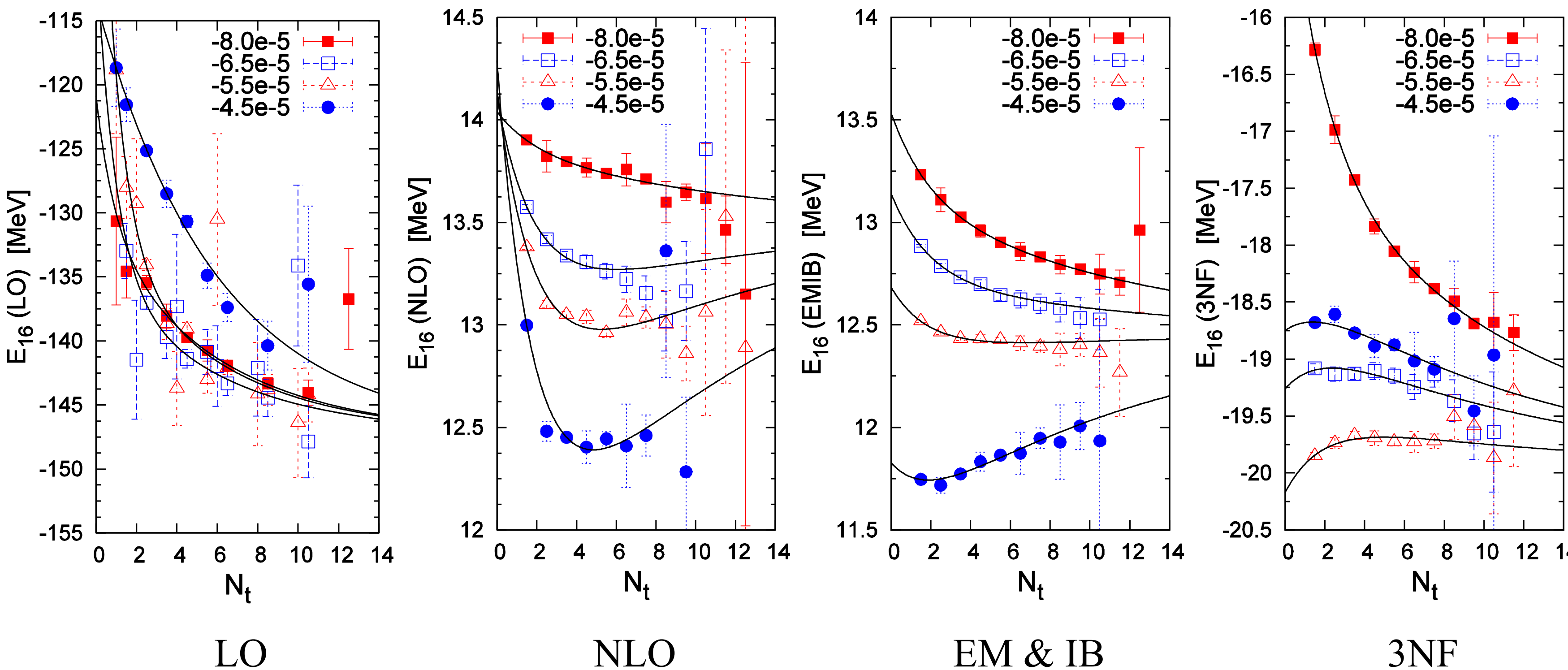
	LO	Experiment
$r(0_1^+)$ [fm]	2.2(2)	2.47(2) [26]
$r(2_1^+)$ [fm]	2.2(2)	–
$Q(2_1^+)$ [$e\,\mathrm{fm}^2$]	6(2)	6(3) [27]
$r(0_2^+)$ [fm]	2.4(2)	–
$r(2_2^+)$ [fm]	2.4(2)	–
$Q(2_2^+)$ [$e\,\mathrm{fm}^2$]	$-7(2)$	–

Improving on large-t extrapolations

Lähde, EE, Krebs, Lee, Meißner, Rupak ’13

To compute heavier & non α-like systems, one needs to improve on large-t extrapolations (and/or reduce sign oscillations) → new „triangulation“ method [use several trial states $|\Psi_A(t')\rangle \equiv \exp(-H_{\mathrm{SU}(4)}t')|\Psi_A^{\mathrm{init}}\rangle$ pre-evolved with different values of $C_{\mathrm{SU}(4)}$]

Large Euclidean time extrapolations for the ^{16}O ground state



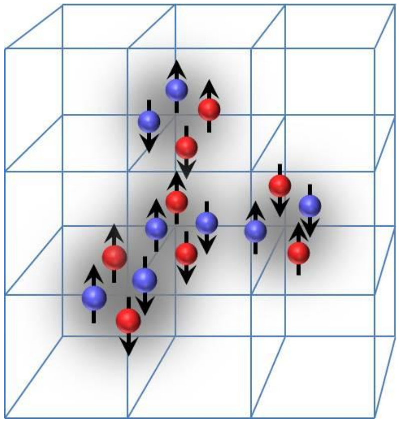
We also calculated GS energies of ^{20}Ne, ^{24}Mg and ^{28}Si finding an increasing overbinding which can be corrected by a single 4N force Lähde, EE, Krebs, Lee, Meißner, Rupak arXiv:1311.0477; to appear in PLB

Lowest even-parity states of ^{16}O

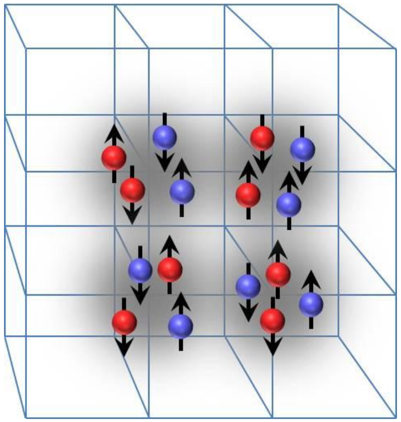
EE, Krebs, Lähde, Lee, Meißner, Rupak ’14

	LO	NNLO (2N)	NNLO (3N)	$+ 4N_{eff}$	Exp.
0_1^+	-147.3(5)	-121.4(5)	-138.8(5)	-131.3(5)	-127.62
0_2^+	-145(2)	-116(2)	-136(2)	-123(2)	-121.57
2_1^+	-145(2)	-116(2)	-136(2)	-123(2)	-120.70

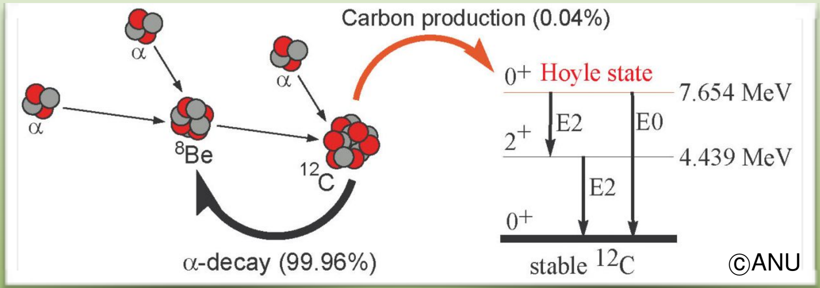
Dominant configuration of the 0_1^+ state



Dominant configuration of the 0_2^+ state



Also looked at radii and transition amplitudes, see EE, Krebs, Lähde, Lee, Meißner, Rupak PRL 112 (2014) 102501



The triple alpha reaction rate as a function of the quark mass

Production of ^{12}C in stars depends sensitively on the energy differences:

$$\Delta E_b \equiv E_8 - 2E_4\,,$$
$$\Delta E_h \equiv E_{12}^\star - E_8 - E_4$$

Reaction rate for the triple alpha process: $r_{3\alpha} \simeq 3^{\frac{3}{2}} N_\alpha^3 \left(\frac{2\pi\hbar^2}{M_\alpha k_\mathrm{B} T}\right)^3 \frac{\Gamma_\gamma}{\hbar} \exp\left(-\frac{\varepsilon}{k_\mathrm{B} T}\right)$

Oberhummer, Csoto, Schlattl, Science 289 (2000) 88

where $\varepsilon \equiv \Delta E_b + \Delta E_h = E_{12}^\star - 3E_4 = 379.47(18)\ \mathrm{keV}$ - crucial control parameter

Changing ε by ~100 keV destroys production of either ^{12}C or ^{16}O Livio et al.'89; Oberhummer, et al.'00

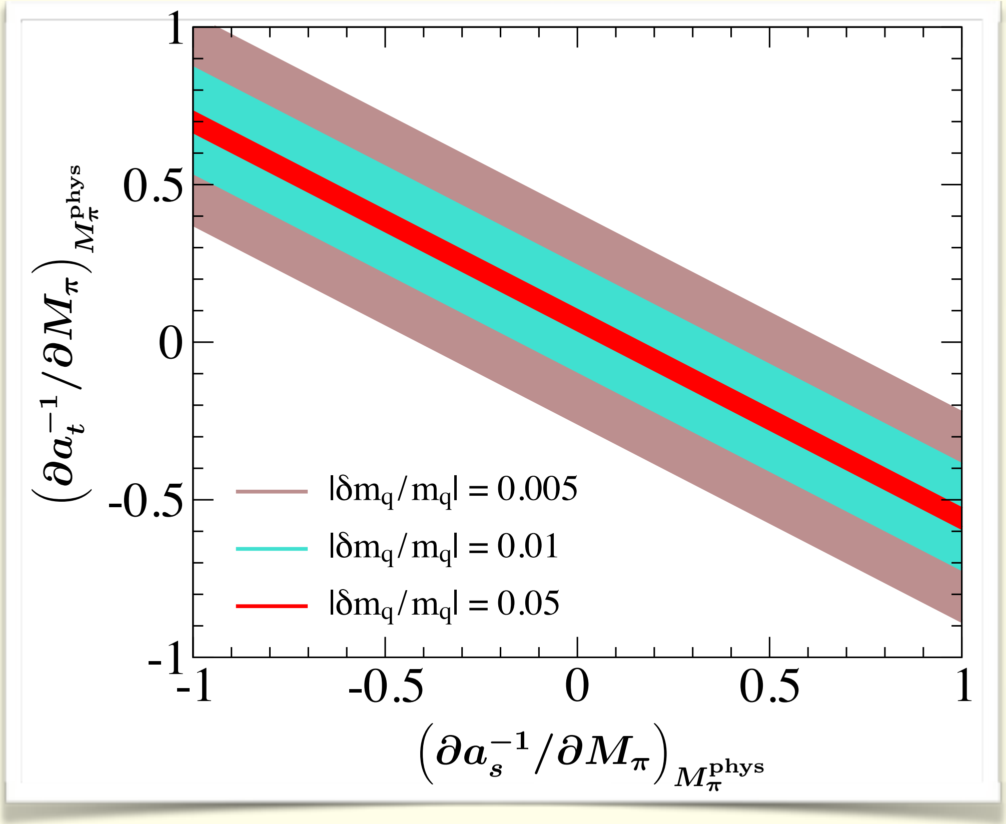
How robust is ε with respect to variations of the light quark mass?

Quark mass dependence of the triple-α reaction rate

EE, Krebs, Lähde, Lee, Meißner, PRL 110 (2013) 112502; EPJA 49 (2013)

$$|\delta\varepsilon| < 100 \text{ keV} \longrightarrow \left| \left(0.771(14) \frac{\partial a_s^{-1}}{\partial M_\pi}\bigg|_{M_\pi^{\text{phys}}} + 0.934(11) \frac{\partial a_t^{-1}}{\partial M_\pi}\bigg|_{M_\pi^{\text{phys}}} - 0.069(6) \right) \frac{\delta m_q}{m_q} \right| < 0.0015$$

„Survivability bands" for carbon-oxygen based life due to 0.5%, 1%, 5% variation of m_q



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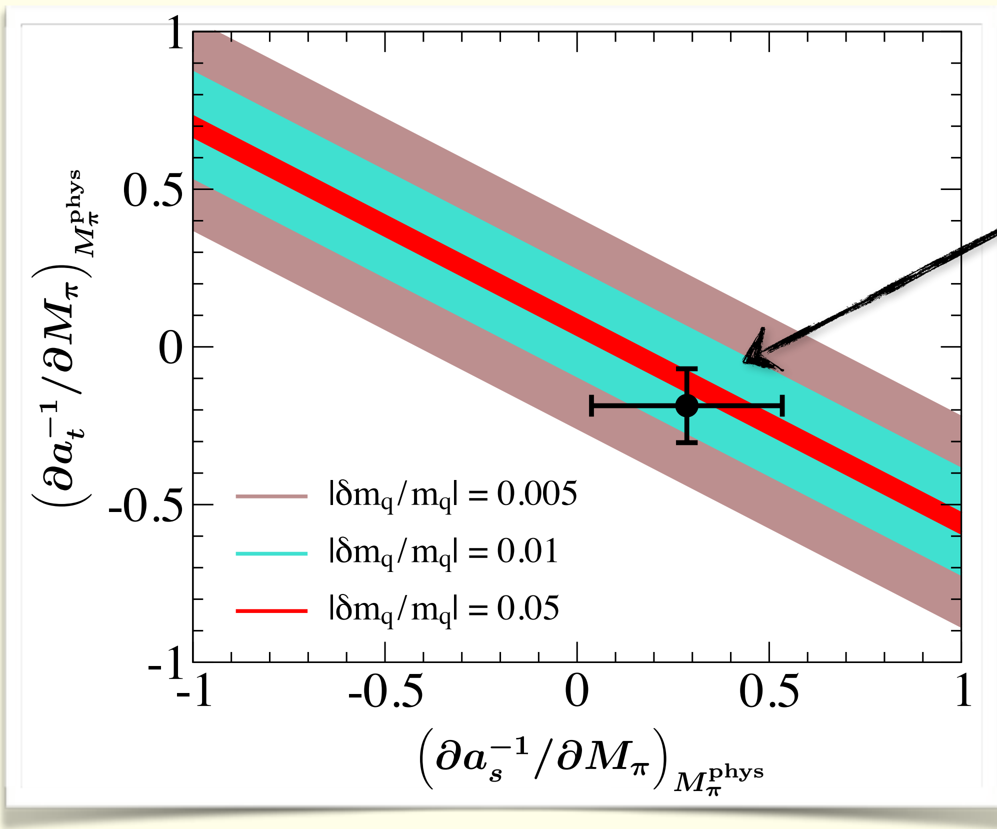

up-to-date chiral EFT calculation (N^2LO):

$K^q_{a_s} = 2.3^{+1.9}_{-1.8}, \quad K^q_{a_t} = 0.32^{+0.17}_{-0.18}$

Berengut et al., PRD 87 (2013)

Summary and outlook

Chiral two-nucleon force

- A new generation of chiral NN potentials is underway…

Chiral three-nucleon force

- Worked out completely at N^3LO and, to a large extent, at N^4LO. The N^4LO contributions are driven by the Δ and appear to be large (as expected)
- Alternatively, calculations in EFT with explicit Δ are being performed. For 2π 3NF both approaches lead to comparable results (with the Δ-full approach showing faster convergence). Δ-contributions to other topologies in progress
- Very good progress on the PWD of the 3NF due to improving on the algorithm

Nuclear lattice simulations:

- Improved method for large-t extrapolations allows to access heavier systems (calculated GS energies up to ^{28}Si)
- Promising results for low-lying states of ^{12}C and ^{16}O, evidence for α-clustering, dominant configurations in terms of α-clusters identified

Future plans: heavier nuclei, smaller lattice spacing, N^3LO, reactions ...