The Tjon Band in Nuclear Lattice EFT

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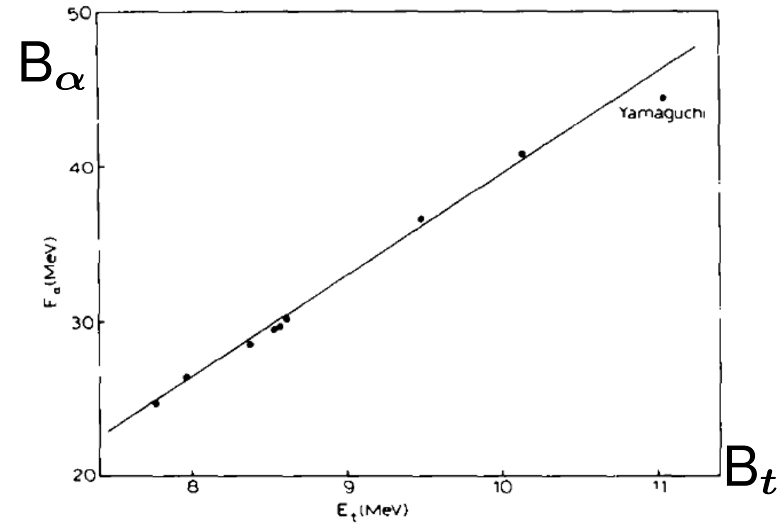
CONTENTS

Introduction

WHAT is the TJON LINE/BAND?

- Tjon observed a correlation between the ^{4}He and triton (^{3}H) binding energies for local interactions

  Tjon, Phys. Lett. 56B (1975) 217

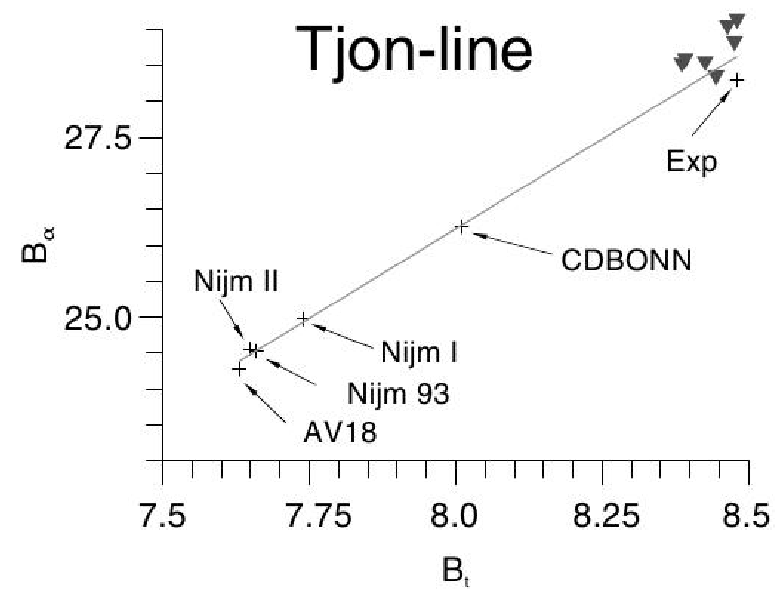


- This correlation, then called the **Tjon line**, was also found using high-precision potentials

  $\hookrightarrow$ three-nucleon forces required

  $\hookrightarrow$ no nedd for additional four-nucleon forces

  Nogga et al., Phys. Rev. Lett. 85 (2000) 944



UNDERSTANDING the TJON LINE/BAND?

- Consider pionless EFT: contact interactions

$$\langle \mathbf{k}'|V|\mathbf{k}\rangle = \mathcal{P}_s\,\lambda_2^s\,g(\mathbf{k}')g(\mathbf{k}) + \mathcal{P}_t\,\lambda_2^t\,g(\mathbf{k}')g(\mathbf{k}) + \ldots\,, \quad g(\mathbf{u}) = \exp(-u^2/\Lambda^2)$$

$\hookrightarrow$ physics independent of cut-off Λ

- Renormalization of the three-nucleon system requires a three-nucleon force (3NF)

  Bedaque, Hammer, van Kolck, Phys. Rev. Lett. 82 (1999) 463

$$V_3 = \mathcal{P}_a \lambda_3 h(u_1, u_2) h(u_1', u_2')\,, \quad h(u_1, u_2) = \exp(-(u_1^2 + \tfrac{3}{4}u_2^2)/\Lambda^2)$$

- This also renormalizes the four-nucleon system!

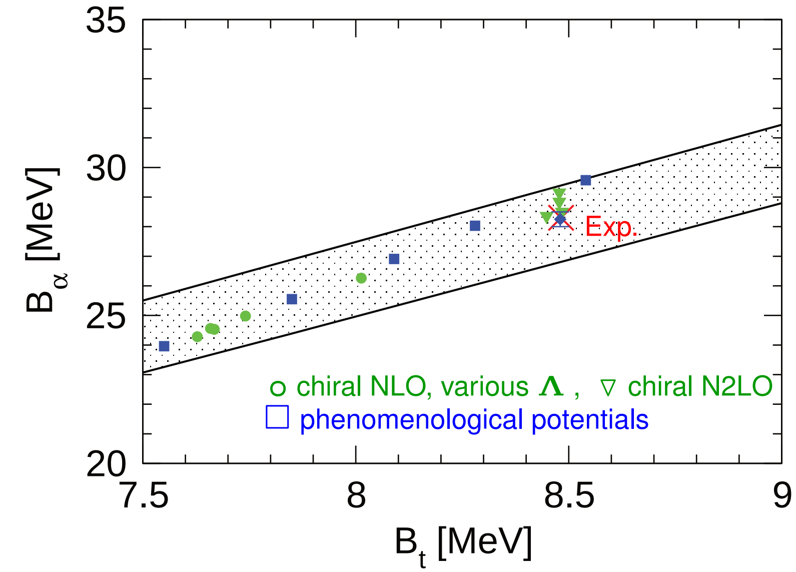
  Platter, Hammer, UGM, Phys. Rev. A70 (2004) 052101

$\hookrightarrow$ explains naturally the correlation

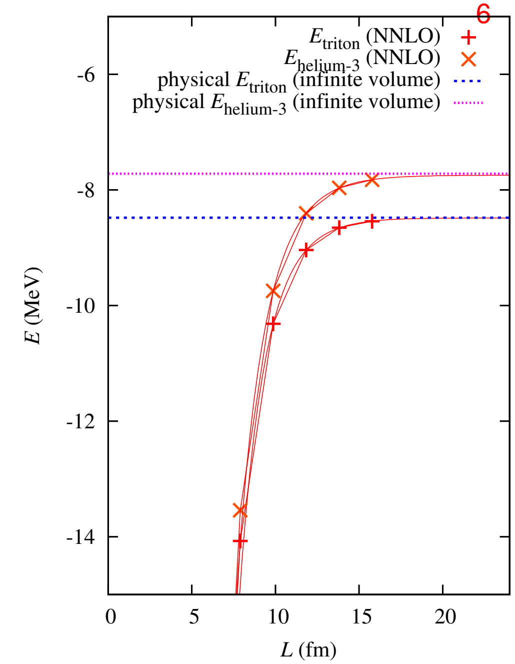
$\hookrightarrow$ it is really a **band** (theor. uncertainty)

  Platter, Hammer, UGM, Phys. Lett. B607 (2005) 254

$\hookrightarrow$ width of the band shrinks at higher orders



STATUS of NUCLEAR LATTICE EFT



- standard coarse lattice at NNLO, $a = 1.97$ fm, $N = 8$

  11+2 LECs, incl. isospin breaking and Coulomb

  local smearing of the LO contact interactions in S-wave

$\hookrightarrow$ ^{3}H and ^{3}He well described

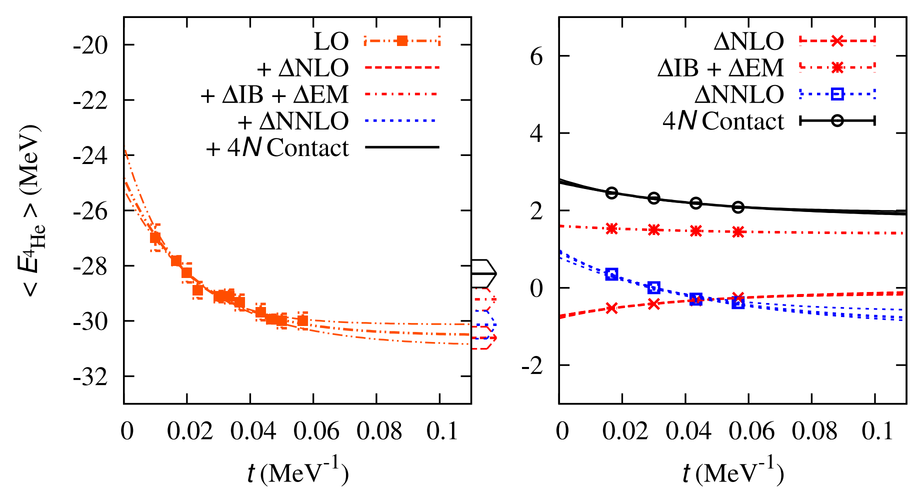
$$E(^3\mathrm{He}) - E(^3\mathrm{H}) = 0.78(5)\ \mathrm{MeV}\quad [\mathrm{exp.: } 0.76\ \mathrm{MeV}]$$

$\hookrightarrow$ ^{4}He overbound by a few MeV

$\hookrightarrow$ cured by an effective four-nucleon interaction

$$V_{\rm eff}^{(4)} = D_{\rm eff}^{(4)} \sum_{\vec{n}} [\rho(\vec{n})]^4$$

Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. 104 (2010) 142501, Eur. Phys. J. A 45 (2010) 335



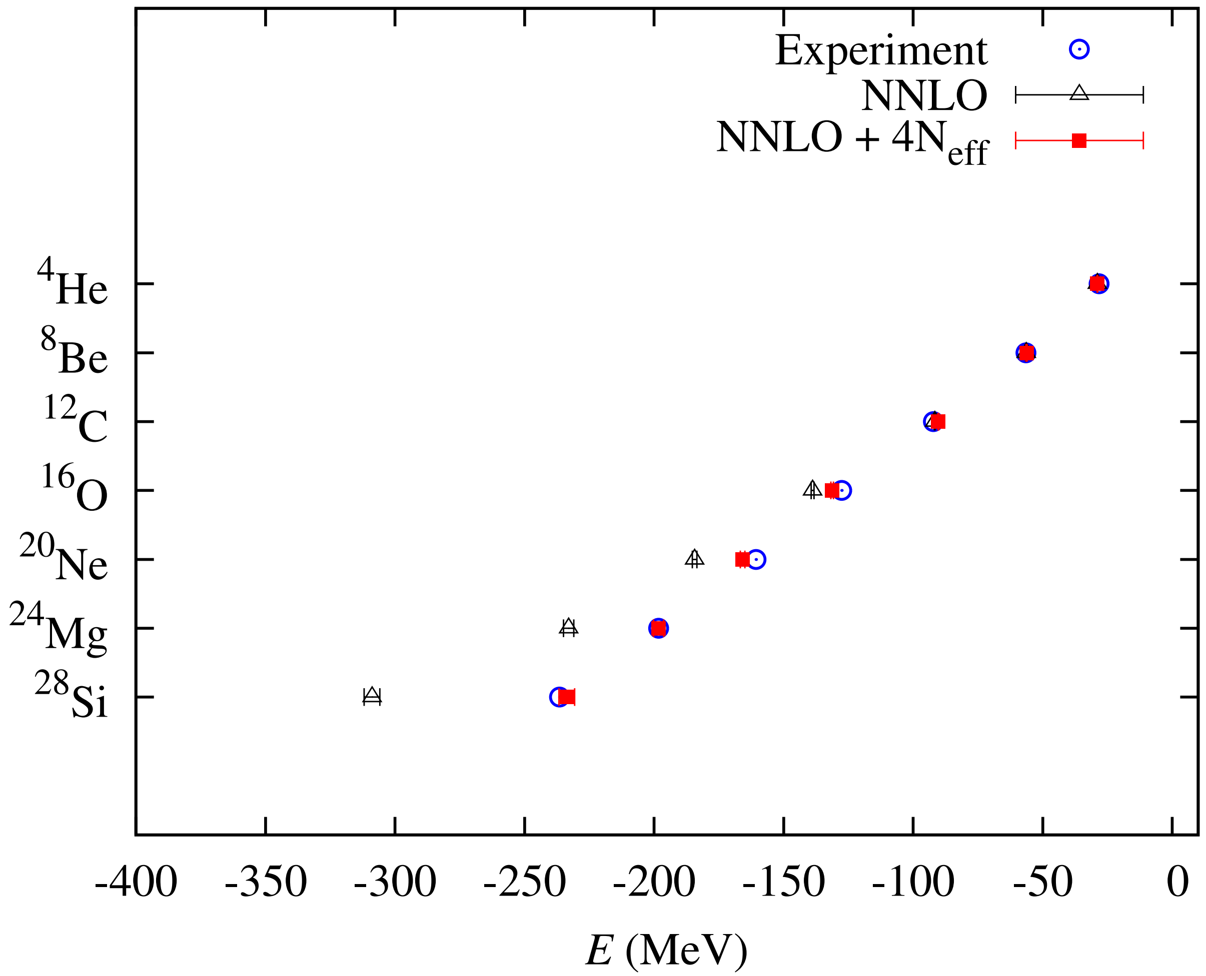
- presumably a lattice artefact?

  or non-local smearing required?

$\hookrightarrow$ investigate this in more detail!

The D-TERM in ACTION

- Works quite well up to the mid-mass region:
  ↪ 1% accuray for ground-state energies



Lähde, Epelbaum, Krebs, Lee, UGM, Rupak, Phys. Lett. B732 (2014) 110

Basics of nuclear lattice simulations

for an easy intro, see: UGM, Nucl. Phys. News **24** (2014) 11

for an early review, see: D. Lee, Prog. Part. Nucl. Phys. **63** (2009) 117

upcoming textbook, see: T. Lähde, UGM, Springer Lecture Notes in Physics

NUCLEAR LATTICE SIMULATIONS

Frank, Brockmann (1992), Koonin, Müller, Seki, van Kolck (2000) , Lee, Schäfer (2004), . . .
Borasoy, Krebs, Lee, UGM, Nucl. Phys. **A768** (2006) 179; Borasoy, Epelbaum, Krebs, Lee, UGM, Eur. Phys. J. **A31** (2007) 105

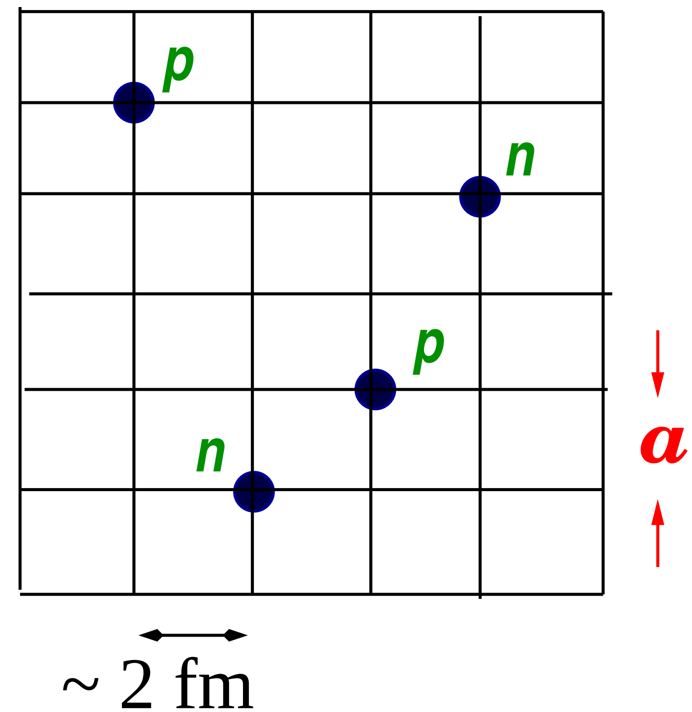
- *new method* to tackle the nuclear many-body problem

- discretize space-time $V = L_s \times L_s \times L_s \times L_t$:
  nucleons are point-like particles on the sites

- discretized chiral potential w/ pion exchanges
  and contact interactions + Coulomb

  → see Epelbaum, Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773

- typical lattice parameters



$$p_{\max} = \frac{\pi}{a} \simeq 314 \text{ MeV [UV cutoff]}$$

- strong suppression of sign oscillations due to approximate Wigner SU(4) symmetry

  J. W. Chen, D. Lee and T. Schäfer, Phys. Rev. Lett. **93** (2004) 242302, T. Lähde et al., EPJA **51** (2015) 92

- hybrid Monte Carlo & transfer matrix (similar to LQCD)

TRANSFER MATRIX METHOD

- Correlation–function for A nucleons: $Z_A(\tau) = \langle \Psi_A | \exp(-\tau H) | \Psi_A \rangle$

  with Ψ_A a Slater determinant for A free nucleons
  [or a more sophisticated (correlated) initial/final state]

  Euclidean time

- Transient energy

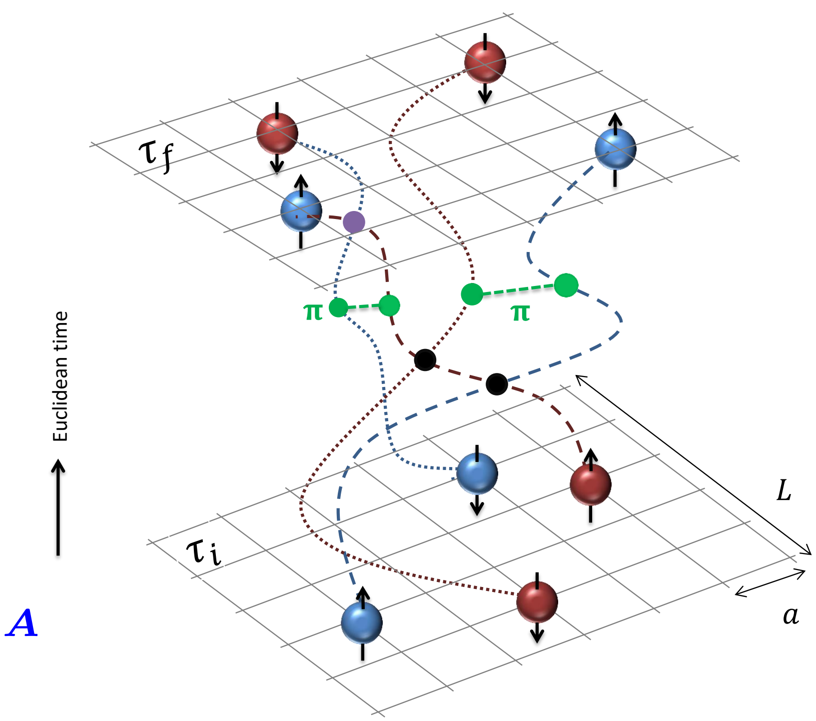
$$E_A(\tau) = -\frac{d}{d\tau} \ln Z_A(\tau)$$

→ ground state: $E_A^0 = \lim_{\tau \to \infty} E_A(\tau)$

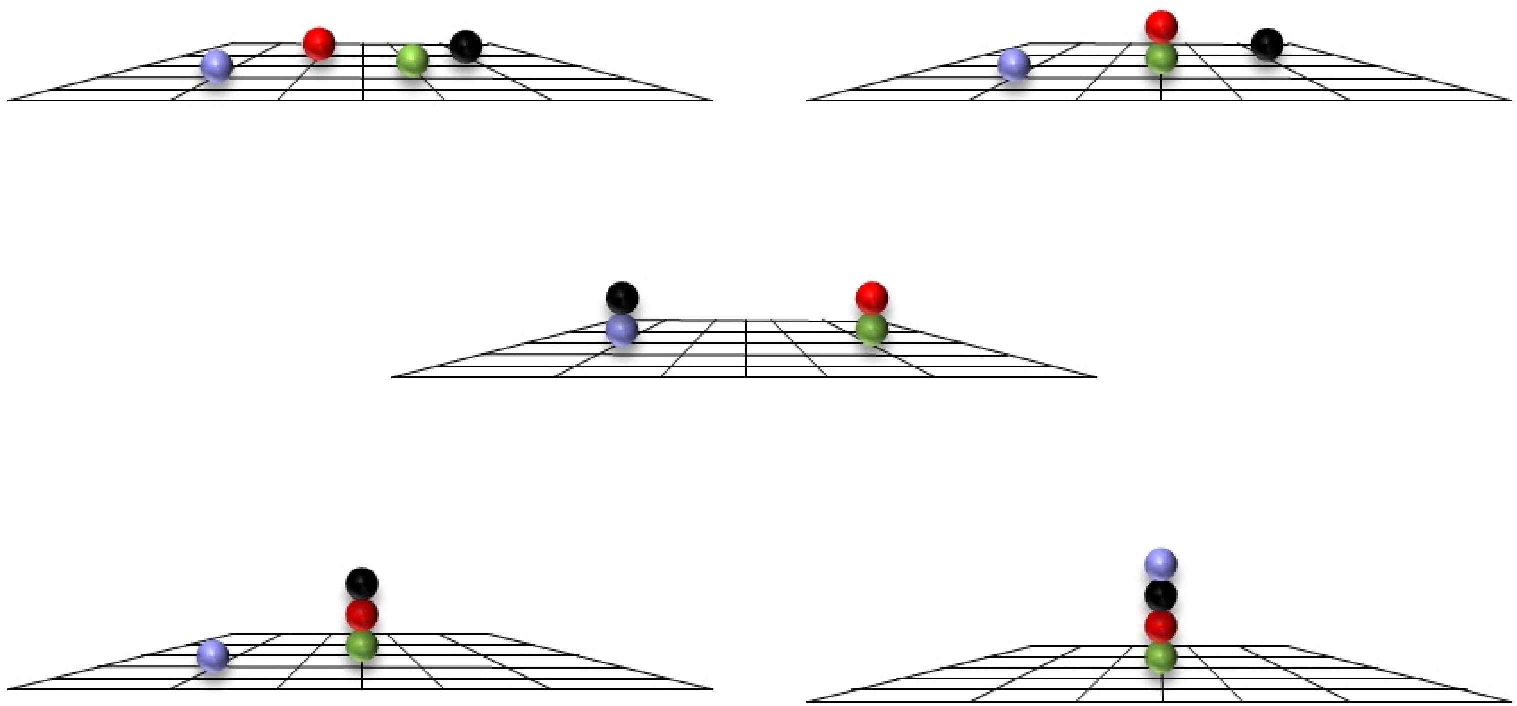
- Exp. value of any normal–ordered operator $\mathcal{O}$

$$Z_A^{\mathcal{O}} = \langle \Psi_A | \exp(-\tau H/2)\, \mathcal{O}\, \exp(-\tau H/2)\, | \Psi_A$$

$$\lim_{\tau \to \infty} \frac{Z_A^{\mathcal{O}}(\tau)}{Z_A(\tau)} = \langle \Psi_A | \mathcal{O} | \Psi_A \rangle$$



CONFIGURATIONS



⇒ all *possible* configurations are sampled

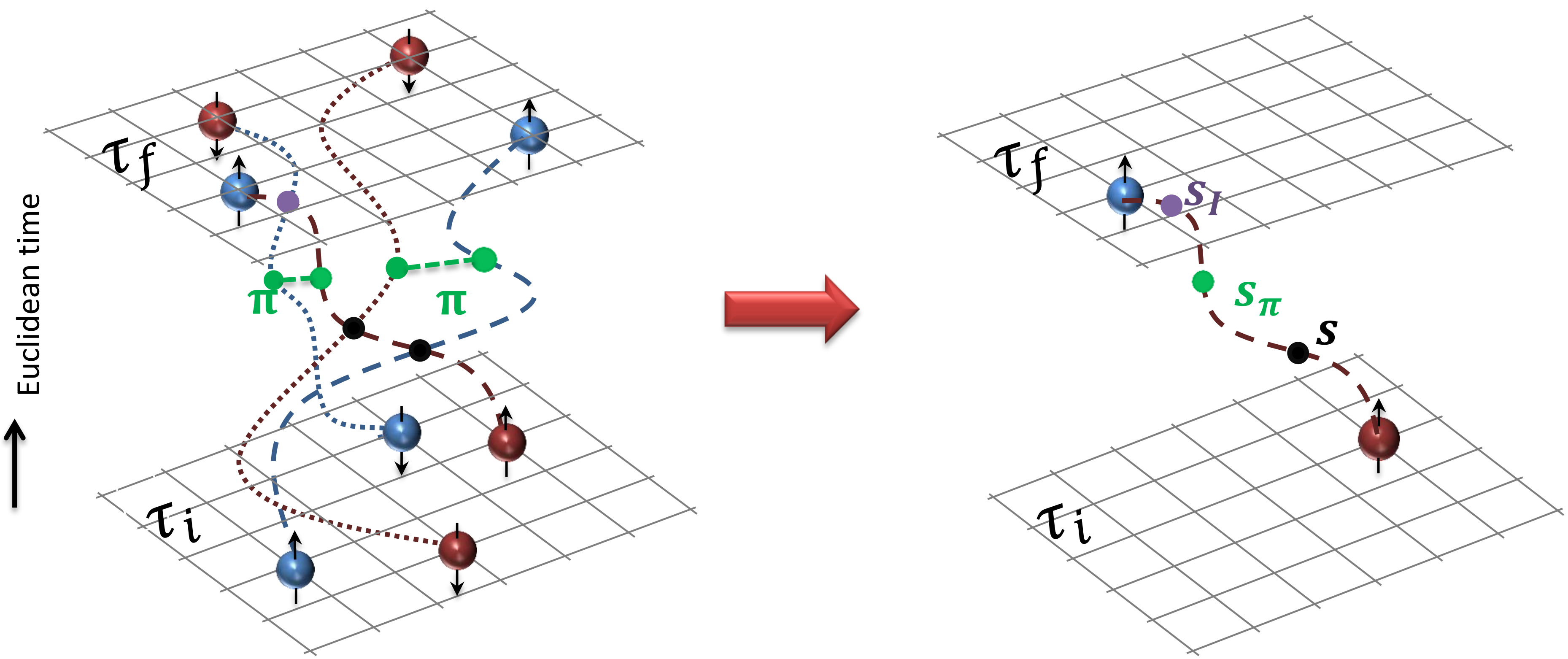
⇒ preparation of *all possible* initial/final states

⇒ *clustering* emerges *naturally*

AUXILIARY FIELD METHOD

- Represent interactions by auxiliary fields:

$$\exp\left[-\frac{C}{2}\left(N^\dagger N\right)^2\right] = \sqrt{\frac{1}{2\pi}}\int ds \exp\left[-\frac{s^2}{2} + \sqrt{C}\; s\left(N^\dagger N\right)\right]$$



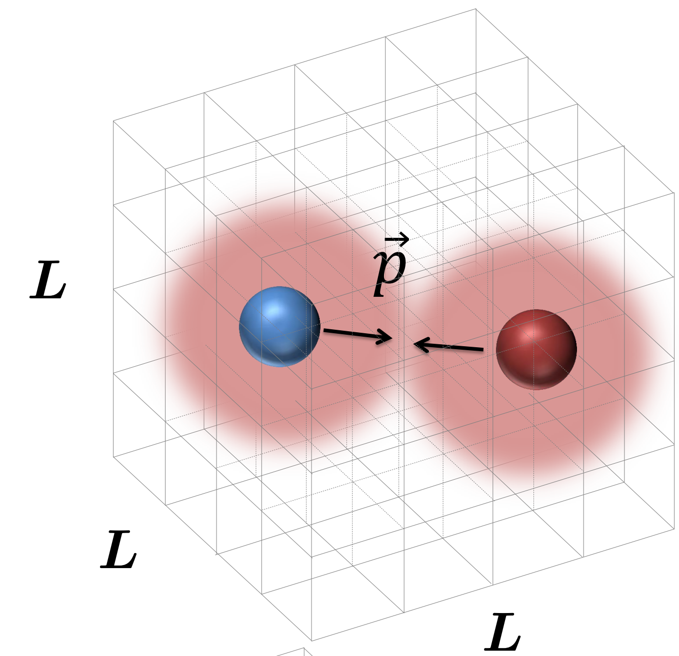
EXTRACTING PHASE SHIFTS on the LATTICE

- Lüscher's method:

  Two-body energy levels below the inelastic threshold on a periodic lattice are related to the phase shifts in the continuum

  Lüscher, Comm. Math. Phys. **105** (1986) 153

  Lüscher, Nucl. Phys. B **354** (1991) 531



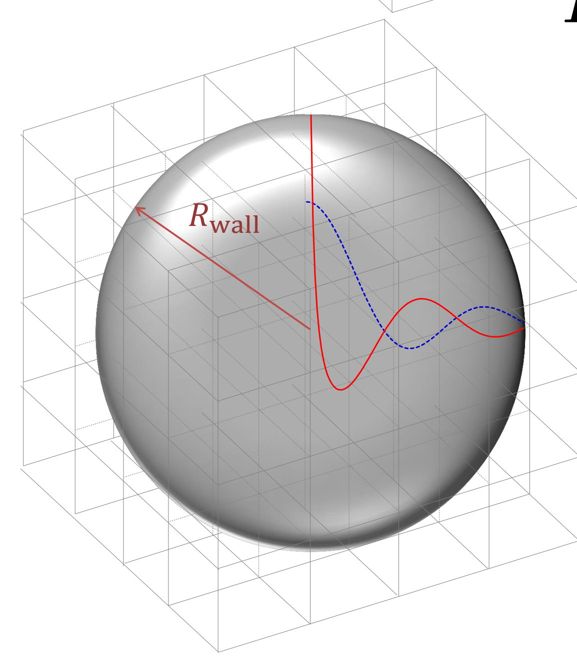
- Spherical wall method:

  Impose a hard wall on the lattice and use the fact that the wave function vanishes for $r = R_{\text{wall}}$:

$$\psi_\ell(r) \sim [\cos \delta_\ell(p) F_\ell(pr) + \sin \delta_\ell(p) G_\ell(pr)]$$

Borasoy, Epelbaum, Krebs, Lee, UGM, EPJA **34** (2007) 185

Carlson, Pandharipande, Wiringa, NPA **424** (1984) 47

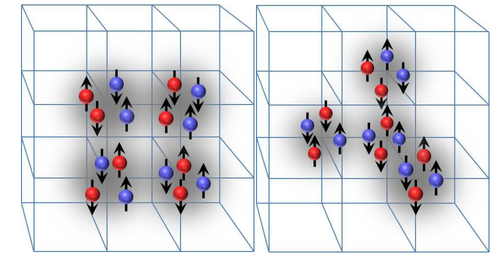


COMPUTATIONAL EQUIPMENT

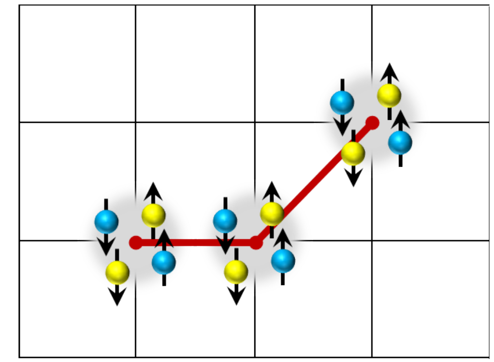
- Mostly used = JUQUEEN (BlueGene/Q)

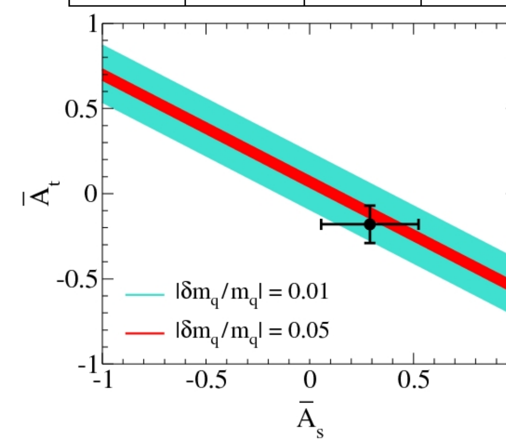
6 Pflops

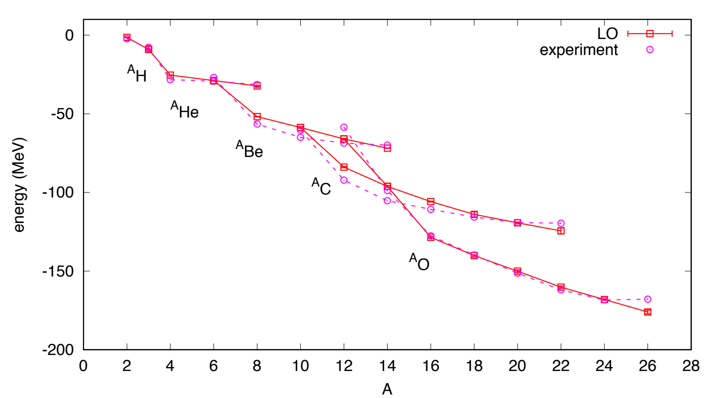
RESULTS from LATTICE NUCLEAR EFT

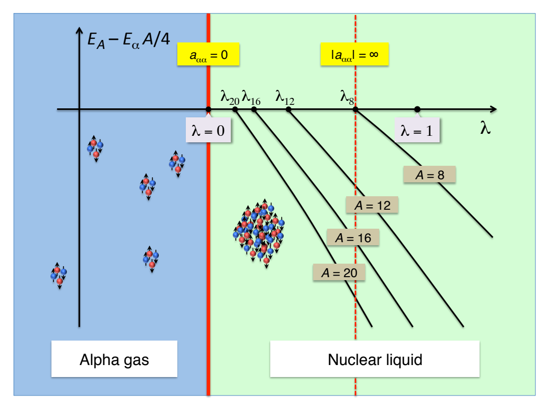


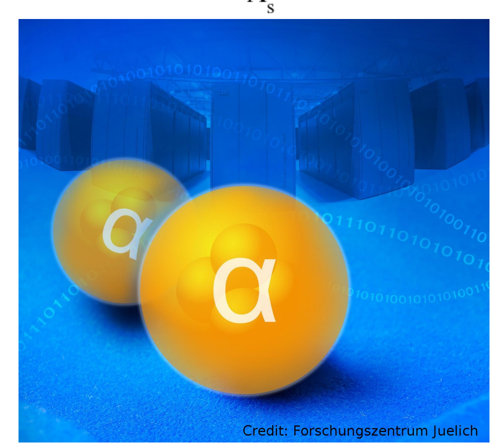
- ☐ Lattice EFT calculations for A=3,4,6,12 nuclei, PRL 104 (2010) 142501
- ☐ *Ab initio* calculation of the Hoyle state, PRL 106 (2011) 192501
- ☐ Structure and rotations of the Hoyle state, PRL 109 (2012) 142501
- ☐ Validity of Carbon-Based Life as a Function of the Light Quark Mass
  PRL 110 (2013) 142501
- ☐ *Ab initio* calculation of the Spectrum and Structure of ^{16}O,
  PRL 112 (2014) 142501
- ☐ *Ab initio* alpha-alpha scattering, Nature 528 (2015) 111
- ☐ Nuclear Binding Near a Quantum Phase Transition, PRL 117 (2016) 132501
- ☐ *Ab initio* calculations of the isotopic dependence of nuclear clustering,
  PRL 119 (2017) 222505











The two-body system at NNLO

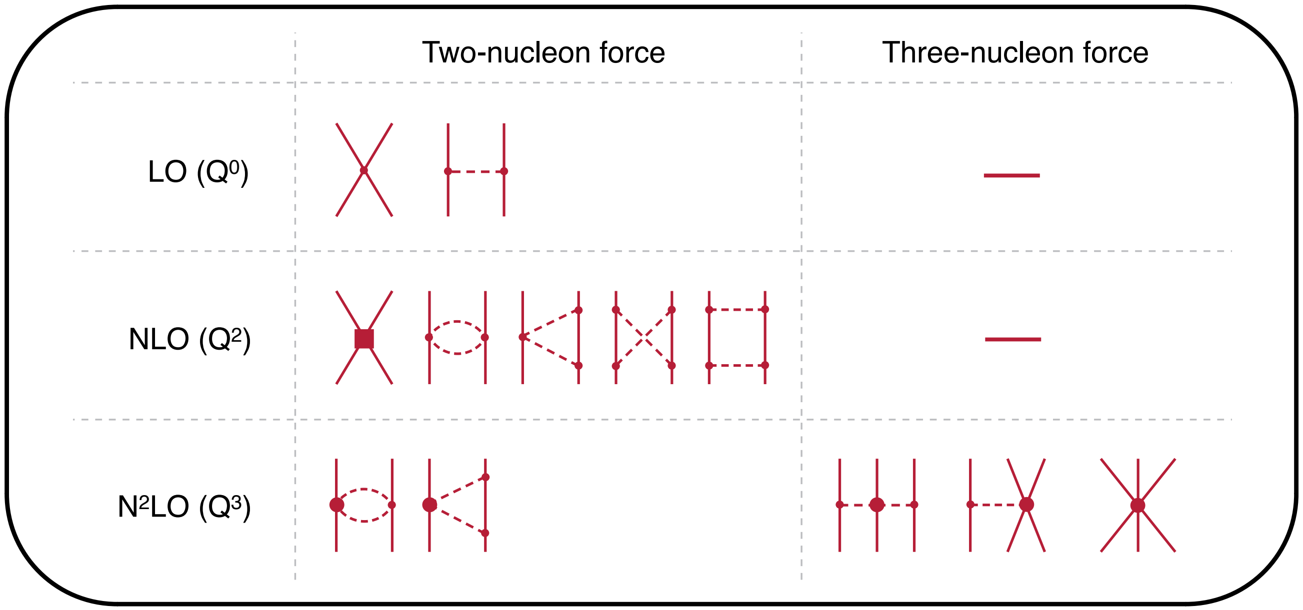
Alarcón, Du, Klein, Lähde, Lee, Li, Luu, UGM
Eur. Phys. J. **A 53** (2017) 83 [arXiv:1702.05319]

Klein, Elhatisari, Lähde, Lee, UGM [arXiv:1803.04231]

NUCLEAR FORCES at NNLO

for details, see: Epelbaum, Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773

- Potential at next-to-next-to-leading order $[Q = \{p/\Lambda, M_\pi/\Lambda\}]$:



- NN potential to NNLO [all πN and $\pi\pi N$ LECs fixed from πN scattering]:

$$V_{\rm NN} = V_{\rm LO}^{(0)} + V_{\rm NLO}^{(2)} + V_{\rm NNLO}^{(3)}$$

$$= V_{\rm LO}^{\rm cont} + V_{\rm LO}^{\rm OPE} + V_{\rm NLO}^{\rm cont} + V_{\rm NLO}^{\rm TPE} + V_{\rm NNLO}^{\rm TPE}$$

NUCLEAR FORCES at NNLO continued

- Analytic expressions [2+7 LECs]:

$$V_{\rm LO}^{\rm cont} = C_S + C_T \, (\vec\sigma_1 \cdot \vec\sigma_2)$$

$$V_{\rm LO}^{\rm OPE} = -\frac{g_A^2}{4F_\pi^2}\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{(\vec\sigma_1\cdot\vec q\,)\,(\vec\sigma_2\cdot\vec q\,)}{q^{\,2} + M_\pi^2}$$

$\vec q$ = t-channel mom. transfer

$$V_{\rm NLO}^{\rm cont} = C_1 q^2 + C_2 k^2 + \left(C_3 q^2 + C_4 k^2\right)(\vec\sigma_1\cdot\vec\sigma_2) + iC_5 \tfrac{1}{2}\,(\vec\sigma_1 + \vec\sigma_2)\cdot(\vec q\times\vec k\,) + C_6\,(\vec\sigma_1\cdot\vec q\,)\,(\vec\sigma_2\cdot\vec q\,) + C_7(\vec\sigma_1\cdot\vec k\,)(\vec\sigma_2\cdot\vec k\,)$$

$\vec k$ = u-channel mom. transfer

$$V_{\rm NLO}^{\rm TPE} = -\frac{\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{384\pi^2F_\pi^4}L(q)\Big[4M_\pi^2\left(5g_A^4 - 4g_A^2 - 1\right) + q^2\left(23g_A^4 - 10g_A^2 - 1\right) + \frac{48g_A^4M_\pi^4}{4M_\pi^2+q^2}\Big] - \frac{3g_A^4}{64\pi^2F_\pi^4}L(q)\left[(\vec q\cdot\vec\sigma_1)\,(\vec q\cdot\vec\sigma_2) - q^2\,(\vec\sigma_1\cdot\vec\sigma_2)\right]$$

- Loop function: $L(q) = \frac{1}{2q}\sqrt{4M_\pi^2 + q^2}\,\ln\frac{\sqrt{4M_\pi^2+q^2}+q}{\sqrt{4M_\pi^2+q^2}-q}$

$$\to 1 + \frac{1}{3}\frac{q^2}{4M_\pi^2} + \cdots \text{ for } q \ll \Lambda$$

$\to$ for coarse lattices $a \simeq 2$ fm, the TPE at N(N)LO can be absorbed in the LECs C_i

$\to$ no longer true as a decreases, need to account for the TPE explicitly

A FEW DETAILS ON THE FITS

- Fits in large & fixed volumes, vary a from 1 to 2 fm:

a^{-1} [MeV]	a [fm]	L	La [fm]
100	1.97	32	63.14
120	1.64	38	62.48
150	1.32	48	63.14
200	0.98	64	63.14

- OPE and TPE LECs completely fixed ($g_A \sim g_{\pi NN}$ and $c_{1,2,3,4}$ from RS analysis)

Hoferichter, Ruiz de Elvira, Kubis, UGM, Phys. Rev. Lett. **115** (2015) 092301

- Smeared LO S-wave contact interactions: $f(\vec{q}) \equiv f_0^{-1} \exp\left(-b_s \frac{\vec{q}^4}{4}\right)$

- Partial-wave projection of the contact interactions

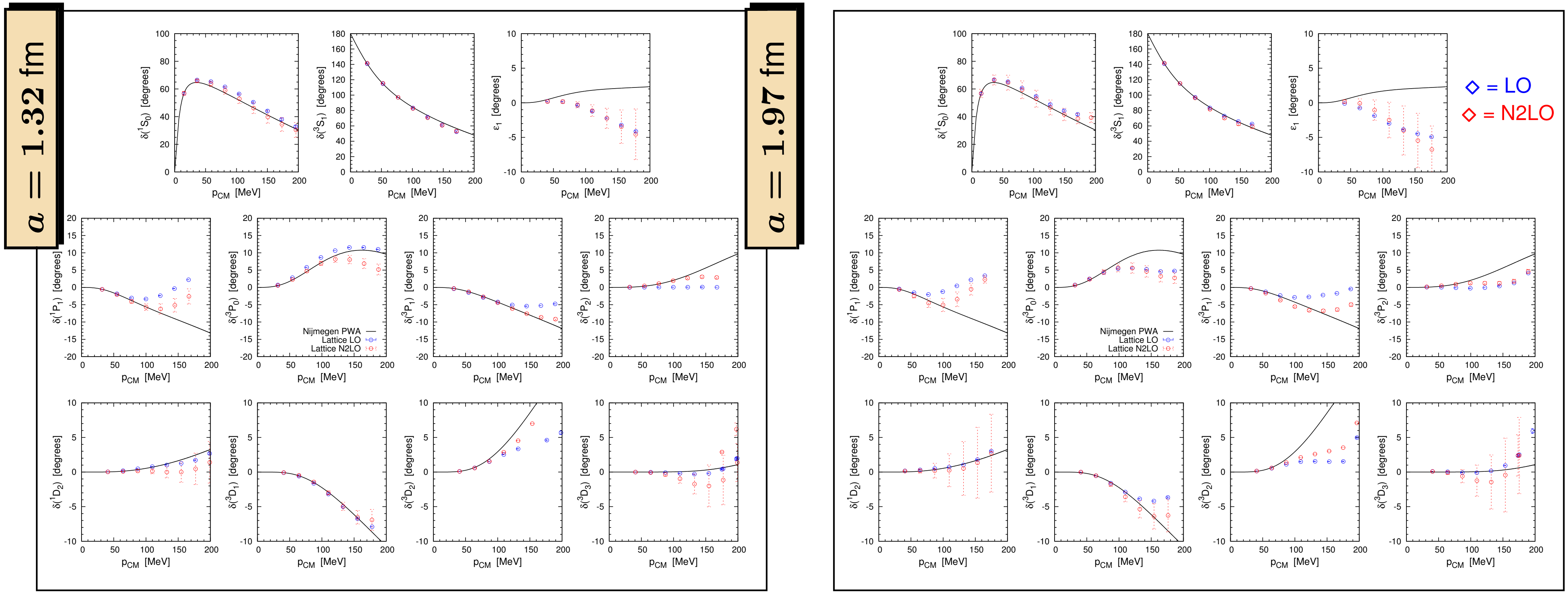
$\rightarrow$ fit b_s and two S-wave LECs C_i at LO up to $p_{\rm cm} = 100$ MeV

$\rightarrow$ w/ b_s fixed, fit two/seven S/P-wave LECs C_i at NLO/NNLO up to $p_{\rm cm} = 150$ MeV

$\rightarrow$ treat NLO and NNLO corrections perturbatively

NEUTRON–PROTON SCATTERING

- neutron-proton scattering



$\rightarrow$ up to $p_{\rm cm} \simeq 150$ MeV, physics largely independent of a $\surd$

$\rightarrow$ description consistent with the continuum within error bands $\surd$

$\rightarrow$ however: there are some differences in the P-waves description improves with decreasing a

$\rightarrow$ errors appear large due to recaling of χ^2

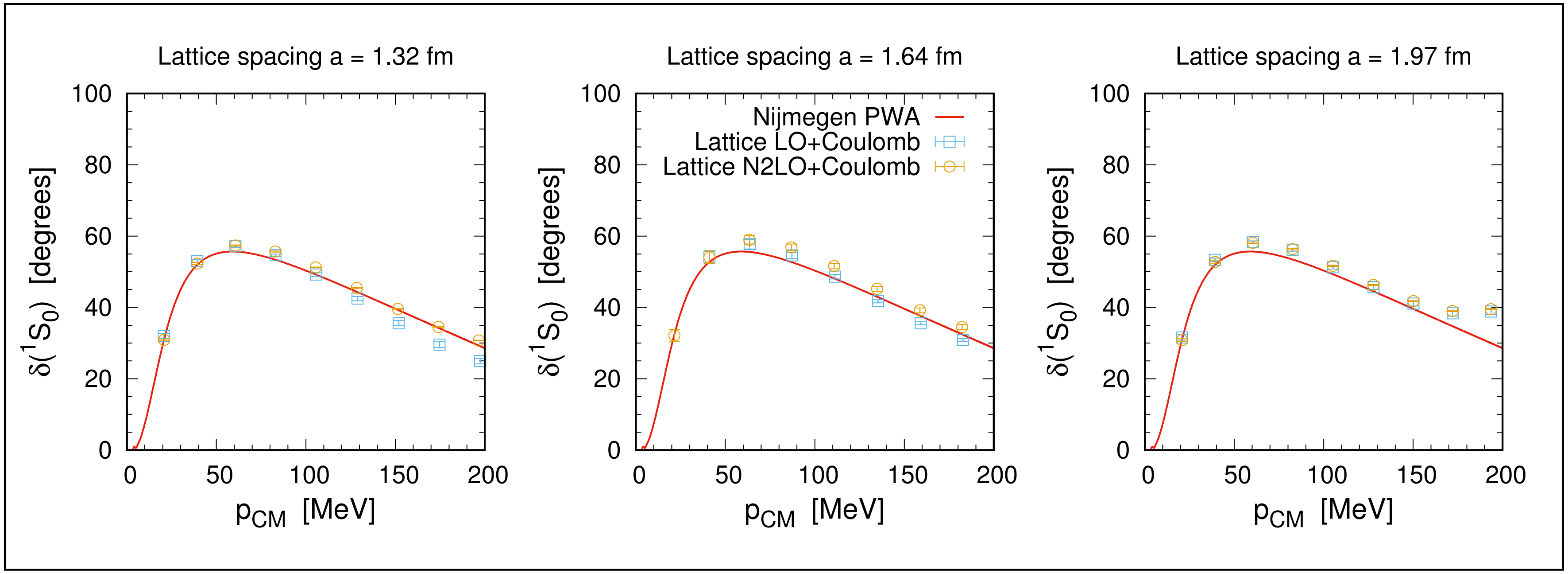
PROTON-PROTON SCATTERING

- proton-proton scattering

$\hookrightarrow$ only in $I = 1$, so $L + S$ must be even

$\hookrightarrow$ fit to Coulomb functions on the spherical wall at $R_{\rm wall} \simeq 28$ fm

$\hookrightarrow$ fit additional LEC to proton-proton scattering length



- somewhat too large for the coarse a, improving with decreasing a

- other phases only differ by Coulomb, very small effect

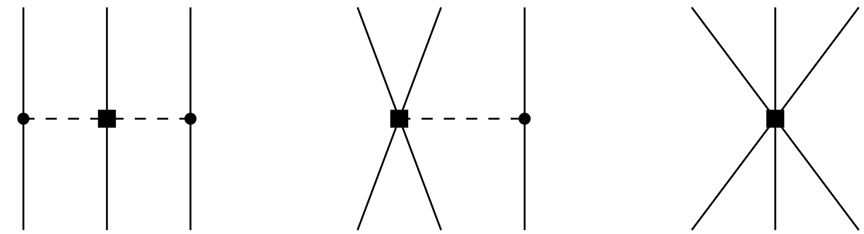
The three-body system at NNLO

Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. 104 (2010) 142501 [arXiv:0912.4195]

Klein, Elhatisari, Lähde, Lee, UGM [arXiv:1803.04231]

BASICS of the THREE NUCLEON SYSTEM

- At NNLO, the first non-vanishing 3NF appears



$c_{1,3,4}$ c_D c_E

- $c_{1,3,4}$ from RS analysis of $\pi N \to \pi N$

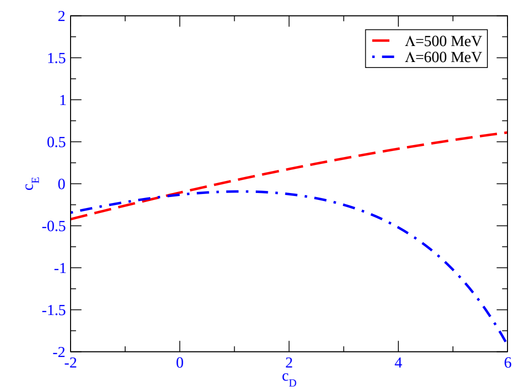
Hoferichter, Ruiz de Elvira, Kubis, UGM, Phys. Rev. Lett. 115 (2015) 092301

$\hookrightarrow$ longest range part determined from chiral symmetry

$\hookrightarrow$ corresponds to the renowned Fujita-Miyazawa force

Fujita, Miyazawa, Prog. Theor. Phys. 17 (1957) 360; UGM, AIP Conf. Proc. 1011 (2008) 49

- c_D and c_E are correlated



Epelbaum, Nogga, Glöckle, Kamada, UGM, Witala, Phys. Rev. C66 (2002) 064001

$\hookrightarrow$ fix $c_D = -0.79$ and determine c_E from the ^{3}H binding energy

CALCULATING the THREE-NUCLEON SYSTEM

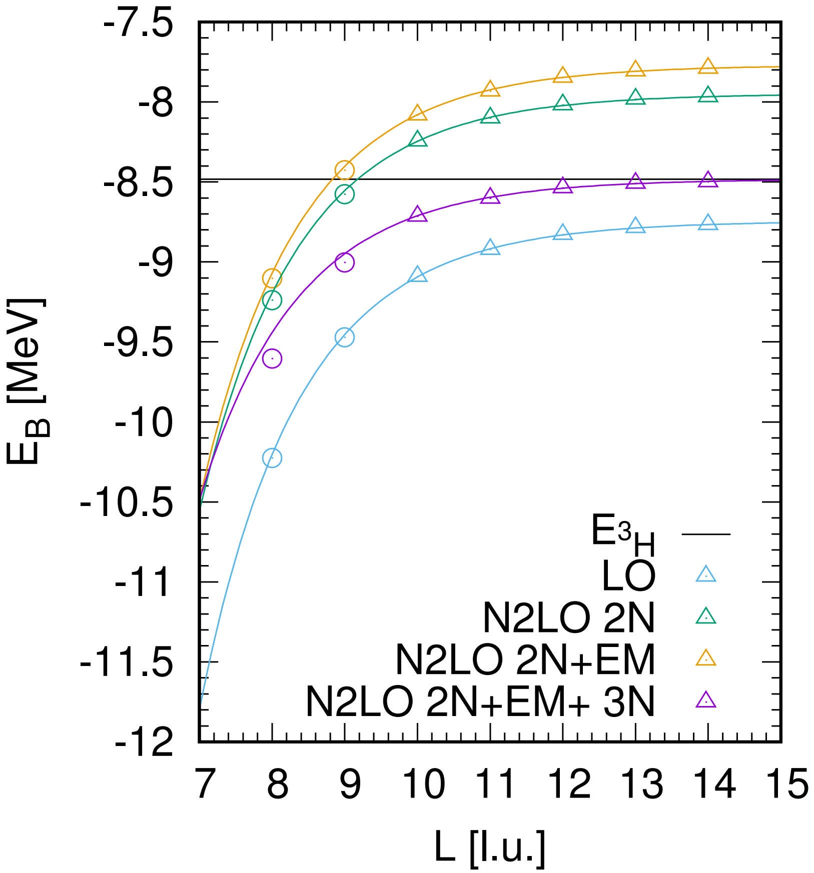
- can use MC methods or Lanzcos, here: Lanczos plus finite volume corrections
- no finite volume corrections for $V \simeq (10 \cdot 1.97\,\mathrm{fm})^3 \simeq (12 \cdot 1.64\,\mathrm{fm})^3 \simeq (20\,\mathrm{fm})^3$
- scaling as L^6, so for $a = 1.32$ fm, we must include finite volume corrections
- ground state energy:

$$E^{3\mathrm{N}}(L) = E^{3\mathrm{N}}_\infty + \mathcal{A}\frac{\exp\left(\frac{2\kappa L}{\sqrt{3}}\right)}{(\kappa L)^{\frac{3}{2}}}$$

$$\kappa = \sqrt{-mE^{3\mathrm{N}}_\infty}$$

- higher order corrections:

$$\langle \mathcal{O}_i \rangle (L) = \langle \mathcal{O}_i \rangle_\infty + \mathcal{A}_i \frac{\exp\left(\frac{2\kappa L}{\sqrt{3}}\right)}{(\kappa L)^{\frac{3}{2}}}$$



UGM, Rios, Rusetsky,
Phys. Rev. Lett. 114 (2015) 091602

Hammer et al., JHEP 1709 (2017) 109

RESULTS for the THREE-NUCLEON SYSTEM

- Triton binding energy at various orders:

	$a = 1.97$ fm	$a = 1.64$ fm	$a = 1.32$ fm
$E_{\rm LO}$ [MeV]	-7.80	-8.29	-8.74
$E_{\rm N2LO}$ [MeV]	$-7.846(4)$	$-8.11(2)$	$-7.95(2)$
$E_{\rm N2LO}^{\rm +EM}$ [MeV]	$-7.68(2)$	$-7.91(3)$	$-7.77(2)$
$E_{\rm N2LO}^{\rm +EM+3N}$ [MeV]	$-8.48(3)$	$-8.48(3)$	$-8.48(2)$
c_E	$0.5309(2)$	$0.3854(3)$	$1.0386(5)$

- different overbinding at LO can be mostly traced back to the ${}^3P_0$ wave
- N2LO w/o 3NF similar to phenomenological potentials
- c_E of natural size, that is of $\mathcal{O}(1)$

The four-body system at NNLO & the Tjon band

Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. 104 (2010) 142501 [arXiv:0912.4195];
Eur. Phys. J. A 45 (2010) 335 [arXiv:1003.5697]

Klein, Elhatisari, Lähde, Lee, UGM [arXiv:1803.04231]

CALCULATING the FOUR-NUCLEON SYSTEM

- must use MC methods $\longrightarrow$ correlation function:

$$Z_{^4\text{He}}(t) = \langle \psi_{^4\text{He}} \mid \exp(-tH_{\rm LO}) \mid \psi_{^4\text{He}} \rangle$$

- Transient energy: $E_{\rm LO}(t) = -\dfrac{d \log Z_{^4\text{He}}(t)}{dt}$

- preparation of initial states utilizing the Wigner SU(4) symmetry:

$$H_0 = H_{\rm free} + \frac{1}{2} C_0 \sum_{\vec{n}_1, \vec{n}_2} \underbrace{f(\vec{n}_1 - \vec{n}_2)}_{\text{Gaussian smearing}} \rho(\vec{n}_1)\, \rho(\vec{n}_2)$$

$\hookrightarrow$ create initial states close to the physical one: from the antisymm. free-particle solution:

$$|\psi_{^4\text{He}}\rangle = \exp(-t_0 H_0)\, |\psi_0\rangle$$

$\hookrightarrow$ leading order Hamiltonian:

$$H_{\rm LO} = H_{\rm free} + H_{\rm LO,contact} + H_{\rm OPE}$$

$\hookrightarrow$ higher orders as perturbative corrections:

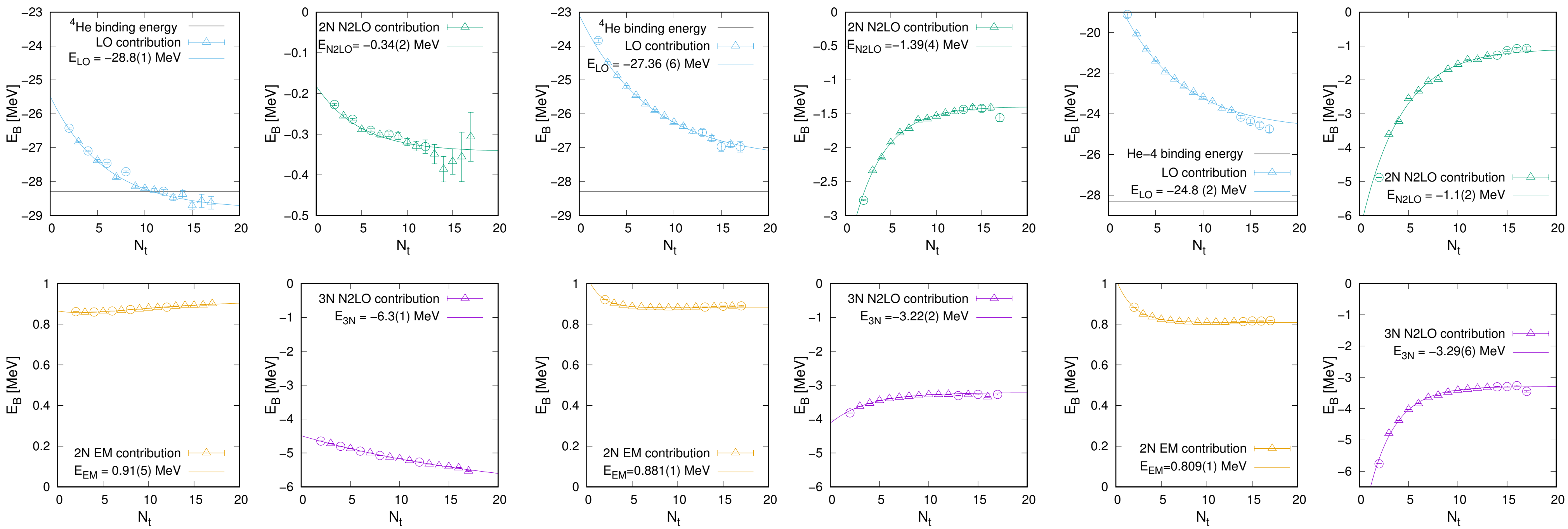
$$Z_{\mathcal{O}}(t) = \langle \psi_{^4\text{He}} \mid \exp\left(\frac{-tH}{2}\right) \mathcal{O} \exp\left(\frac{-tH}{2}\right) \mid \psi_{^4\text{He}} \rangle$$

$$\langle \mathcal{O} \rangle (t) = \frac{Z_{\mathcal{O}}(t)}{Z_{^4\text{He}}(t)}$$

RESULTS for the FOUR-NUCLEON SYSTEM

- Time extrapolations for various lattice spacings

$a = 1.97$ fm $\qquad a = 1.64$ fm $\qquad a = 1.32$ fm



- agrees with the earlier calculation at $a = 1.97$ fm, much faster $\surd$

Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. 104 (2010) 142501, Eur. Phys. J. A 45 (2010) 335

RESULTS for the FOUR-NUCLEON SYSTEM

- ^{4}He binding energy at various orders for various lattice spacings

	$a = 1.97$ fm	$a = 1.64$ fm	$a = 1.32$ fm
$E_{\rm LO}$	$-28.81(11)$	$-27.36(6)$	$-24.81(19)$
$E_{\rm N2LO}$	$-29.15(11)(3)$	$-28.75(7)(5)$	$-25.89(27)(3)$
$E_{\rm N2LO}^{\rm +EM}$	$-28.23(12)(3)$	$-27.87(7)(6)$	$-25.08(27)(3)$
$E_{\rm N2LO}^{\rm +EM+3N}$	$-34.55(18)(3)$	$-31.09(7)(6)$	$-28.37(28)(3)$

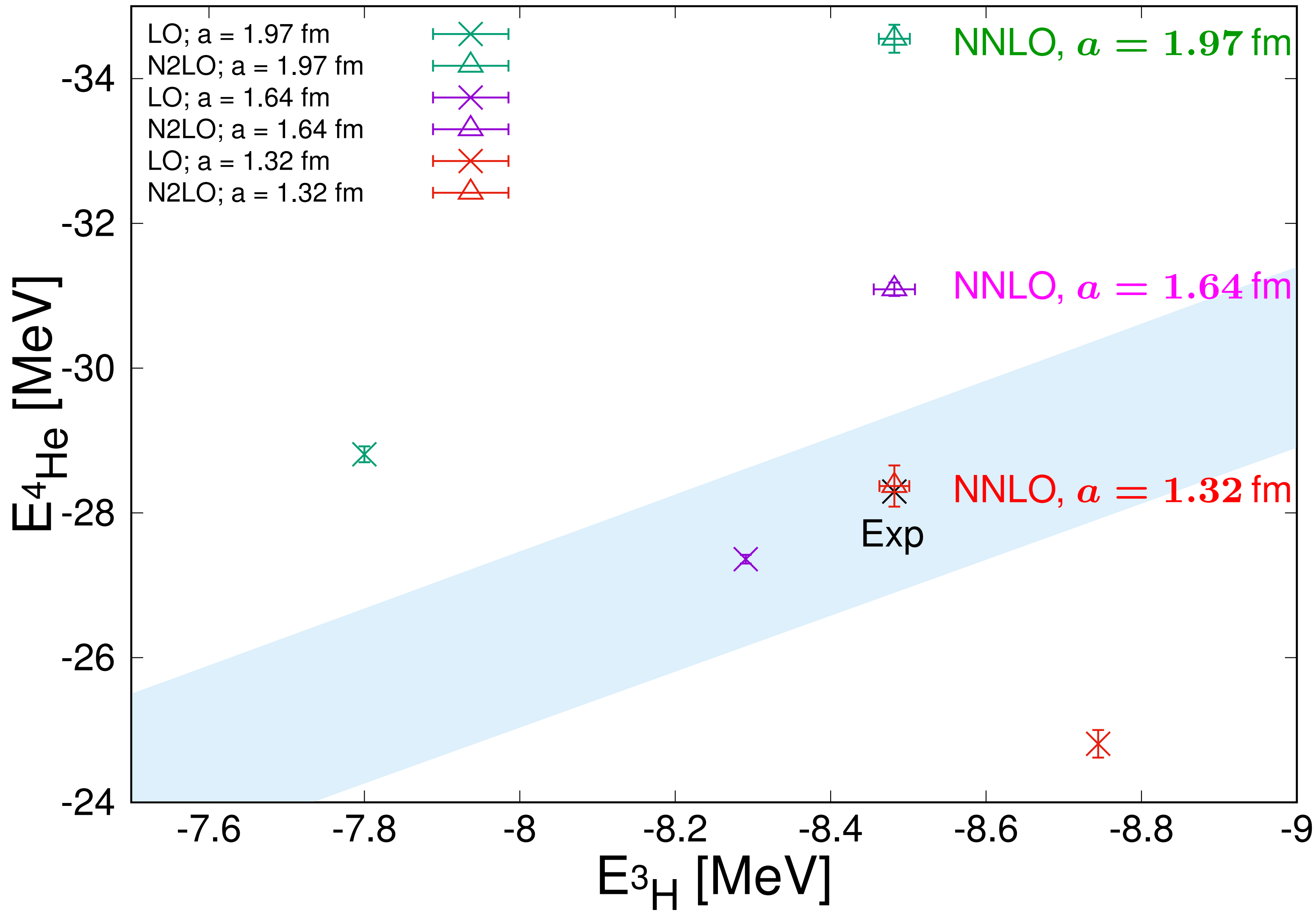
$\hookrightarrow$ revover the overbinding for the coarse lattice already found in 2010

$\hookrightarrow$ overbinding decreases with with decreasing lattice spacing

$\hookrightarrow$ look at the Tjon band plot

RESULTS for the FOUR-NUCLEON SYSTEM cont'd

- Tjon band plot



DISCUSSION

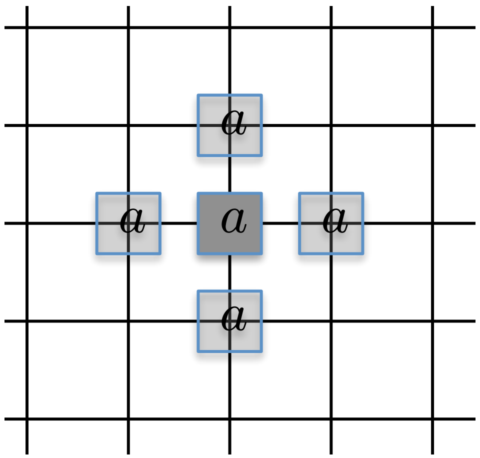
- Deviation from the Tjon band confirmed for the coarse lattice spacing $a = 1.97$ fm

- Deviation decreases with decreasing a,
  perfect agreement for the smallest $a = 1.32$ fm

$\hookrightarrow$ no more need for four-nucleon interactions for these smaller values of a

$\hookrightarrow$ intrinsic problem of the standard action resolved $\surd$

- how about improved actions and or higher orders?



non-local smearing: mimimizes remaining

sign oscillations / higher-body forces

Elhatisari et al., Phys. Rev. Lett. 117 (2016) 132501

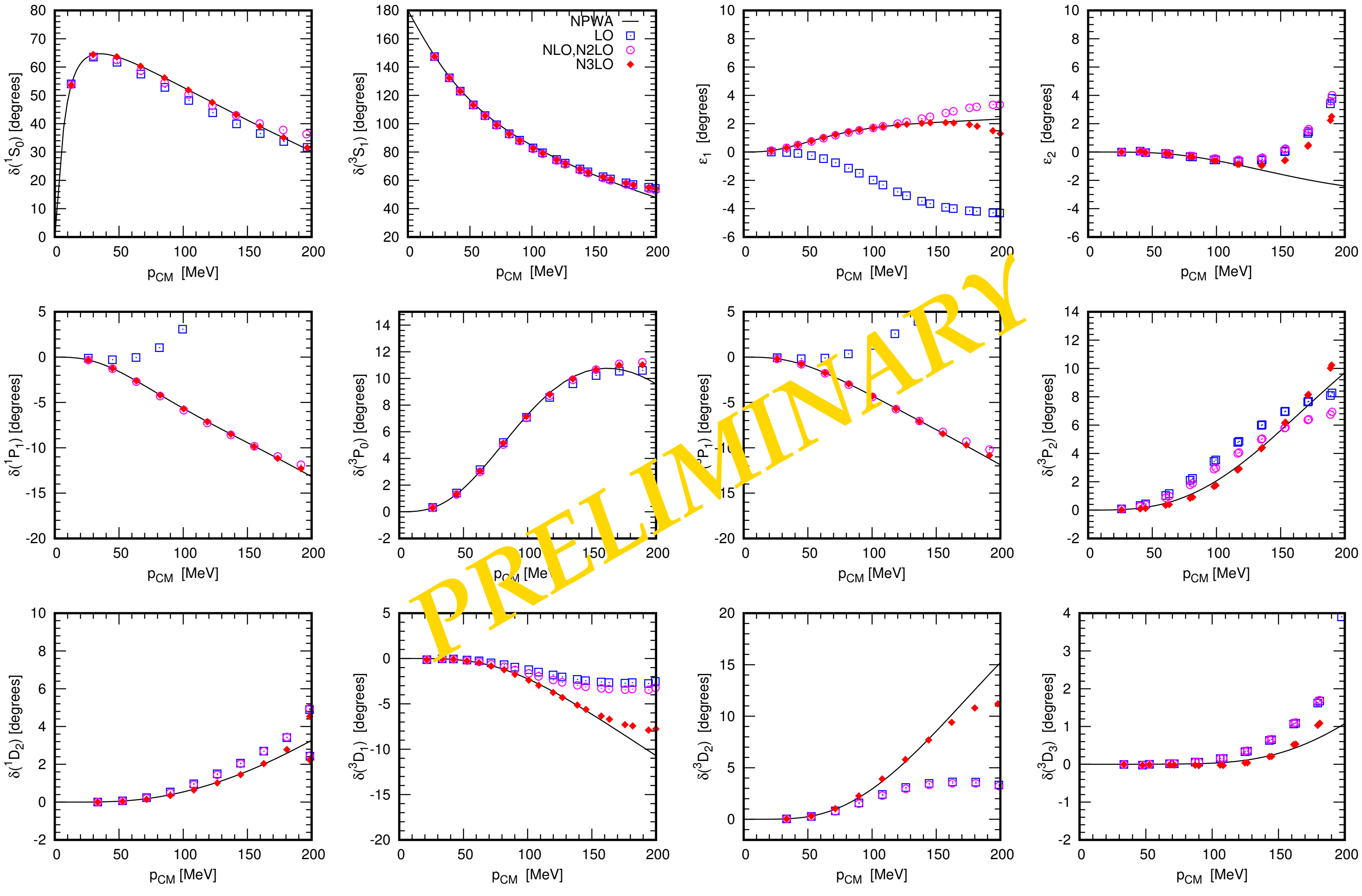
N3LO is required for a better precision

$\rightarrow$ better 2NFs and 3NFs

Li, Lu, ... et al., in preparation

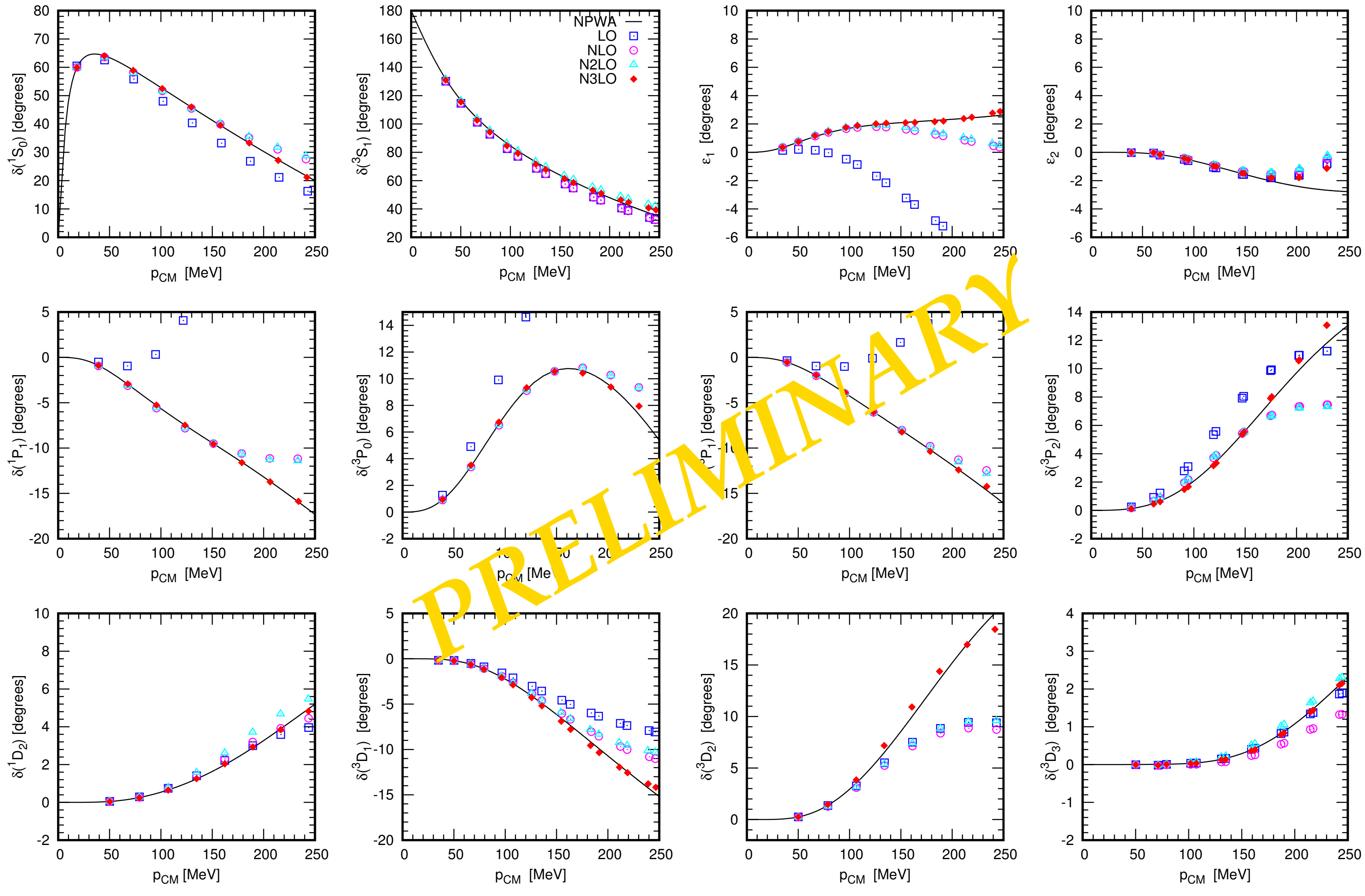
SCATTERING at N3LO

- coarse lattice $a = 1.97$ fm, TPE absorbed in the local operators



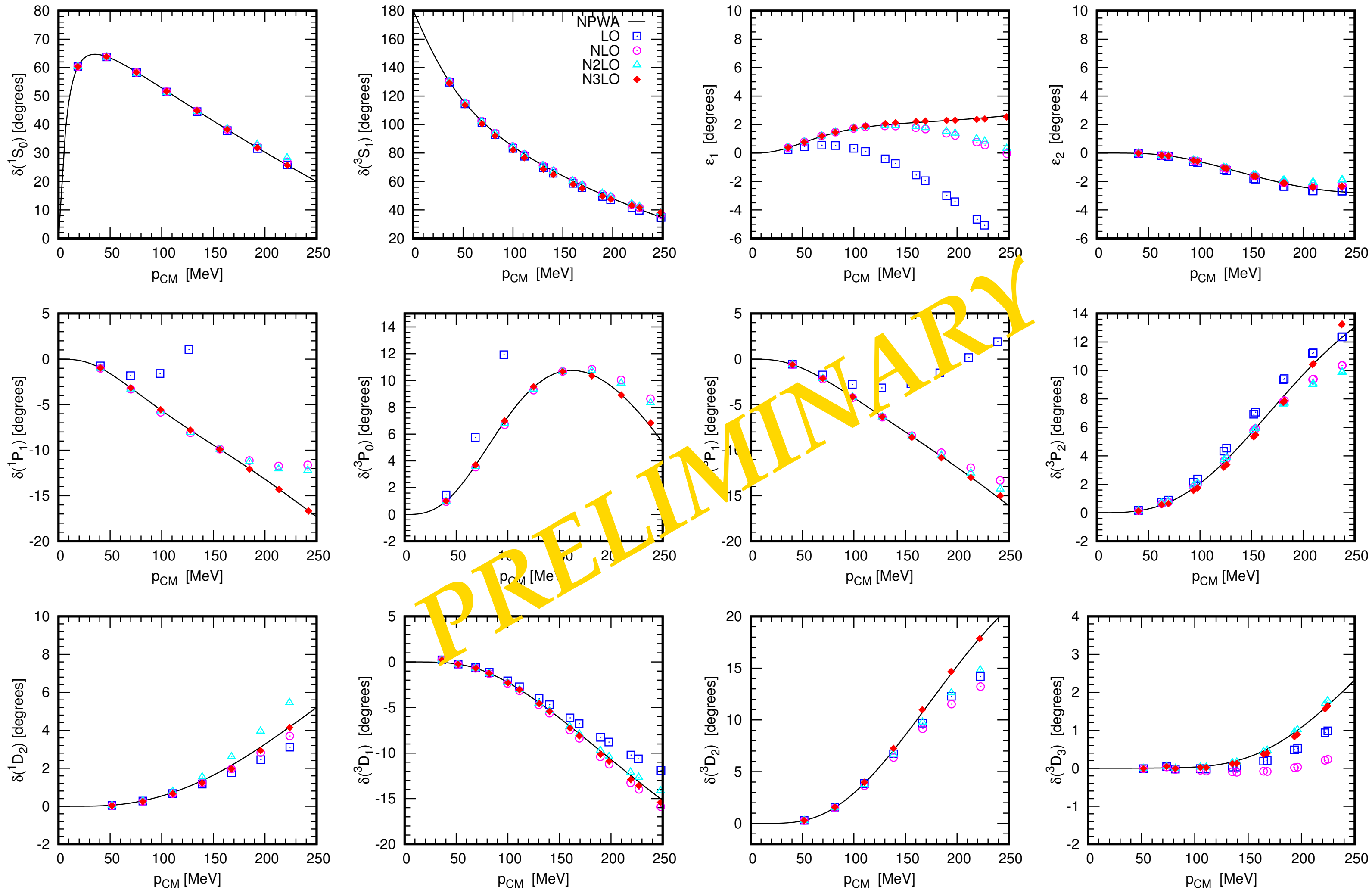
SCATTERING at N3LO

- finer lattice $a = 1.32$ fm, TPE fully included



SCATTERING at N3LO

- very fine lattice $a = 0.98$ fm, TPE fully included



DISCUSSION and OUTLOOK

- All phases well described, no more differences for the various a values

$\hookrightarrow$ Calculations of the binding energies of ^{3}H and ^{4}He will recover the Tjon band

- Upcoming papers:

  Neutron-proton scattering at N3LO

  Light and medium-mass nuclei at N3LO up $A = 30$ with N3LO forces

  Excited spectrum of nuclei using the pinhole algorithm

  How nuclei boil: Simulations of nuclei at nonzero temperature

$\hookrightarrow$ with increased precision for the nuclear forces,
many exciting results in nuclear structure & reactions to come

SPARES

NUCLEAR FORCES: OPEN ENDS

- Why is there this hierarchy $\boxed{V_{2N} \gg V_{3N} \gg V_{4N}}$?
- Gauge and chiral symmetries difficult to include (meson-exchange currents)

Brown, Riska, Gari, . . .

- Connection to QCD ?

most models have one-pion-exchange, but not necessarily respect chiral symmetry

some models have two-pion exchange reconstructed via dispersion relations from $\pi N \to \pi N$

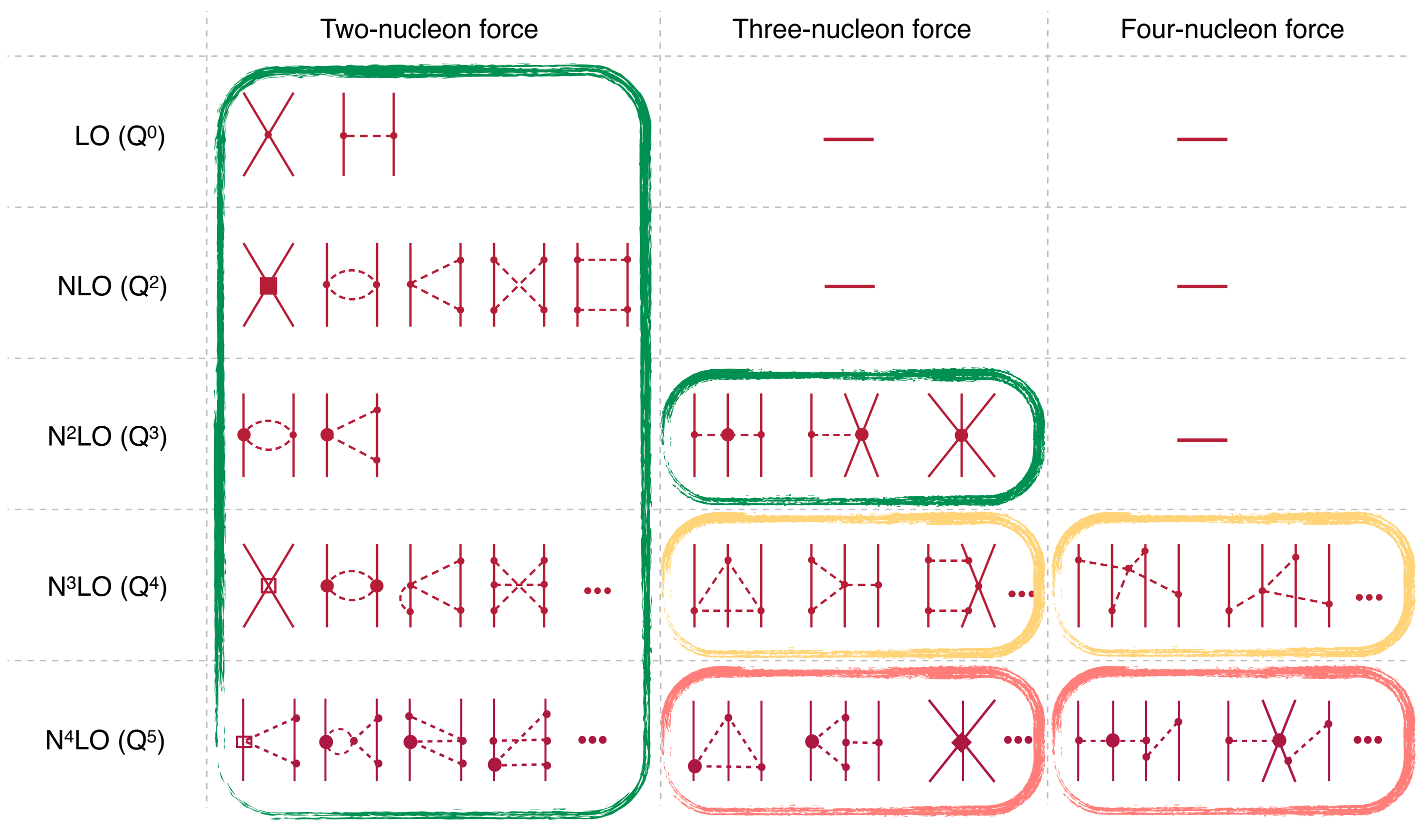
⇒ We want an approach that

– is linked to QCD via its symmetries
– allows for systematic calc's with a controlled theoretical error
– explains the observed hierarchy of the nuclear forces
– matches nucleon structure to nuclear dynamics
– allows for a lattice formulation / chiral extrapolations
– puts nuclear physics on a sound basis

NUCLEAR FORCES in CHIRAL NUCLEAR EFT

- expansion of the potential in powers of Q [small parameter]: $\{p/\Lambda_b,\, M_\pi/\Lambda_b\}$
- explains observed hierarchy of the nuclear forces
- extremely successful in few-nucleon systems

Epelbaum, Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773

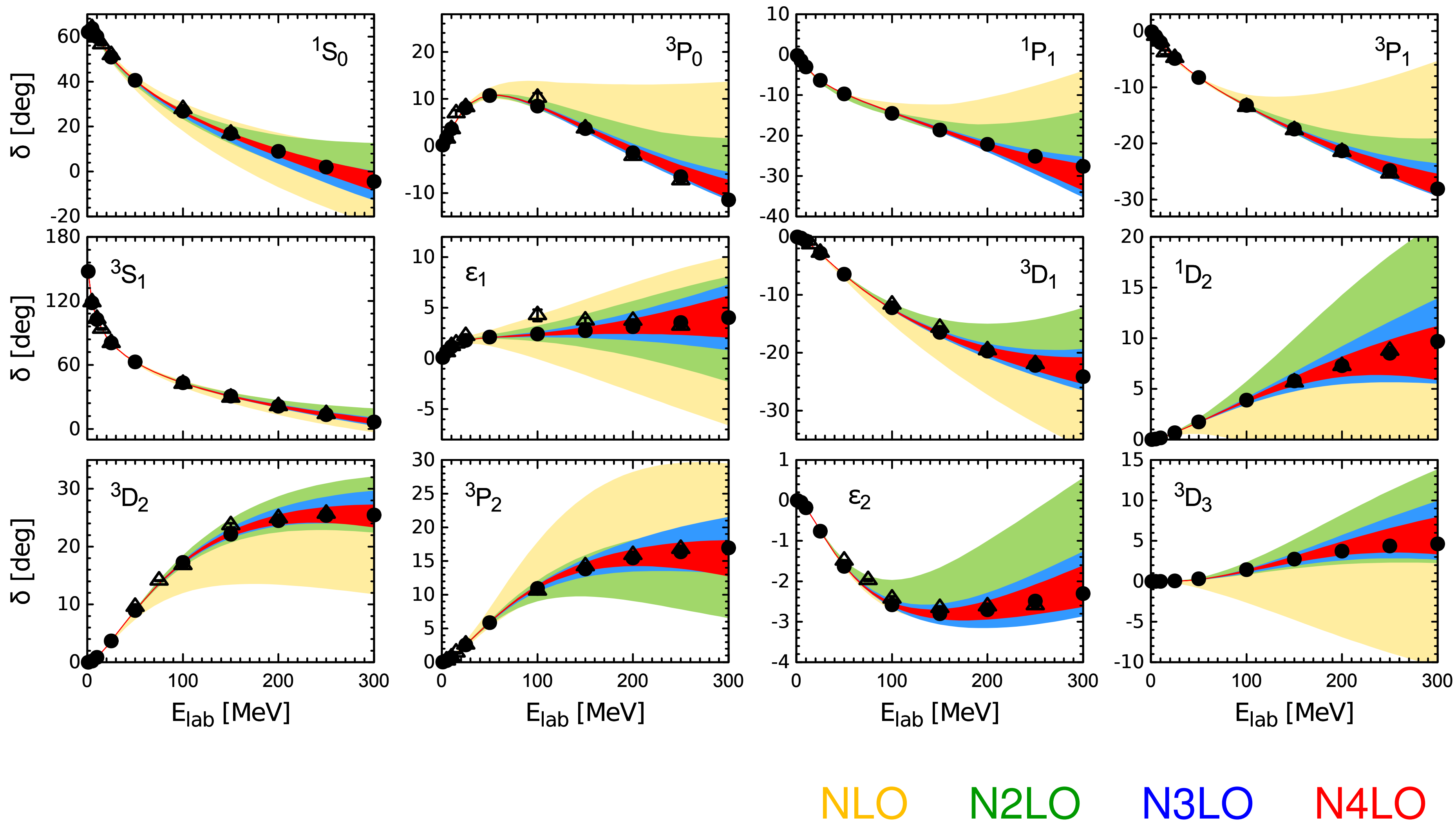


worked out and applied   worked out and to be applied   calculations in progress

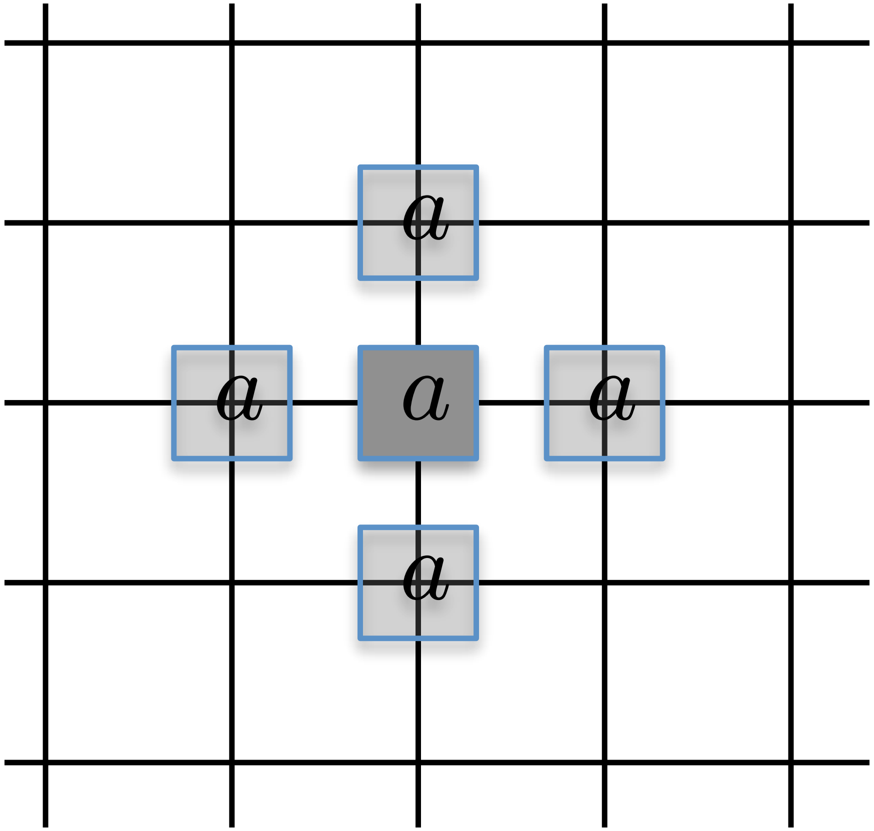
PHASE SHIFTS at N4LO

⇒ Precision phase shifts with small uncertainties up to $E_{\mathrm{lab}} = 300$ MeV

Epelbaum, Krebs, UGM, Phys. Rev. Lett. **115** (2015) 122301



LOCAL/NON–LOCAL INTERACTIONS on the LATTICE



- Local operators/densities:

$a(\mathbf{n}), a^\dagger(\mathbf{n})$ [$\mathbf{n}$ denotes a lattice point]

$$\rho_{\rm L}(\mathbf{n}) = a^\dagger(\mathbf{n}) a(\mathbf{n})$$

- Non-local operators/densities:

$$a_{\rm NL}(\mathbf{n}) = a(\mathbf{n}) + s_{\rm NL} \sum_{\langle \mathbf{n}'\,\mathbf{n}\rangle} a(\mathbf{n}')$$

$$a^\dagger_{\rm NL}(\mathbf{n}) = a^\dagger(\mathbf{n}) + s_{\rm NL} \sum_{\langle \mathbf{n}'\,\mathbf{n}\rangle} a^\dagger(\mathbf{n}')$$

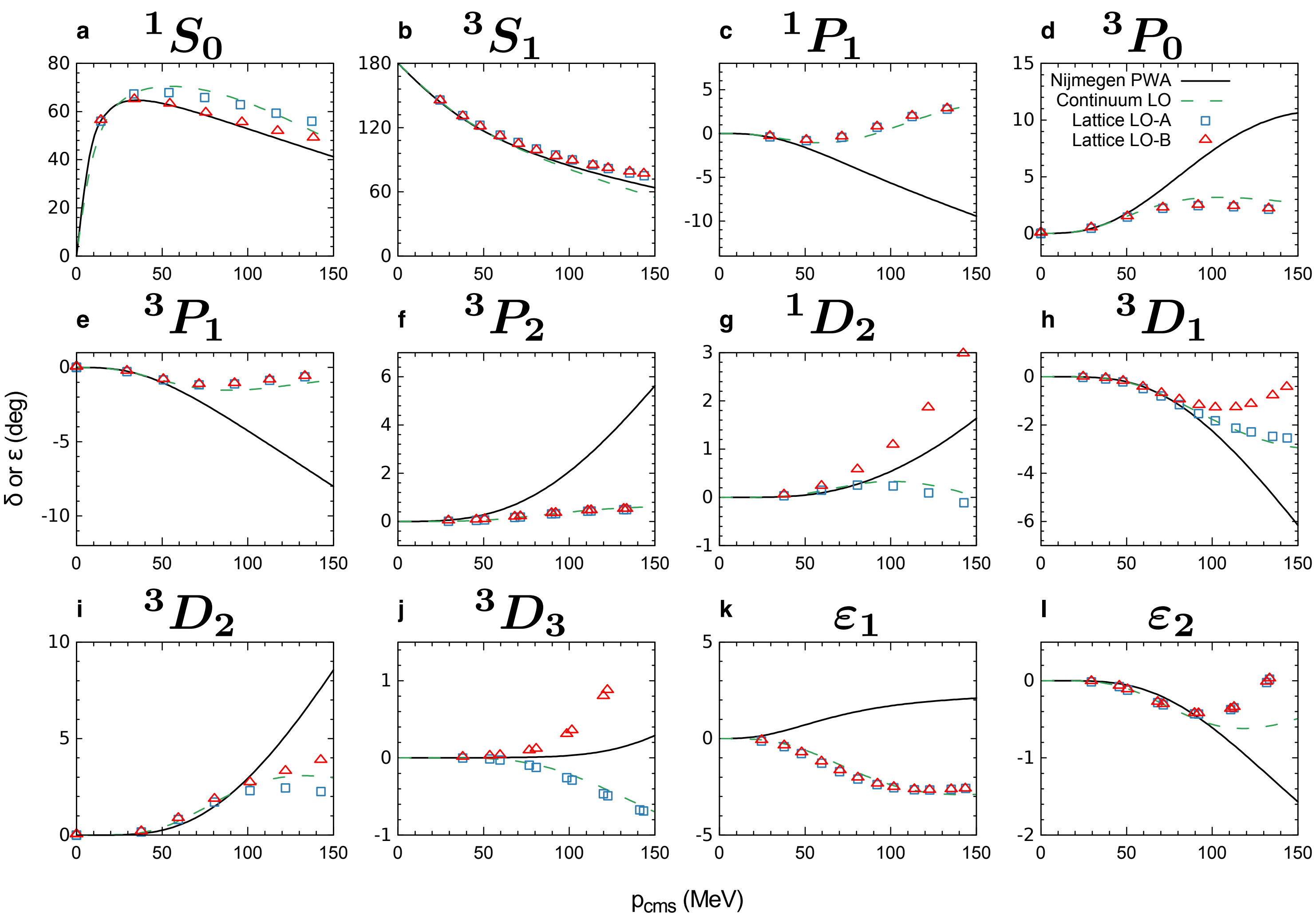
$$\rho_{\rm NL}(\mathbf{n}) = a^\dagger_{\rm NL}(\mathbf{n}) a_{\rm NL}(\mathbf{n})$$

$\rightarrow$ where $\sum_{\langle \mathbf{n}'\,\mathbf{n}\rangle}$ denotes the sum over nearest-neighbor lattice sites of $\mathbf{n}$

$\rightarrow$ the smearing parameter $s_{\rm NL}$ is determined when fitting to the phase shifts

NUCLEON–NUCLEON PHASE SHIFTS

- Show results for NN [and α-α] phase shifts for both interactions:



→ both interactions very similar