NUCLEAR PHYSICS as PRECISION SCIENCE

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- Continuum physics
- Lattice physics
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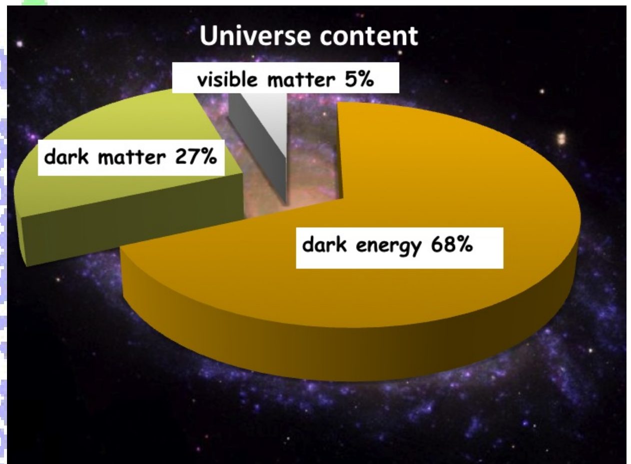
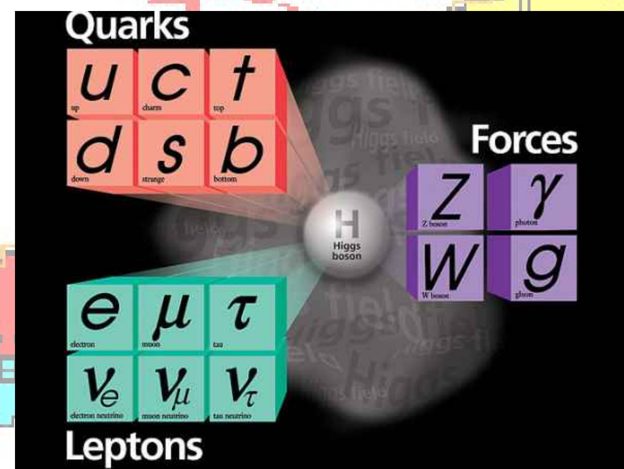
Introduction

WHY NUCLEAR PHYSICS?

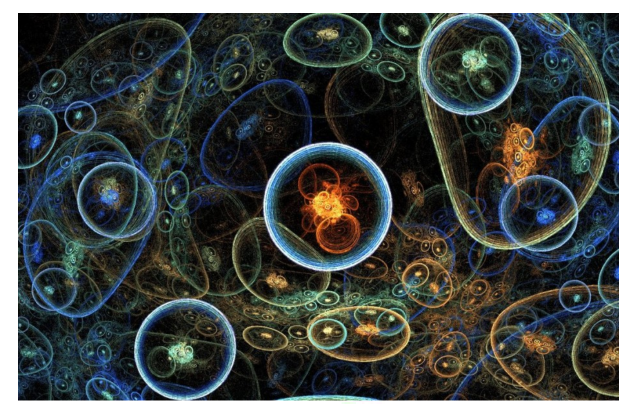
- The matter we are made off



- The last frontier of the SM



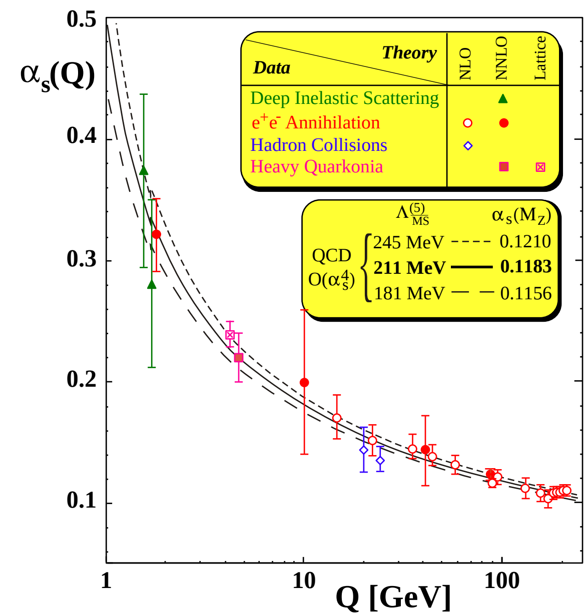
- Access to the Multiverse



EMERGENCE of STRUCTURE in QCD

- The strong interactions are described by QCD:

$$\mathcal{L} = -\frac{1}{4g^2} G_{\mu\nu} G^{\mu\nu} + \sum_{f=u,d,s,c,b,t} \bar{q}_f \left(i\gamma_\mu D^\mu - m_f \right) q_f + \ldots$$



- **up** and **down** quarks are very light, a few MeV

- Quarks and gluons are confined within **hadrons**

  $\hookrightarrow$ playground of lattice QCD

- Protons and neutrons form **atomic nuclei**

$\Rightarrow$ This requires the inclusion of electromagnetism described by QED with $\alpha_{\rm EM} \simeq 1/137$ [+ weak int.]

- How can one describe nuclei *ab initio*? $\rightarrow$ chiral EFT
- How sensitive are these strongly interacting composites to variations of the fundamental parameters of QCD+QED?

NUCLEAR CHIRAL EFFECTIVE FIELD THEORY

- The silver jubilee of Weinberg's work extending chiral EFTs to nuclear physics

S. Weinberg,
"Nuclear forces from chiral Lagrangians,"
Phys. Lett. B **251** (1990) 288 [submitted 14 August 1990].
954 citations counted in INSPIRE as of 09 November 2015

S. Weinberg,
"Effective chiral Lagrangians for nucleon - pion interactions and nuclear forces,"
Nucl. Phys. B **363** (1991) 3 [submitted 02 April 1991].
915 citations counted in INSPIRE as of 09 November 2015

- after 25 years, a mature field? Epelbaum, Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773 [534 cites]

- yes *and* no $\rightarrow$ let's discuss some recent developments

Continuum chiral EFT physics

LENPIC

The NUCLEAR HAMILTONIAN

- Nucleons in nuclei are slow-moving particles, binding momenta $\sim M_\pi$

$$\left(-\sum_{i=1}^{A} \frac{\nabla_i^2}{2m_N} + V \right) |\Psi\rangle = E|\Psi\rangle \,, \qquad V = V_{NN} + V_{NNN} + \ldots$$

- The nuclear potential V classically build from meson exchange models

- Weinberg's idea: use chiral perturbation theory to construct V

  ↪ direct link to QCD via symmetries and their breakings

  ↪ systematic approach that can be improved order-by-order

  ↪ allows for a consistent calculation of two-, three- and four-body forces

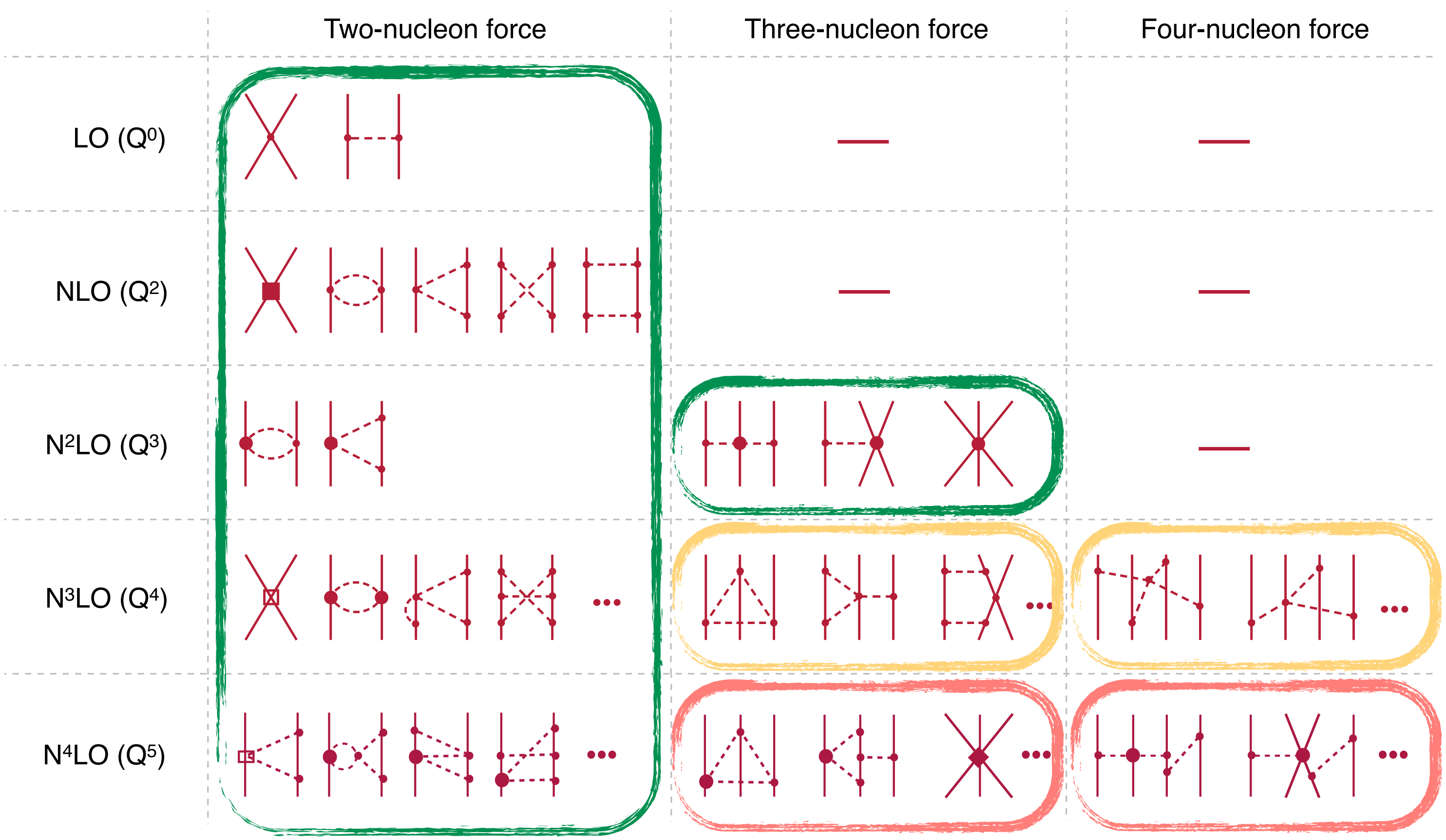
  ↪ allows for a consistent calculation of forces and currents

  ↪ systematic approach that allows for uncertainty quantifications

  ↪ gives access to the multiverse

NUCLEAR FORCES in CHIRAL NUCLEAR EFT

- expansion of the potential in powers of Q [small parameter]
- explains observed hierarchy of the nuclear forces



worked out and applied   worked out and being applied   calculations in progress

NN FORCES to FOURTH ORDER

Epelbaum, Krebs, UGM, Eur. Phys. J. **A 51**: 53 (2015)

- so far: momentum space cut-off regularization, works but not ideal
- new regularization of long-range physics [coordinate space cut-off]:

$$V_{\rm long-range}^{\rm reg}(\vec{r}) = V_{\rm long-range}(\vec{r}) f_{\rm reg}\left(\frac{r}{R}\right) \ , \quad f_{\rm reg} = \left[1 - \exp\left(-\frac{r^2}{R^2}\right)\right]^6$$

$\Longrightarrow$ No distortion of the long-range potential $\to$ better at higher energies

$\Longrightarrow$ Study of the chiral expansion of multi-pion exchanges: $R = 0.8 \cdots 1.2$ fm

Baru et al., EPJ A48 (12) 69

- new way of estimation the theoretical uncertainty [before: only cut-off variations]

$\Longrightarrow$ Expansion parameter depending on the region: $Q = \max\left(\frac{M_\pi}{\Lambda_b}, \frac{p}{\Lambda_b}\right)$

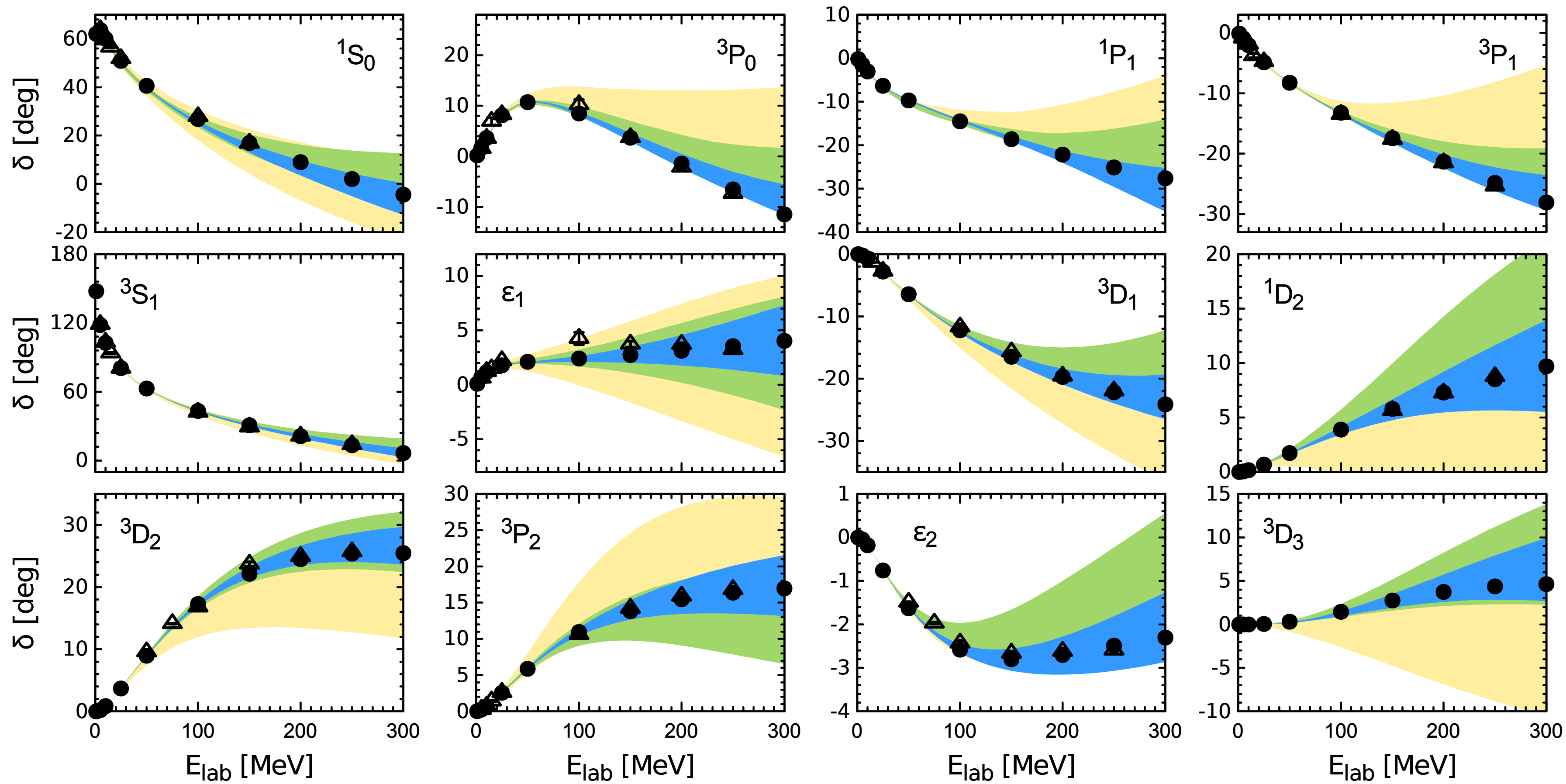
$\Longrightarrow$ Breakdown scale $\Lambda_b = 600$ MeV for $R = 0.8 \cdots 1.0$ fm

NN FORCES at FOURTH ORDER

- clear improvement comp. to earlier N^3LO potentials [momentum space reg.]

Entem, Machleidt; Epelbaum, Glöckle, UGM

- uncertainties show expected pattern



NLO N^2LO N^3LO

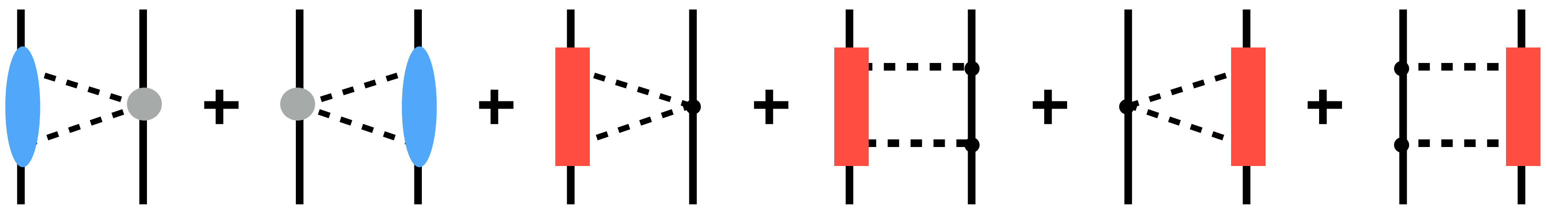
NN FORCES to FIFTH ORDER

Epelbaum, Krebs, UGM, Phys. Rev. Lett. **115** (2015) 122301 [arXiv:1412.4623]

- No contact interactions at this order - odd in Q

- New contributions fixed from πN scattering, LECs c_i, d_i, e_i:

Büttiker, Fettes, UGM, Steininger (1998-2000); Krebs, Gasparian, Epelbaum (2012)



$$\mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)}(c_i) + \mathcal{L}_{\pi N}^{(3)}(d_i) + \mathcal{L}_{\pi N}^{(4)}(e_i)$$

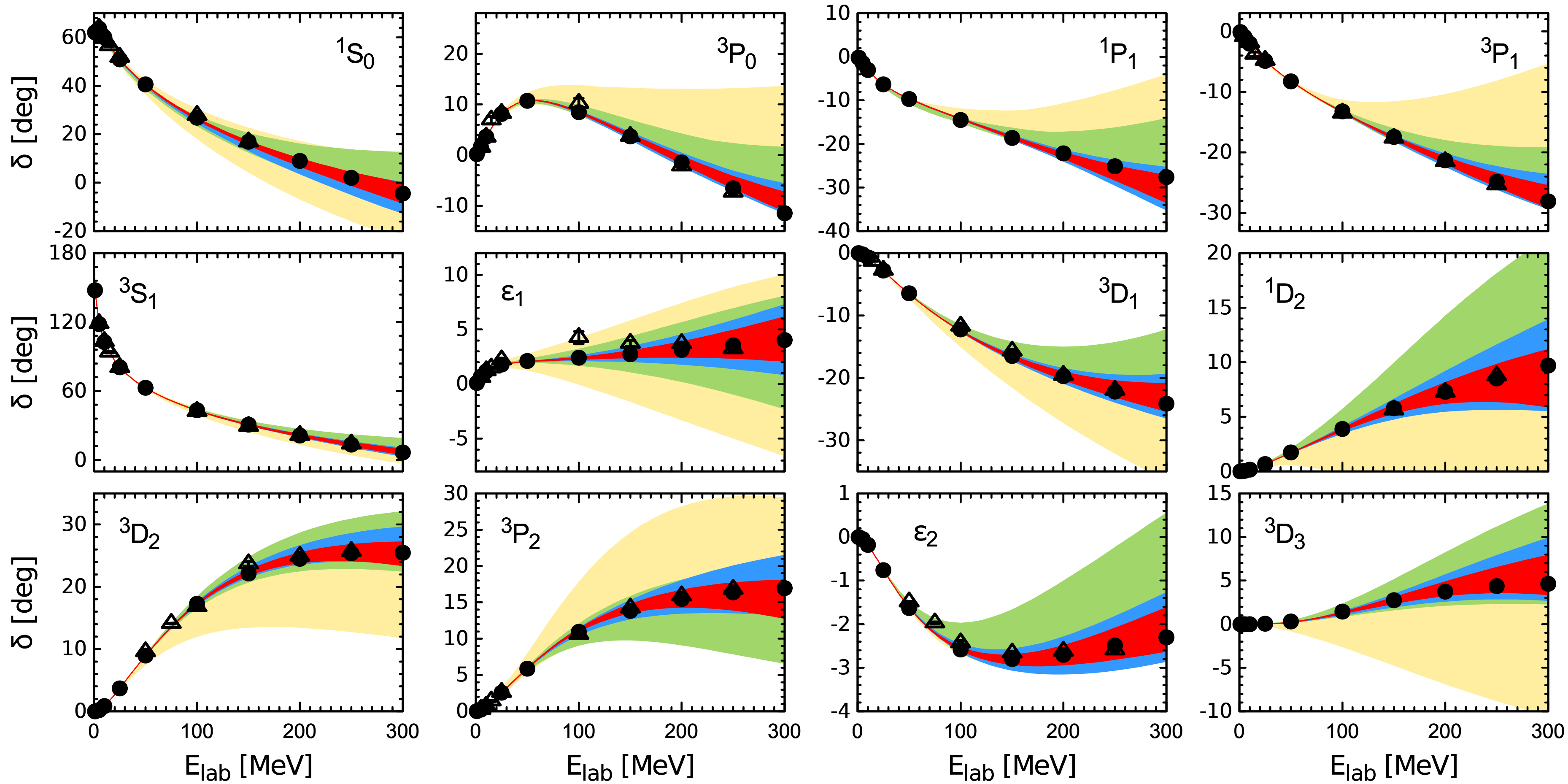
- Three-pion exchange can be neglected

  $\to$ explicit calculation of the dominant NLO contribution — Kaiser (2001)

  $\to$ no influence on phase shifts or deuteron properties

PHASE SHIFTS at N^4LO

$\Rightarrow$ Precision phase shifts with small uncertainties up to $E_{\rm lab} = 300$ MeV

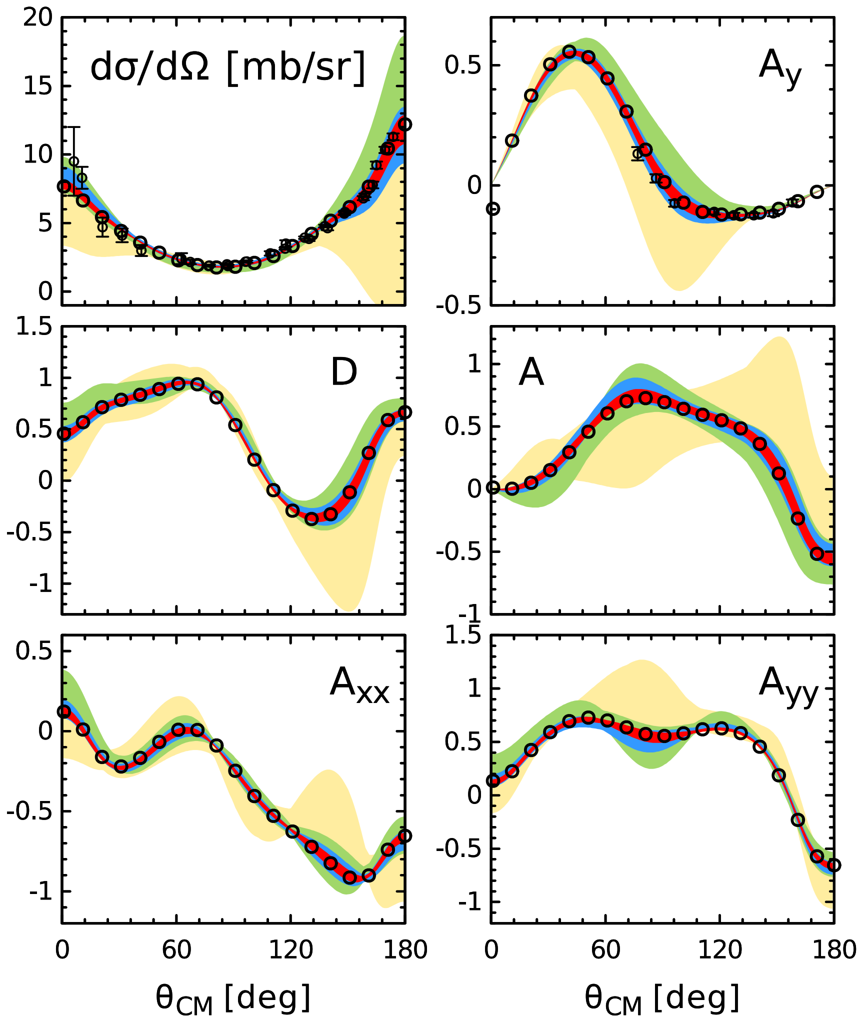


NLO N^2LO N^3LO N^4LO

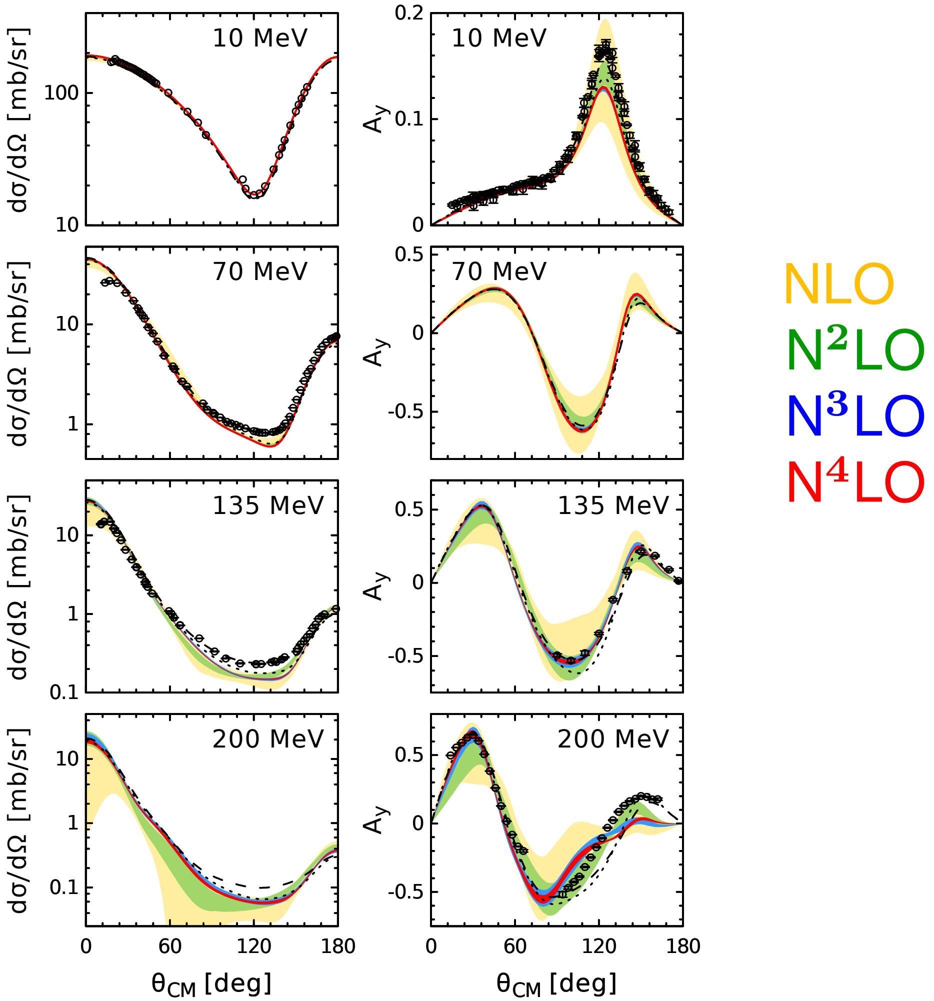
EVIDENCE for THREE-NUCLEON FORCES

- Two-nucleon system under control, three-nucleon system requires 3NFs!
  → being implemented [LENPIC collaboration]

- np scattering at 200 MeV



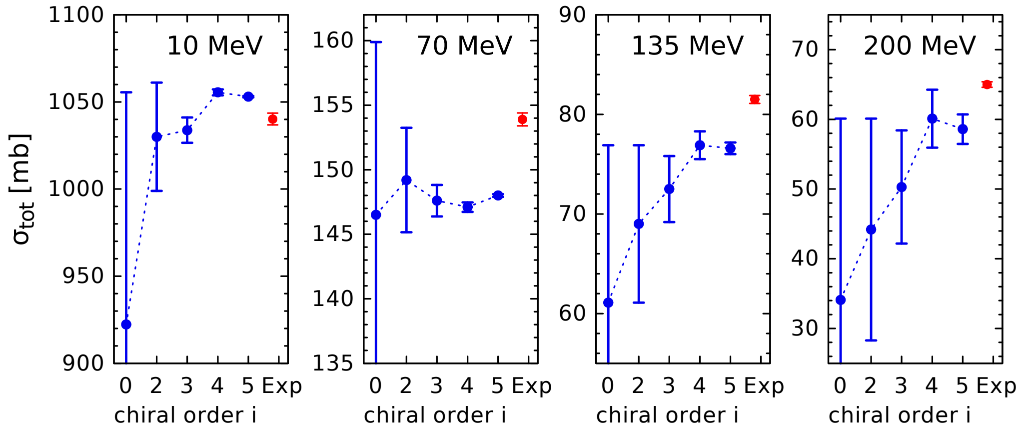
- nd scattering [2NFs only]



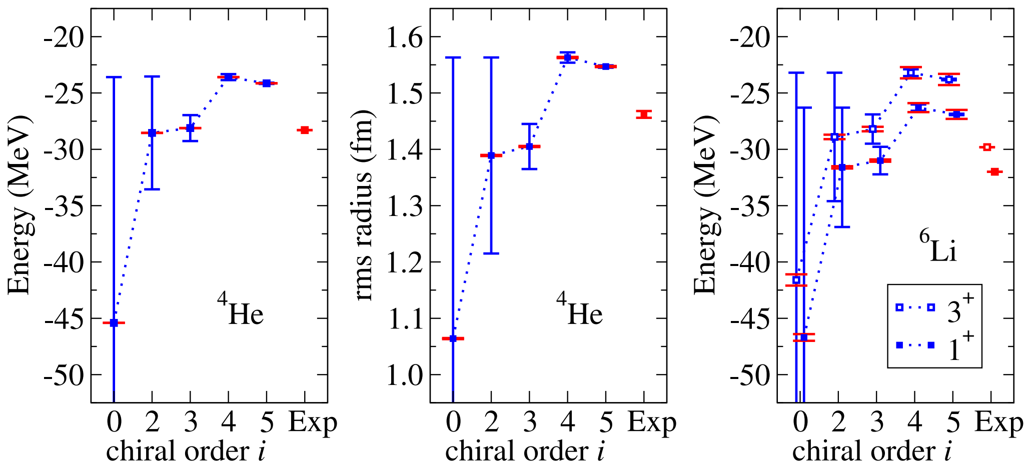
MORE EVIDENCE for THREE-NUCLEON FORCES

Binder et al. [LENPIC collaboration], arXiv:1505.07218

- Total cross section for Nd scattering [2NFs only]



- Binding energy and rms radius of ^{4}He, lowest levels in ^{6}Li [2NFs only]



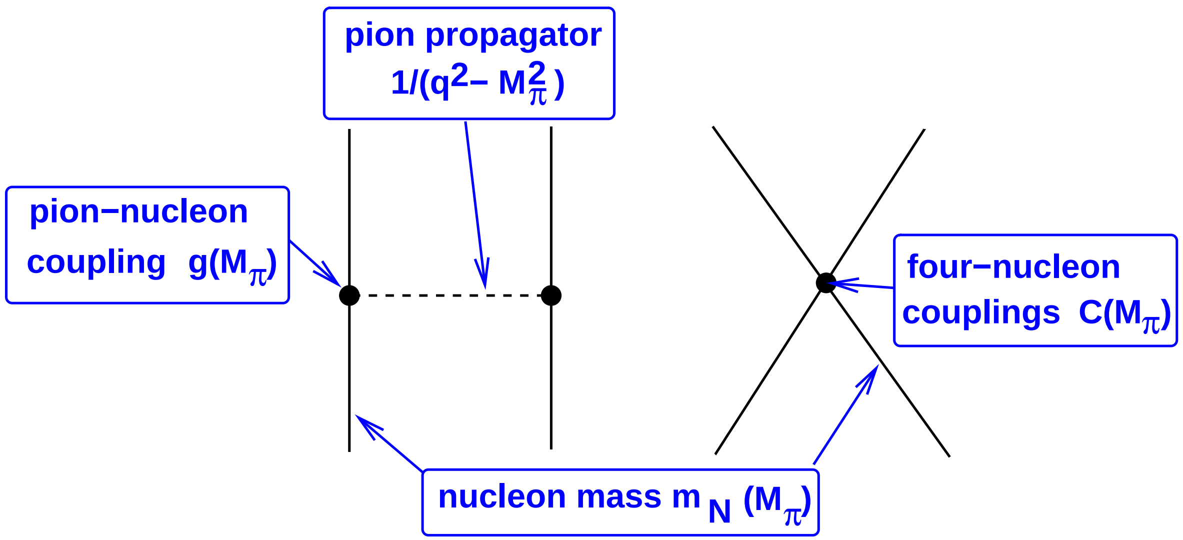
Quark mass dependence of the nuclear forces and impact on BBN

Berengut, Epelbaum, Flambaum, Hanhart, UGM, Nebreda, Pelaez,
Phys. Rev. D **87** (2013) 085018

INGREDIENTS

- Nuclear forces: Pion-exchange contributions & short-distance multi-N operators

- graphical representation of the quark mass dependence of the LO potential



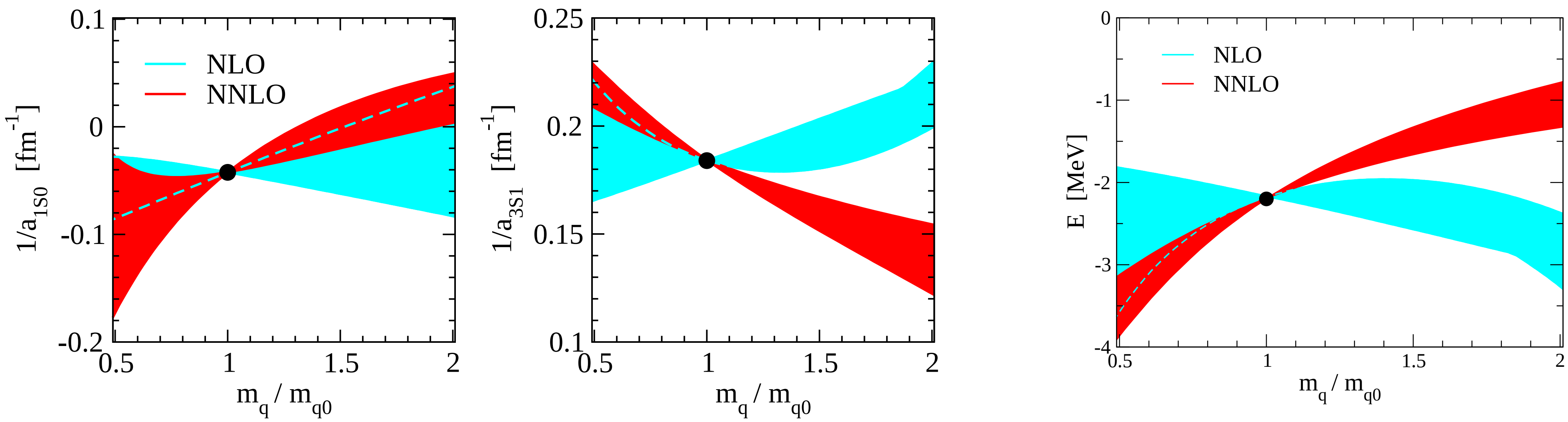
- always use the Gell-Mann–Oakes–Renner relation: $M^2_{\pi^\pm} \sim (m_u + m_d)$

- fulfilled in QCD to better than 94% Colangelo, Gasser, Leutwyler 2001

$\Rightarrow$ Quark mass dependence of hadron properties from lattice QCD, contact interaction require modelling

RESULTS for the NN SYSTEM

- Putting pieces together for the two-nucleon system:



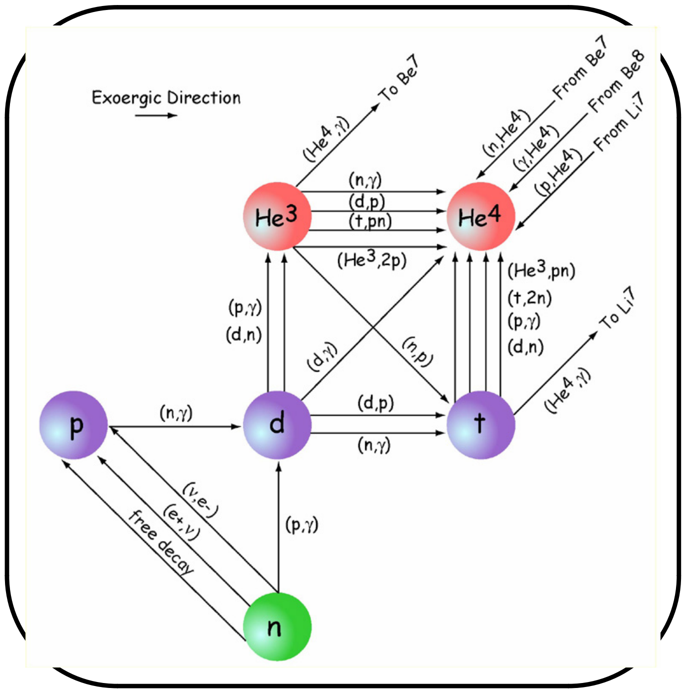
- uncertainties mostly due to modelling the contact interactions

  ↪ contact to lattice QCD required — Baru et al.,Phys.Rev. C92 (2015) 014001

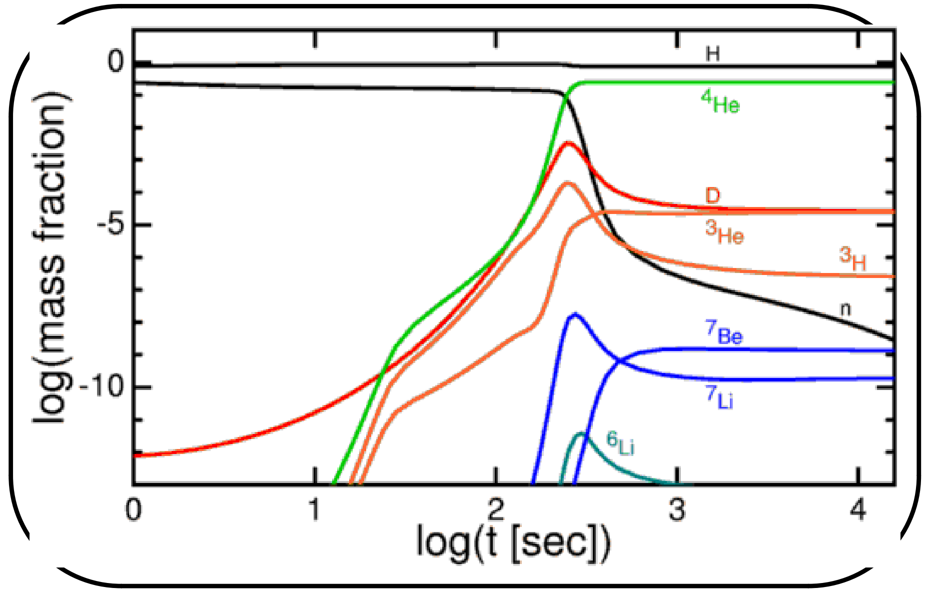
- extends and improves earlier work based on EFTs and models

  Beane, Savage (2003), Epelbaum, UGM, Glöckle (2003), Mondejar, Soto (2007), Flambaum, Wiringa (2007), Bedaque, Luu, Platter (2011), . . .

BBN NETWORK & ELEMENT ADUNDANCES



from Cococubed.com



from Burles, Nollett & Turner

- consider element generation in the Big Bang up to ^{7}Li, ^{7}Be
- how does this network / the abundances of the elements change under variations of the quark masses?

$\Rightarrow$ use results just shown, extended also to $A = 3, 4$

LIMITS for the QUARK MASS VARIATION

- Work first in the isospin limit: Average of [deut/H] and ^{4}He(Y_p):

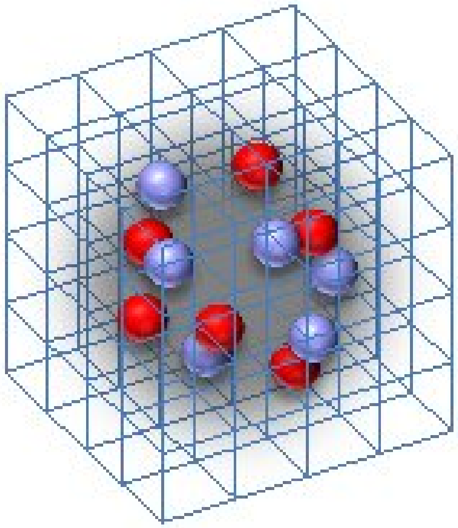
$$\frac{\delta m_q}{m_q} = 0.02 \pm 0.04$$

- in contrast to earlier studies, we provide reliable error estimates (EFT)
- Isospin breaking: stronger constraint due to the neutron life time (affects $Y(^4\mathrm{He})$)
- re-evaluate this under the model-independent assumption that *all* quark & lepton masses vary with the Higgs VEV v

$\Rightarrow$ results are dominated by the ^{4}He abundance:

$$\left|\frac{\delta v}{v}\right| = \left|\frac{\delta m_q}{m_q}\right| \leq 0.9\%$$

Lattice chiral EFT physics



NLEFT

THE TOOL: NUCLEAR LATTICE SIMULATIONS

Frank, Brockmann (1992), Koonin, Müller, Seki, van Kolck (2000) , Lee, Schäfer (2004), . . .
Borasoy, Krebs, Lee, UGM, Nucl. Phys. **A768** (2006) 179; Borasoy, Epelbaum, Krebs, Lee, UGM, Eur. Phys. J. **A31** (2007) 105

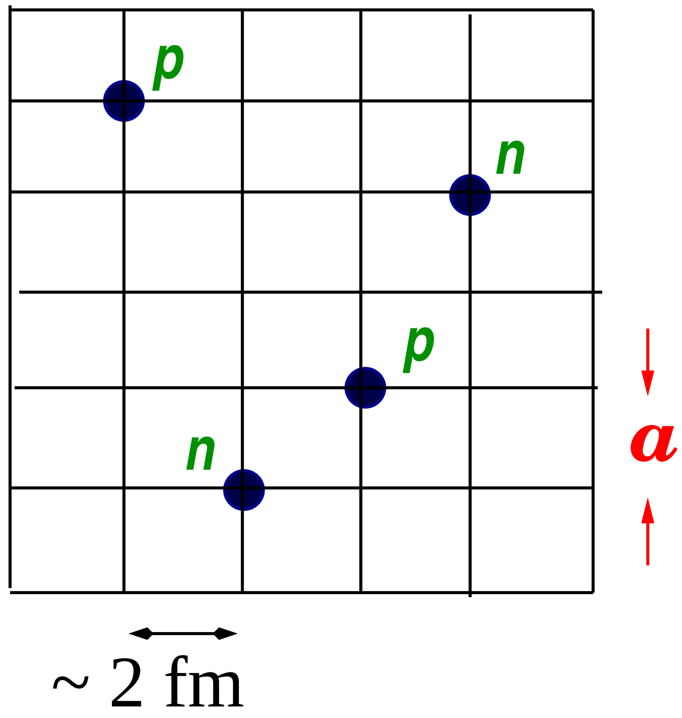
- *new method* to tackle the nuclear many-body problem

- discretize space-time $V = L_s \times L_s \times L_s \times L_t$:
  nucleons are point-like fields on the sites

- discretized chiral potential w/ pion exchanges
  and contact interactions + Coulomb

- typical lattice parameters

$$\Lambda = \frac{\pi}{a} \simeq 300\,\text{MeV [UV cutoff]}$$

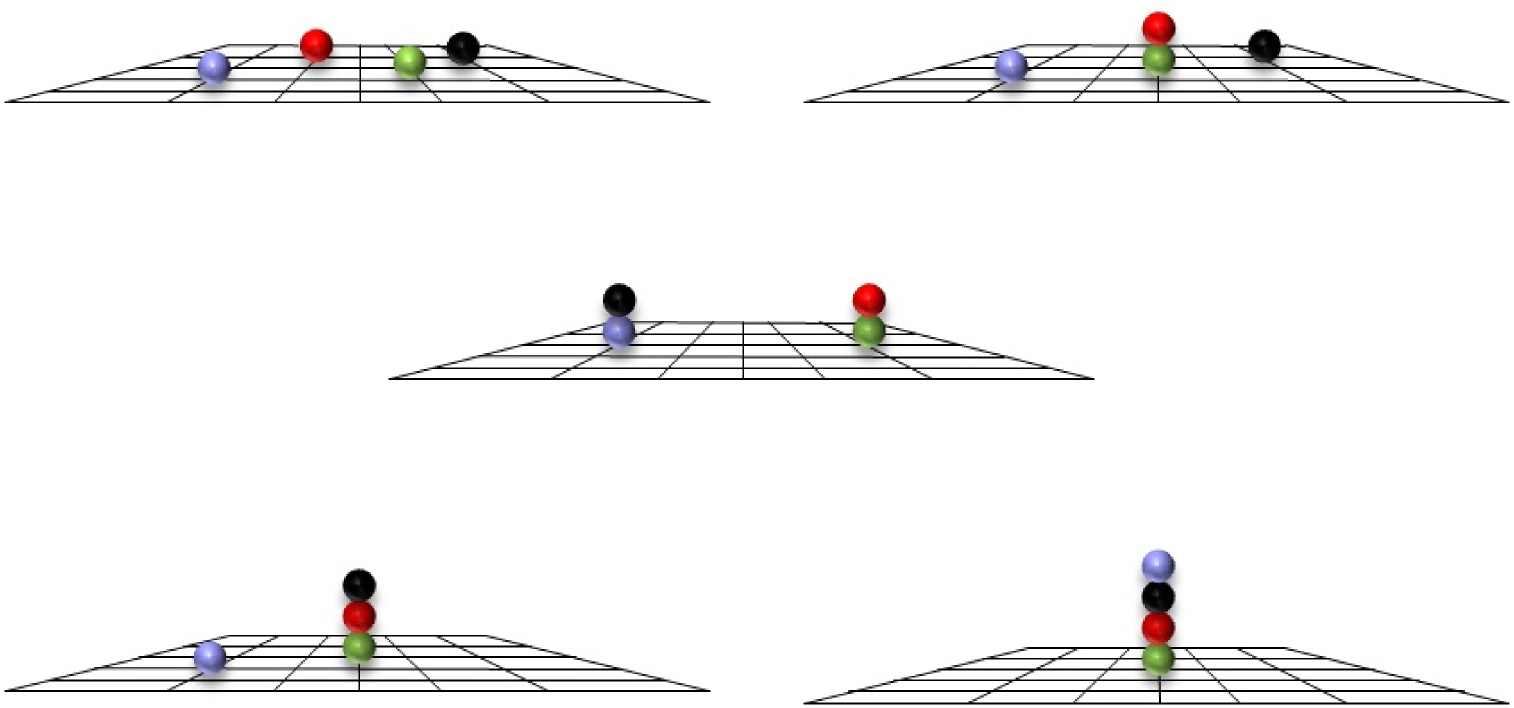


- strong suppression of sign oscillations due to approximate Wigner SU(4) symmetry

J. W. Chen, D. Lee and T. Schäfer, Phys. Rev. Lett. **93** (2004) 242302, T. Lähde et al., *arXiv:1502.06787*

- hybrid Monte Carlo & transfer matrix (similar to LQCD)

CONFIGURATIONS



⇒ all *possible* configurations are sampled

⇒ *clustering* emerges *naturally*

COMPUTATIONAL EQUIPMENT

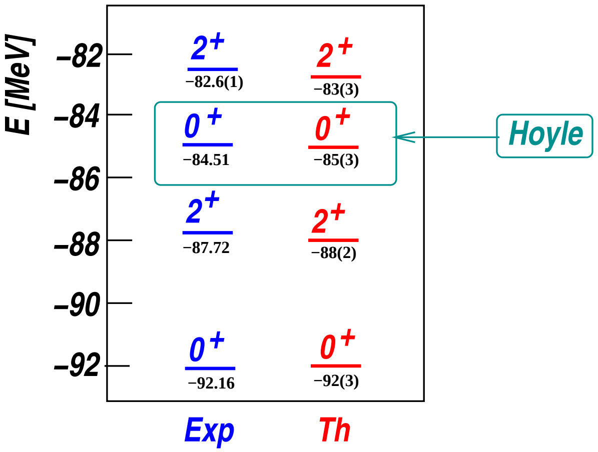
- Present = JUQUEEN (BlueGene/Q)

6 Pflops

RESULTS from LATTICE NUCLEAR EFT

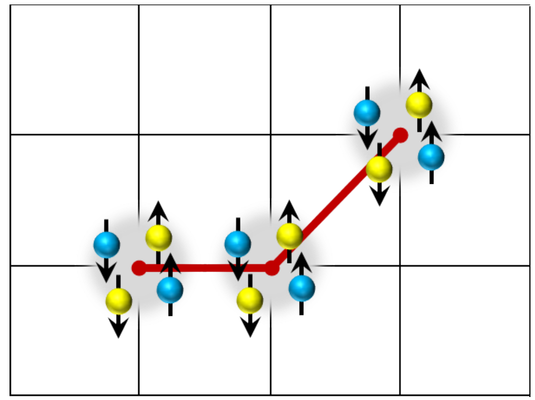
- Hoyle state in ^{12}C

PRL 106 (2011)



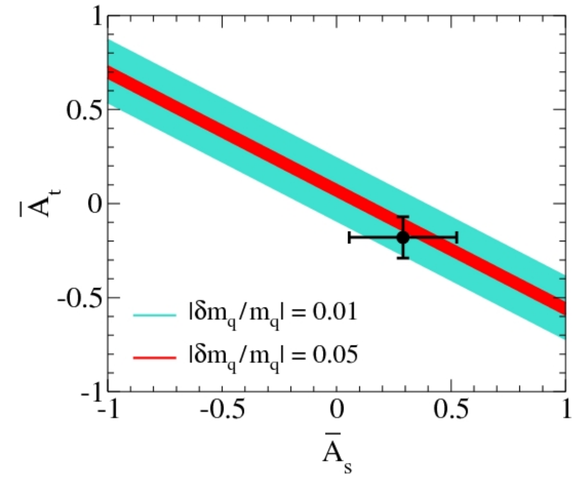
- Structure of the Hoyle state

PRL 109 (2012)



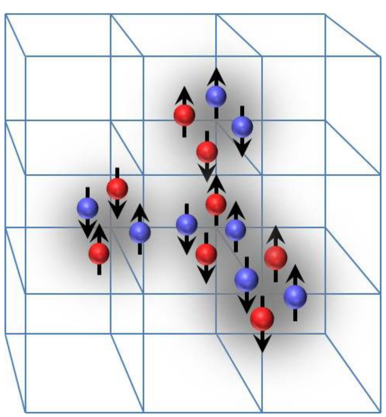
- Fate of carbon-based life

PRL 110 (2013), EPJ A49 (2013)



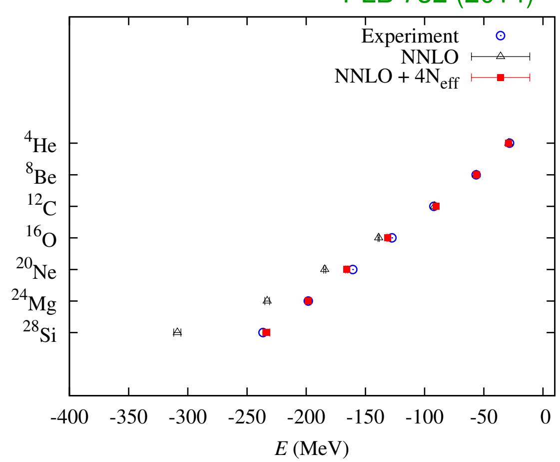
- Spectrum of ^{16}O

PRL 112 (2014)



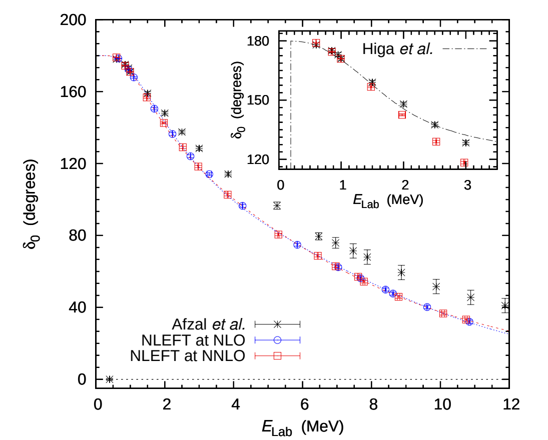
- Going up the α-chain

PLB 732 (2014)



- Ab initio α-α scattering

Nature (2015) in press

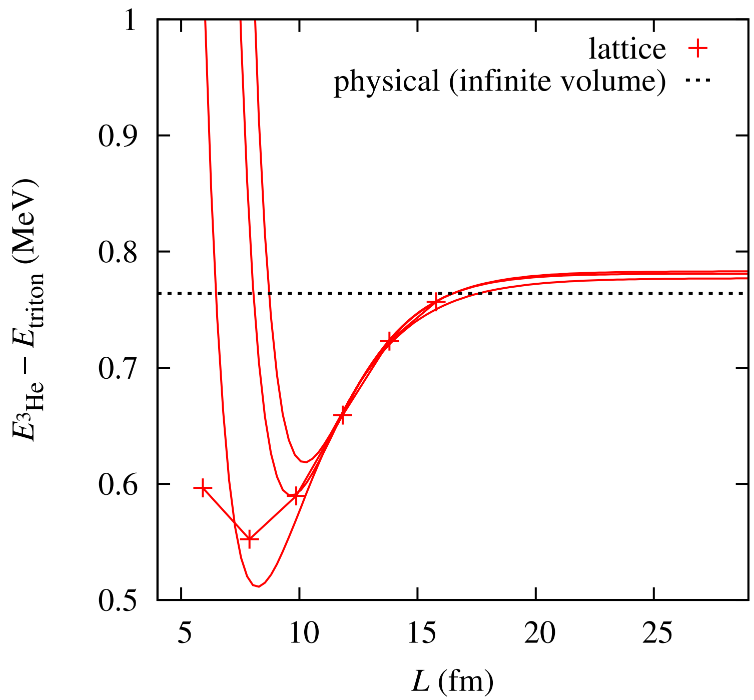


RESULTS

Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. 104 (2010) 142501; Eur. Phys. J. A45 (2010) 335

- some groundstate energies and differences [NNLO, 11+2 LECs]

	E [MeV]	NLEFT	Exp.
old algorithm	^{3}He -^{3}H	0.78(5)	0.76
old algorithm	^{4}He	−28.3(6)	−28.3
old algorithm	^{8}Be	−55(2)	−56.5
old algorithm	^{12}C	−92(3)	−92.2
new algorithm	^{16}O	−131(1)	−127.6
new algorithm	^{20}Ne	−166(1)	−160.6
new algorithm	^{24}Mg	−198(2)	−198.3
new algorithm	^{28}Si	−234(3)	−236.5



- promising results ⇒ uncertainties down to the 1% level

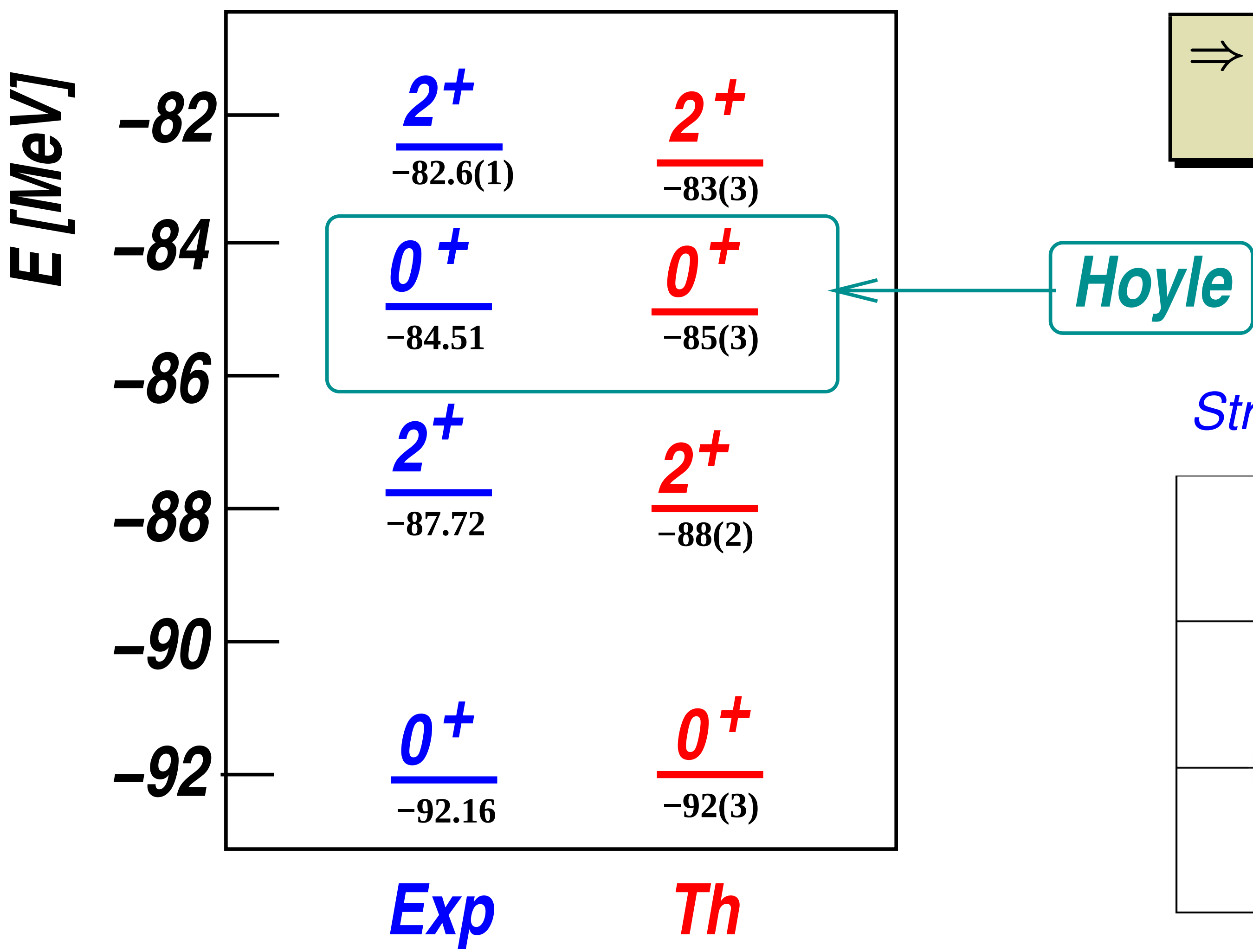
- excited states more difficult ⇒ projection MC method + triangulation

The SPECTRUM of CARBON-12

Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. 106 (2011) 192501

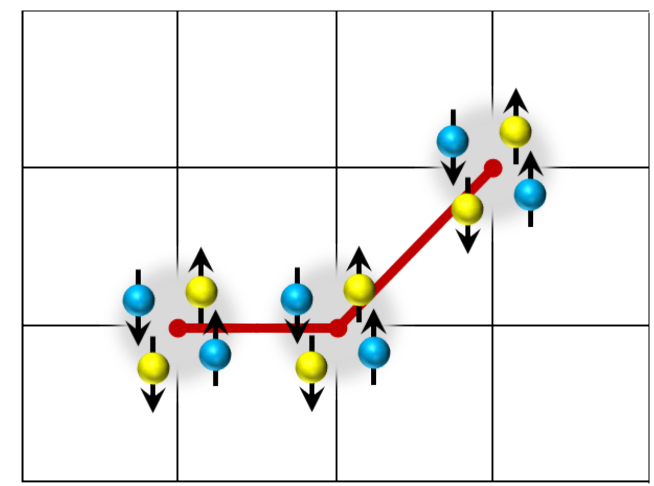
Epelbaum, Krebs, Lähde, Lee, UGM, Phys. Rev. Lett. 109 (2012) 252501

- After $8 \cdot 10^6$ hrs JUGENE/JUQUEEN (and "some" human work)



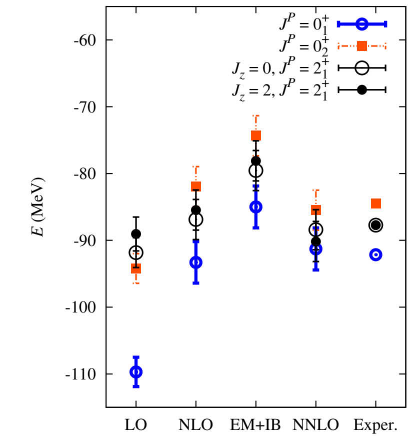
⇒ First ab initio calculation of the Hoyle state ✓

Structure of the Hoyle state:



The RELEVANT QUESTION

Date: Sat, 25 Dec 2010 20:03:42 -0600
From: Steven Weinberg ⟨weinberg@zippy.ph.utexas.edu⟩
To: Ulf-G. Meissner ⟨meissner@hiskp.uni-bonn.de⟩
Subject: Re: Hoyle state in 12C



Dear Professor Meissner,

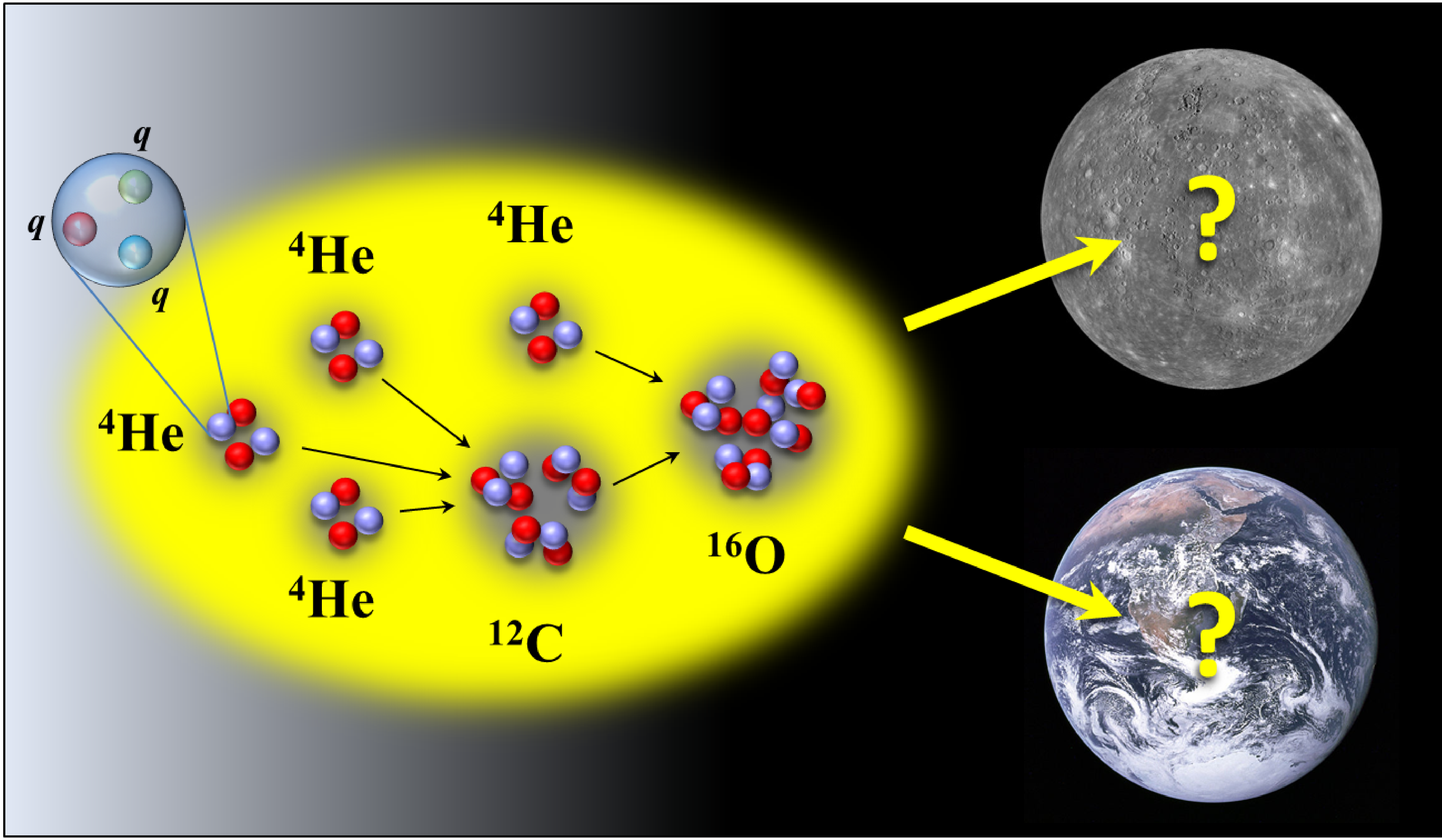
Thanks for the colorful graph. It makes a nice Christmas card. But I have a detailed question. Suppose you calculate not only the energy of the Hoyle state in C12, but also of the ground states of He4 and Be8. How sensitive is the result that the energy of the Hoyle state is near the sum of the rest energies of He4 and Be8 to the parameters of the theory? I ask because I suspect that for a pretty broad range of parameters, the Hoyle state can be well represented as a nearly bound state of Be8 and He4.

All best,
Steve Weinberg

- How does the Hoyle state move relative to the ^{4}He+^{8}Be threshold, if we change the fundamental parameters of QCD+QED?
- not possible in nature, *but on a high-performance computer!*

FINE-TUNING of FUNDAMENTAL PARAMETERS

Fig. courtesy Dean Lee



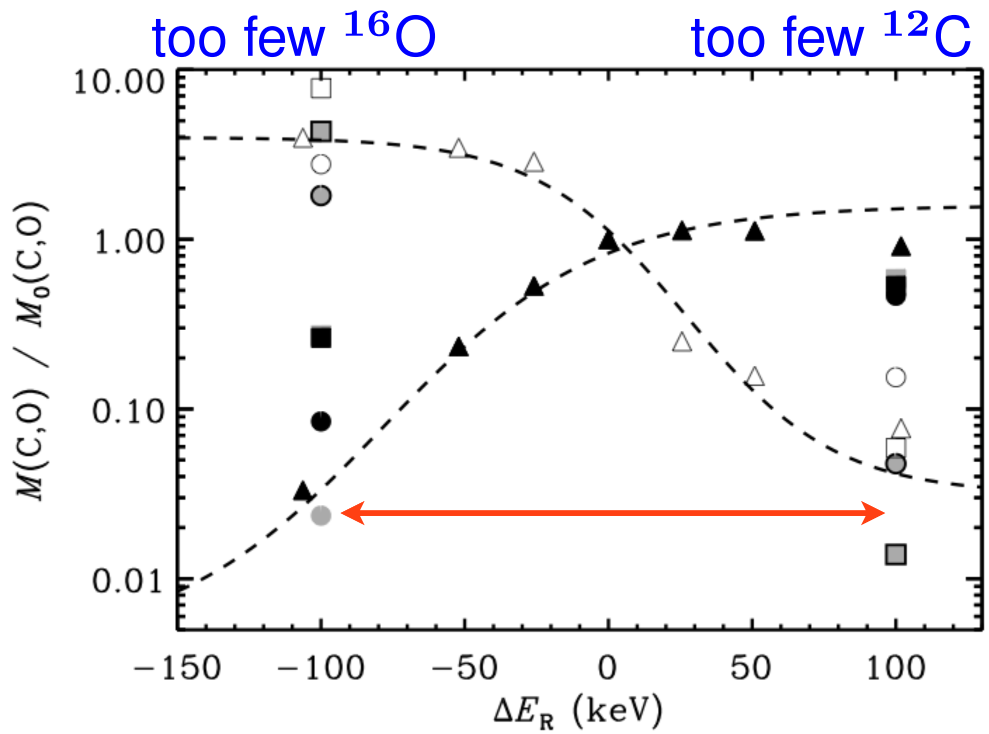
EARLIER STUDIES of the ANTHROPIC PRINCIPLE

- rate of the 3α-process: $r_{3\alpha} \sim \Gamma_\gamma \, \exp\left(-\frac{\Delta E_{h+b}}{kT}\right)$

$$\Delta E_{h+b} = E^\star_{12} - 3E_\alpha = 379.47(18) \text{ keV}$$

- how much can ΔE_{h+b} be changed so that there is still enough ^{12}C and ^{16}O?

$\Rightarrow$ $\delta|\Delta E_{h+b}| \lesssim 100$ keV



Oberhummer et al., Science **289** (2000) 88

Csoto et al., Nucl. Phys. A **688** (2001) 560

Schlattl et al., Astrophys. Space Sci. **291** (2004) 27

[Livio et al., Nature **340** (1989) 281]

FINE-TUNING: MONTE-CARLO ANALYSIS

Epelbaum, Krebs, Lähde, Lee, UGM, PRL **110** (2013) 112502

- consider first QCD only → calculate $\partial \Delta E / \partial M_\pi$

- relevant quantities (energy *differences*)

$$\Delta E_h \equiv E_{12}^* - E_8 - E_4 , \quad \Delta E_b \equiv E_8 - 2E_4$$

- energy differences depend on parameters of QCD (LO analysis)

$$E_i = E_i \left(M_\pi^{\rm OPE}, m_N(M_\pi), g_{\pi N}(M_\pi), C_0(M_\pi), C_I(M_\pi) \right)$$

$$g_{\pi N} \equiv g_A / (2F_\pi)$$

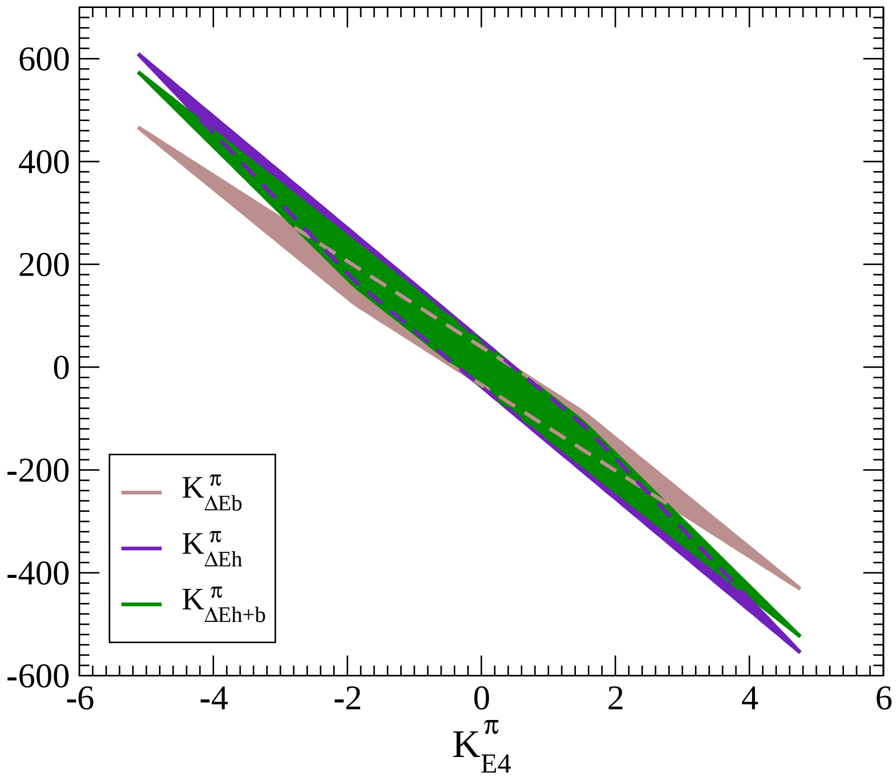
- remember: $M_{\pi^\pm}^2 \sim (m_u + m_d)$ Gell-Mann, Oakes, Renner (1968)

⇒ quark mass dependence ≡ pion mass dependence

CORRELATIONS

- map $C_{0,I}(M_\pi)$ onto $\bar{A}_{s,t} \equiv \partial a_{s,t}^{-1}/\partial M_\pi\big|_{M_\pi^{\rm phys}}$ [singlet/triplet scatt. length]

- vary the derivatives $\bar{A}_{s,t} \equiv \partial a_{s,t}^{-1}/\partial M_\pi\big|_{M_\pi^{\rm phys}}$ within $-1,\ldots,+1$:



$$\Delta E_b = E(^8\mathrm{Be}) - 2E(^4\mathrm{He})$$

$$\Delta E_h = E(^{12}\mathrm{C}^*) - E(^8\mathrm{Be}) - E(^4\mathrm{He})$$

$$\Delta E_{h+b} = E(^{12}\mathrm{C}^*) - 3E(^4\mathrm{He})$$

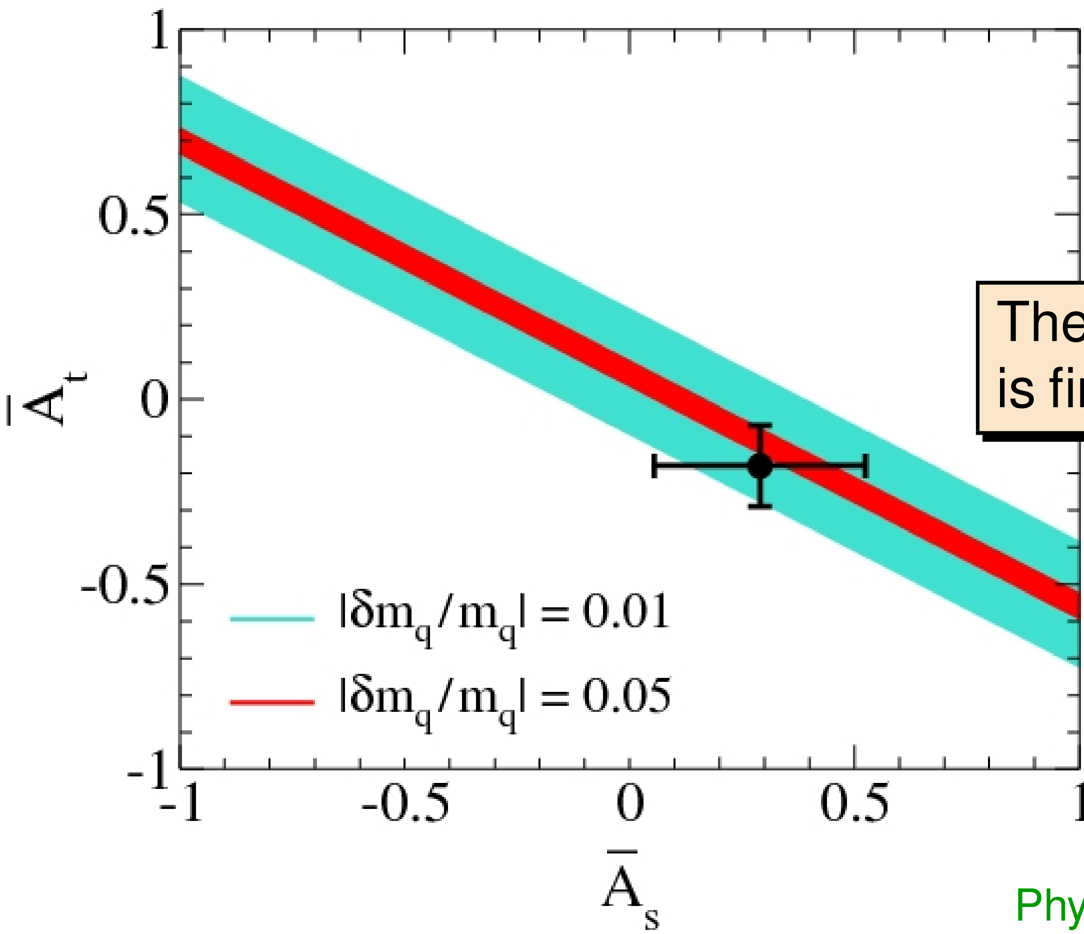
$$\boxed{\frac{\partial O_H}{\partial M_\pi} = K_H^\pi \, \frac{O_H}{M_\pi}}$$

- clear correlations: α-particle BE and the energies/energy differences

THE END-OF-THE-WORLD PLOT

- $|\delta(\Delta E_{h+b})| < 100$ keV [exp: 387 keV] Oberhummer et al., Science (2000)

$$\rightarrow \left| \left(0.571(14)\,\bar{A}_s + 0.934(11)\,\bar{A}_t - 0.069(6) \right) \frac{\delta m_q}{m_q} \right| < 0.0015$$



$$\bar{A}_{s,t} \equiv \left. \frac{\partial a_{s,t}^{-1}}{\partial M_\pi} \right|_{M_\pi^{\rm phys}}$$

The light quark mass is fine-tuned to $\simeq 2-3\,\%$

Similarly: $\alpha_{\rm EM}$ is fine-tuned to $\simeq 2.5\%$

Berengut et al., Phys. Rev. **D 87** (2013) 085018

Ab initio calculation of α-α scattering

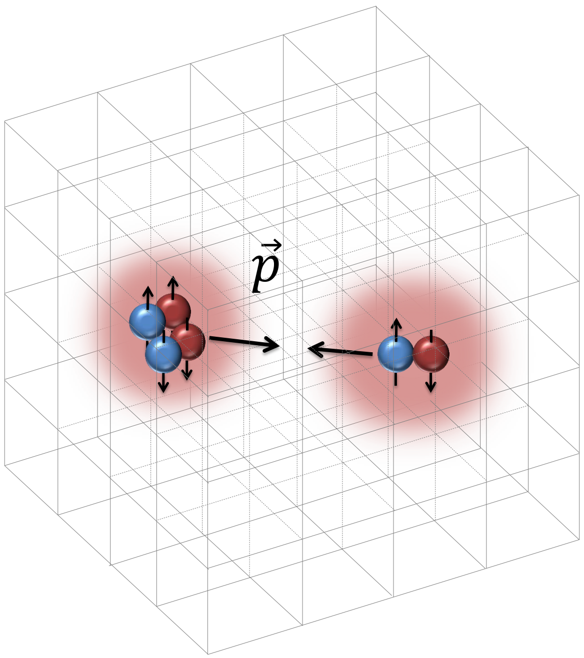
Elhatisari, Lee, Rupak, Epelbaum, Krebs, Lähde, Luu, UGM,

Nature (2015) in press [arXiv:1506.03513]

TWO-BODY SCATTERING on the LATTICE

- Processes involving α-particles and α-type nuclei comprise a major part of stellar nucleosynthesis, and control the production of certain elements in stars

- Ab initio calculations of scattering and reactions using standard many-body methods suffer from computational scaling with the of nucleons in the clusters



Lattice EFT computational scaling $\Rightarrow (A_1 + A_2)^2$

Rupak, Lee, Phys. Rev. Lett. 111 (2013) 032502
Pine, Lee, Rupak, Eur. Phys. J. A49 (2013) 151
Elhatisari, Lee, Phys. Rev. C90 (2014) 064001
Elhatisari, et al., arXiv:1505.02967

ADIABATIC PROJECTION METHOD

- Basic idea to treat scattering and inelastic reactions:
  split the problem into two parts

First part:

use Euclidean time projection to construct an *ab initio* low-energy cluster Hamiltonian, called the **adiabatic Hamiltonian**

Second part:

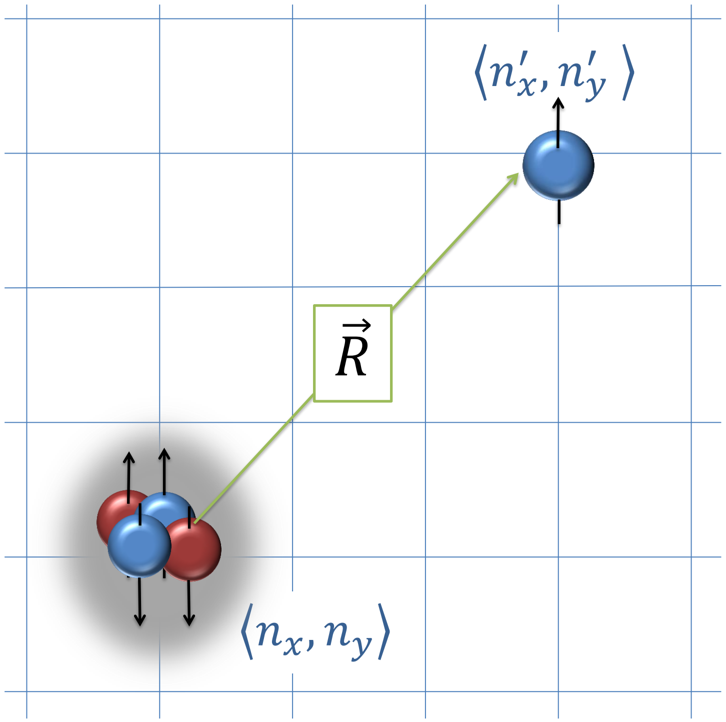
compute the two-cluster scattering phase shifts or reaction amplitudes using the adiabatic Hamiltonian

ADIABATIC PROJECTION METHOD II

- Construct a low-energy effective theory for clusters

- Use initial states parameterized by the relative separation between clusters

$$|\vec{R}\rangle = \sum_{\vec{r}} |\vec{r} + \vec{R}\rangle \otimes \vec{r}$$



- project them in Euclidean time with the chiral EFT Hamiltonian $\boldsymbol{H}$

$$|\vec{R}\rangle_\tau = \exp(-H\tau)|\vec{R}\rangle$$

$\rightarrow$ "dressed cluster states"

- Construct the adiabatic Hamiltonian from the dressed cluster states:

$$[H_\tau]_{\vec{R}\vec{R}'} = {}_\tau\langle\vec{R}|H|\vec{R}'\rangle_\tau$$

SCATTERING CLUSTER WAVE FUNCTIONS

- During Euclidean time interval τ_ε,
  each cluster undergoes spatial diffusion:

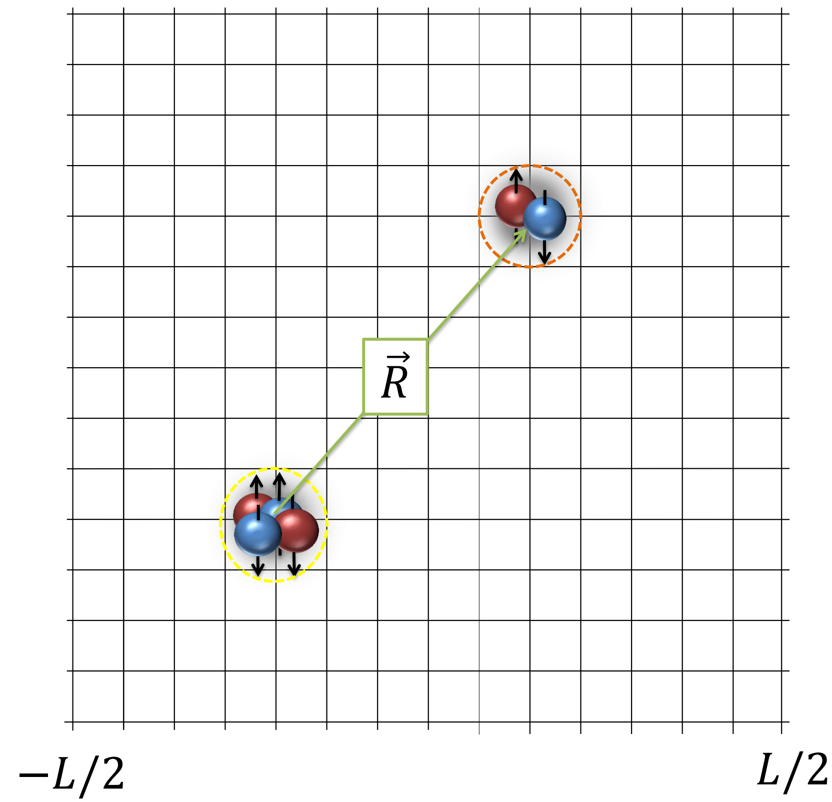
$$d_{\varepsilon,i} = \sqrt{\tau_\varepsilon / M_i}$$

- Only non-overlapping clusters if

$$|\vec{R}| \gg d_{\varepsilon,i} \quad \Rightarrow \quad |\vec{R}\rangle_{\tau_\varepsilon}$$

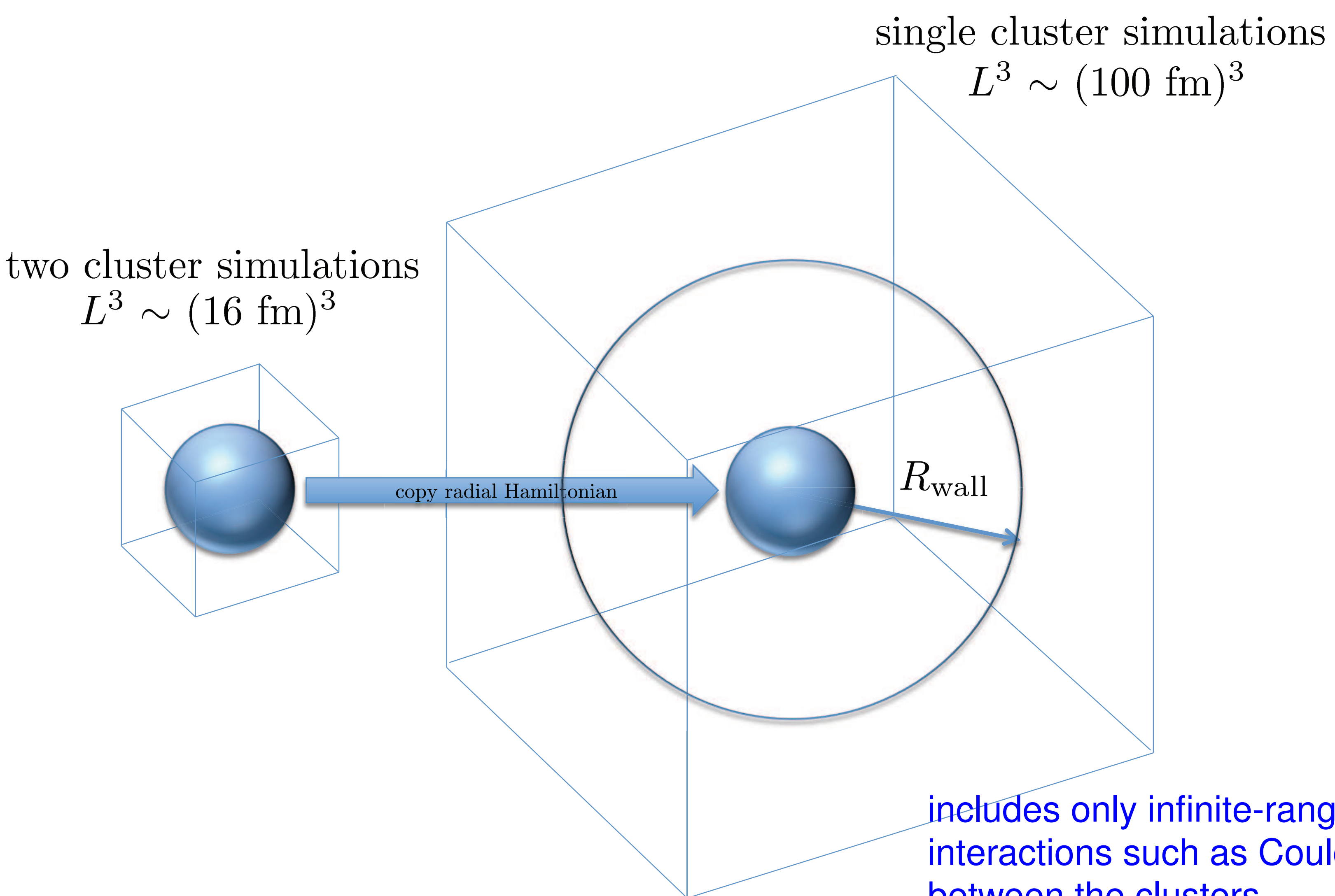
- Defines asymptotic region, where the amount of overlap between clusters is less than ε

$$|\vec{R}| > R_\varepsilon$$



$\Rightarrow$ In the asymptotic region we can describe the system in terms of an effective cluster Hamiltonian (the free lattice Hamiltonian for two clusters) plus infinite-range interactions (like the Coulomb int.)

ADIABATIC HAMILTONIAN plus COULOMB



includes only infinite-range interactions such as Coulomb between the clusters

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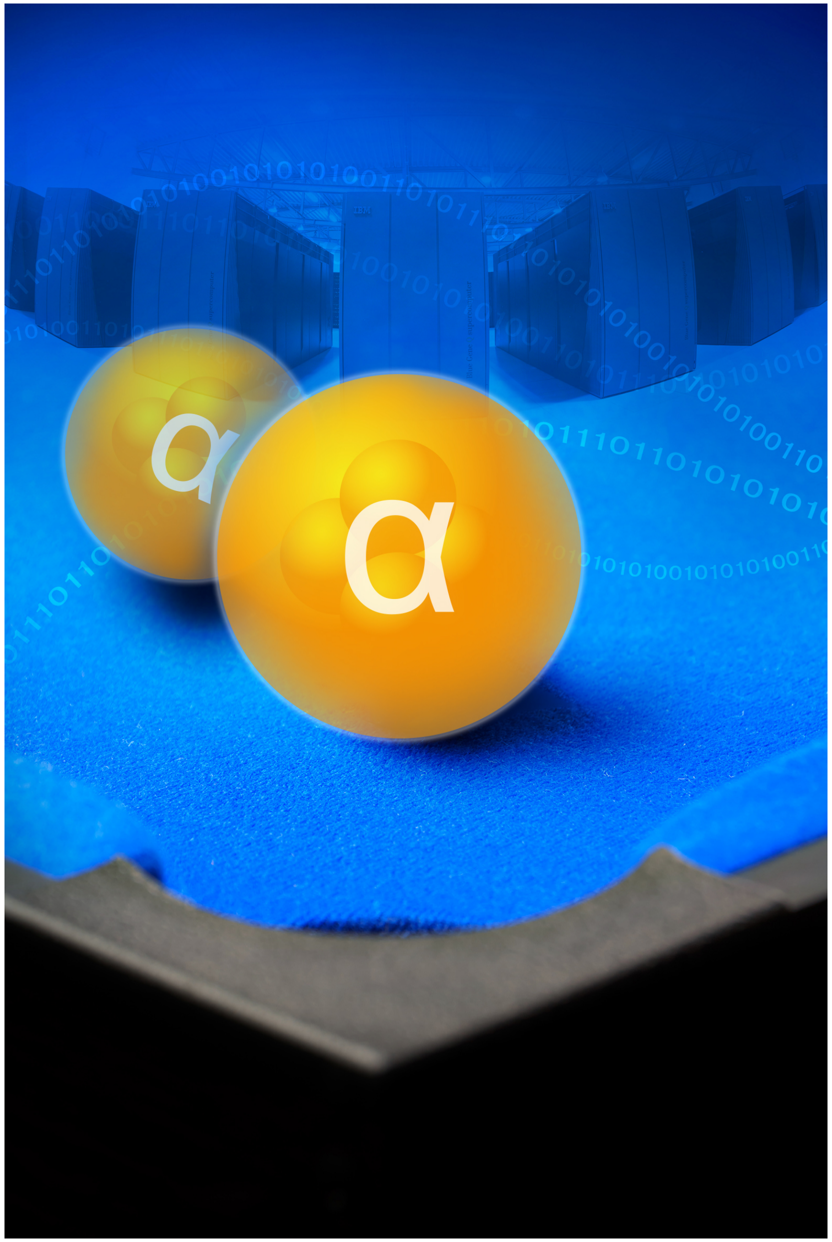
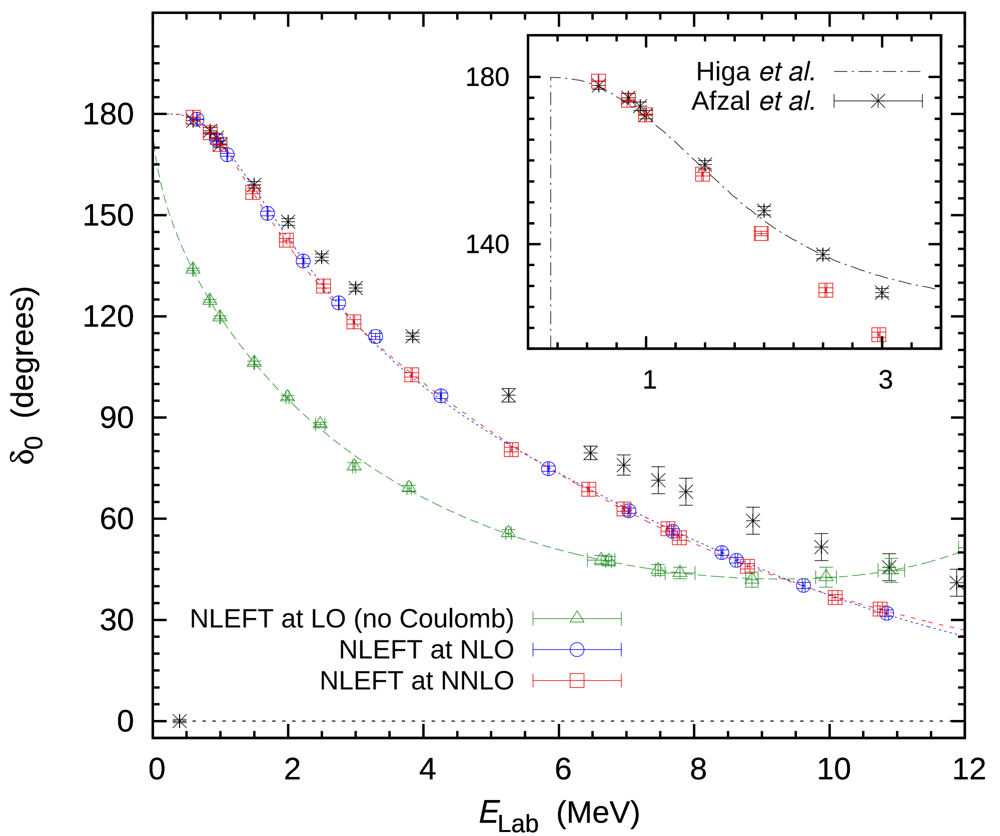
ALPHA–ALPHA SCATTERING

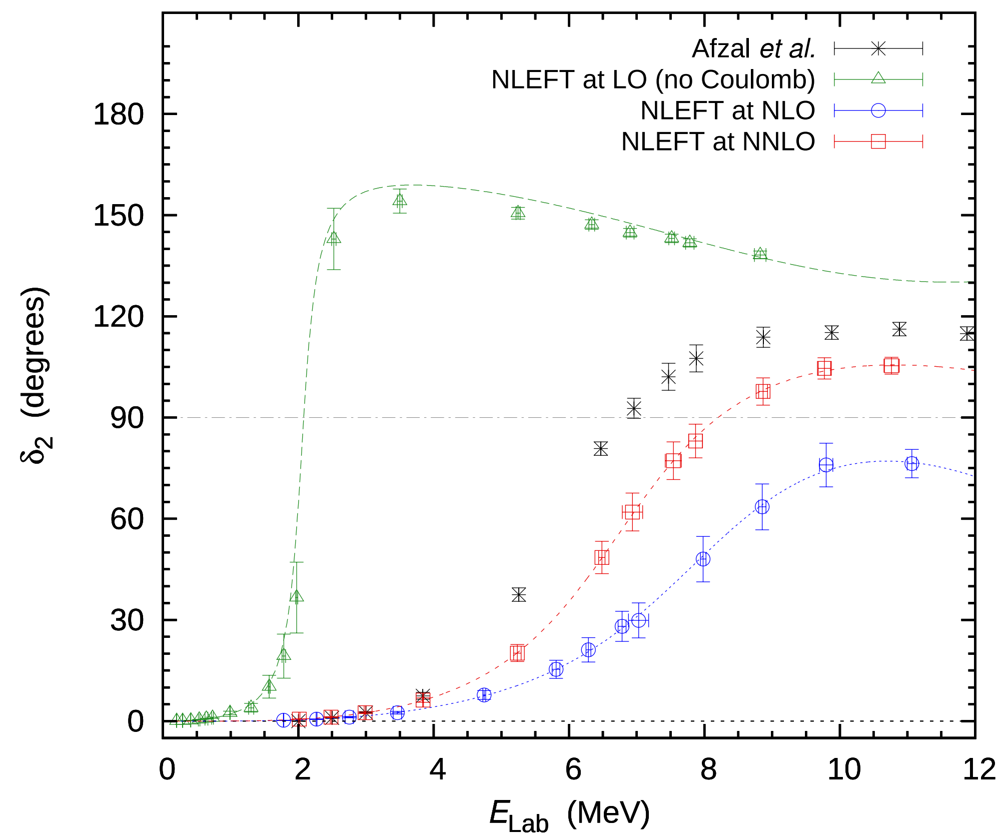
- same lattice action as for the Hoyle state in ^{12}C and the structure of ^{16}O
- 11 NN + 2 3N LECs, coarse lattice $a = 1.97$ fm, $N = 8$
- new algorithm for Monte Carlo updates and alpha clusters
- adiabatic projection method to construct a two-alpha Hamiltonian
- spherical wall method to extract the phase shifts using radial Hamiltonian
  Borasoy, Epelbaum, Krebs, Lee, UGM,
  EPJA 34 (2007) 185



PHASE SHIFTS

- S-wave and D-wave phase shifts (LO has no Coulomb)





$$E_R^{\rm NNLO} = -0.11(1)\ \text{MeV}\ \ [+0.09\ \text{MeV}]$$

$$E_R^{\rm NNLO} = 3.27(12)\ \text{MeV}\ \ [\,2.92(18)\ \text{MeV}]$$

$$\Gamma_R^{\rm NNLO} = 2.09(16)\ \text{MeV}\ \ [1.35(50)\ \text{MeV}]$$

Afzal et al., Rev. Mod. Phys. 41 (1969) 247 [data]; Higa et al., Nucl.Phys. A809 (2008) 171 [halo EFT]

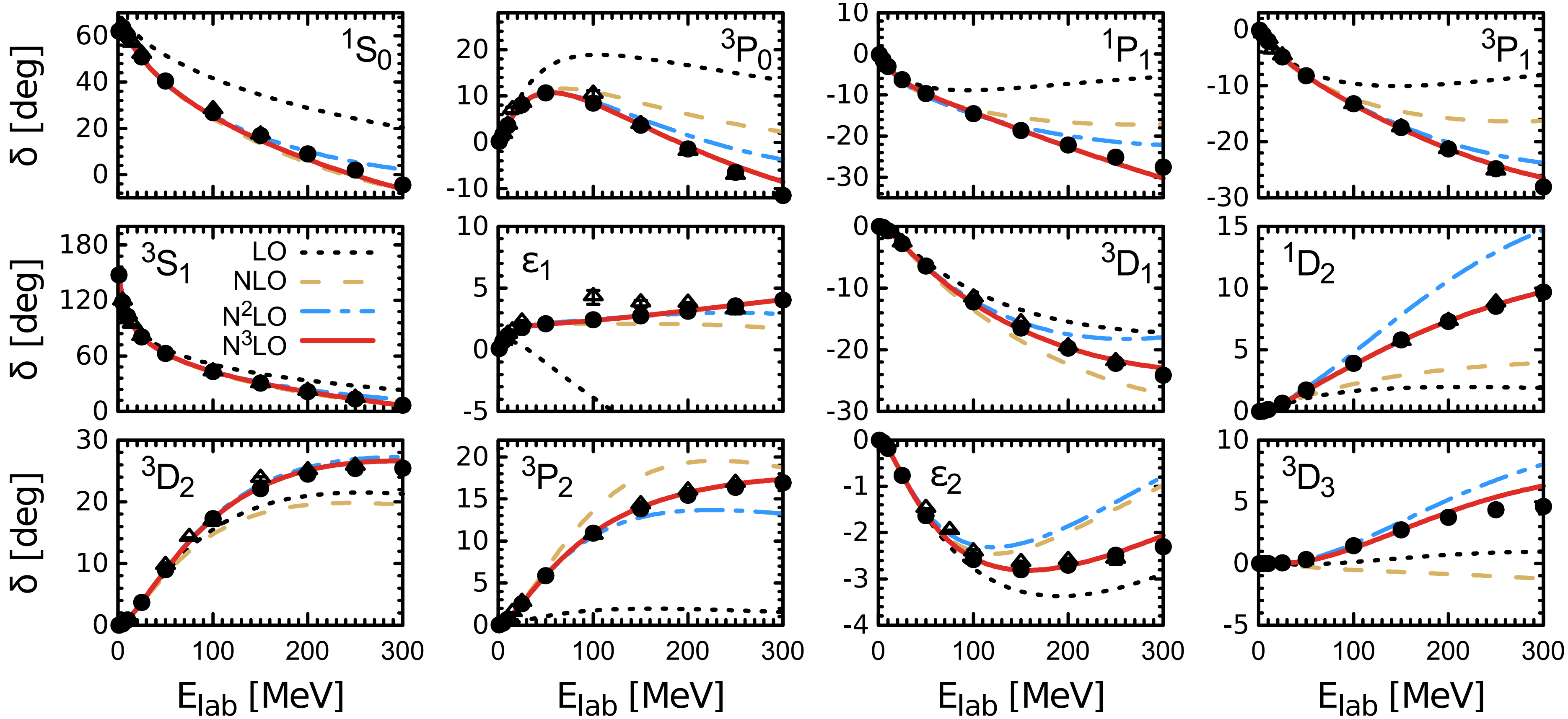
SUMMARY & OUTLOOK

- Chiral nuclear EFT: best approach to nuclear forces and few-body systems
  - $\rightarrow$ new, solid method to estimate the theoretical uncertainties
  - $\rightarrow$ high-precision NN potential to fifth order available
  - $\rightarrow$ pinning down the 3NFs under way
- Nuclear lattice simulations as a new quantum many-body approach
  - $\rightarrow$ many promising results at NNLO using coarse lattices
  - $\rightarrow$ clustering emerges naturally, α-cluster nuclei
  - $\rightarrow$ scattering and inelastic reactions can also be calculated *ab initio*
  - $\rightarrow$ holy grail of nuclear astrophysics (α+^{12}C $\rightarrow$ ^{16}O+γ) in reach

SPARES

CONVERGENCE of the CHIRAL SERIES

- phase shifts show expected convergence [large N^2LO corrections understood]



$\Longrightarrow$ clear improvement comp. to earlier N^3LO potentials [momentum space reg.]

Entem, Machleidt; Epelbaum, Glöckle, UGM

QUARK MASS DEPENDENCE of HADRON MASSES etc

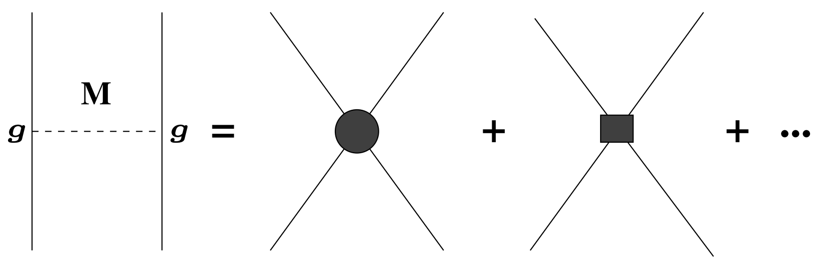
- Quark mass dependence of hadron properties:

$$\frac{\delta O_H}{\delta m_f} \equiv K_H^f \frac{O_H}{m_f}\,, \quad f = u, d, s$$

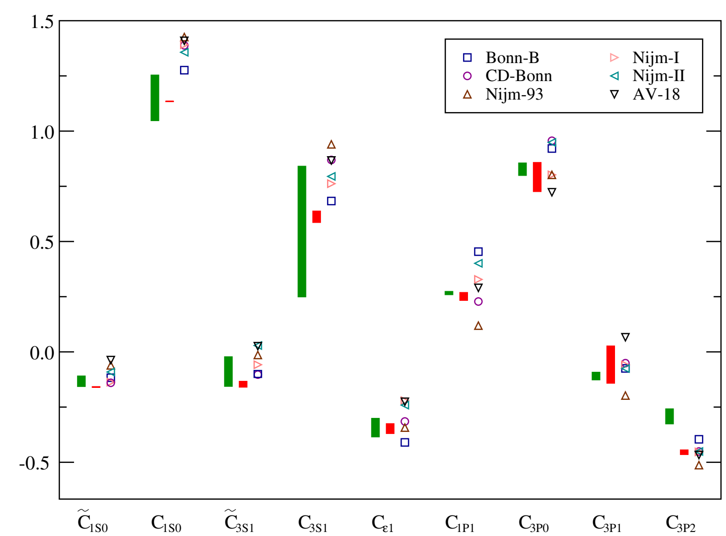
- Pion and nucleon properties from lattice QCD combined with CHPT

- Contact interactions modeled by heavy meson exchanges

Epelbaum, UGM, Glöckle, Elster (2002)



$$\frac{g^2}{t-M^2} = -\frac{g^2}{M^2} - \frac{g^2\,t}{M^4} + \ldots$$



QUARK MASS VARIATIONS of HEAVIER NUCLEI

- In BBN, we also need the variation of ^{3}He and ^{4}He. All other BEs are kept fixed.
- use the method of BLP: Bedaque, Luu, Platter, PRC 83 (2011) 045803

$$K^q_{^A\mathrm{He}} = K^q_{a,\,1\mathrm{S0}} K^{a,\,1\mathrm{S0}}_{^A\mathrm{He}} + K^q_{\mathrm{deut}} K^{\mathrm{deut}}_{^A\mathrm{He}}\ , \quad A = 3, 4$$

with

$$K^{a,\,1\mathrm{S0}}_{^3\mathrm{He}} = 0.12 \pm 0.01\ , \quad K^{\mathrm{deut}}_{^3\mathrm{He}} = 1.41 \pm 0.01$$

$$K^{a,\,1\mathrm{S0}}_{^4\mathrm{He}} = 0.037 \pm 0.011\ , \quad K^{\mathrm{deut}}_{^4\mathrm{He}} = 0.74 \pm 0.22$$

so that

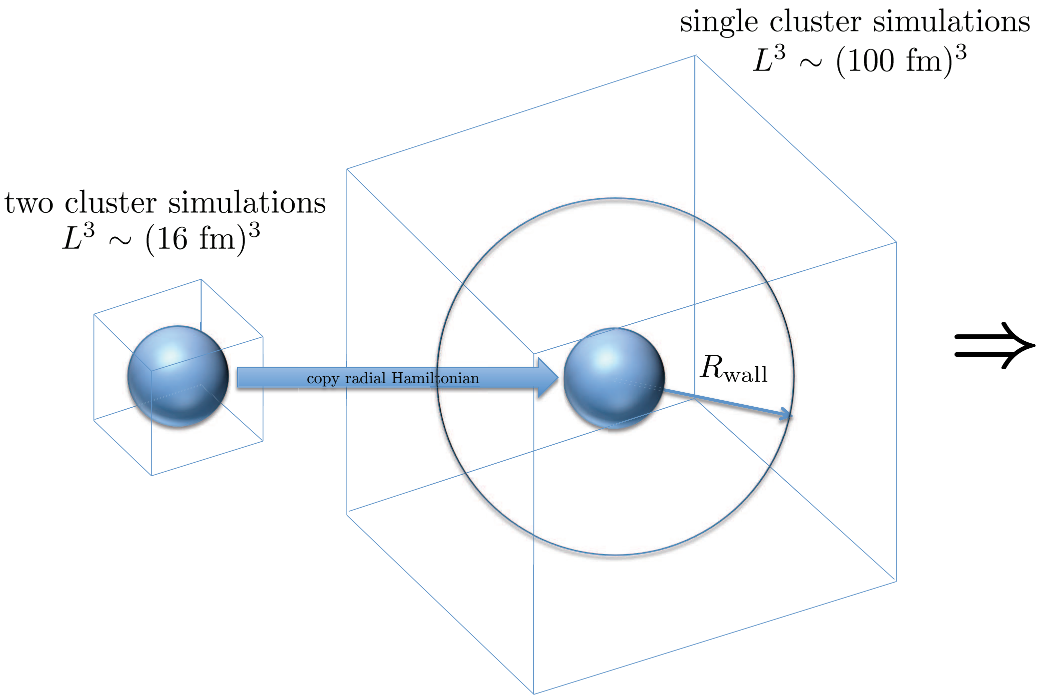
$$\Rightarrow \boxed{K^q_{^3\mathrm{He}} = -0.94 \pm 0.75, \quad K^q_{^4\mathrm{He}} = -0.55 \pm 0.42}$$

- calculate BBN response matrix of primordial abundances Y_a ($a =$ ^{2}H, ^{3}H, ^{3}He, ^{4}He, ^{6}Li, ^{7}Li, ^{7}Be) at fixed baryon-to-photon ratio ($n_B/n_\gamma \simeq 6 \cdot 10^{-10}$)

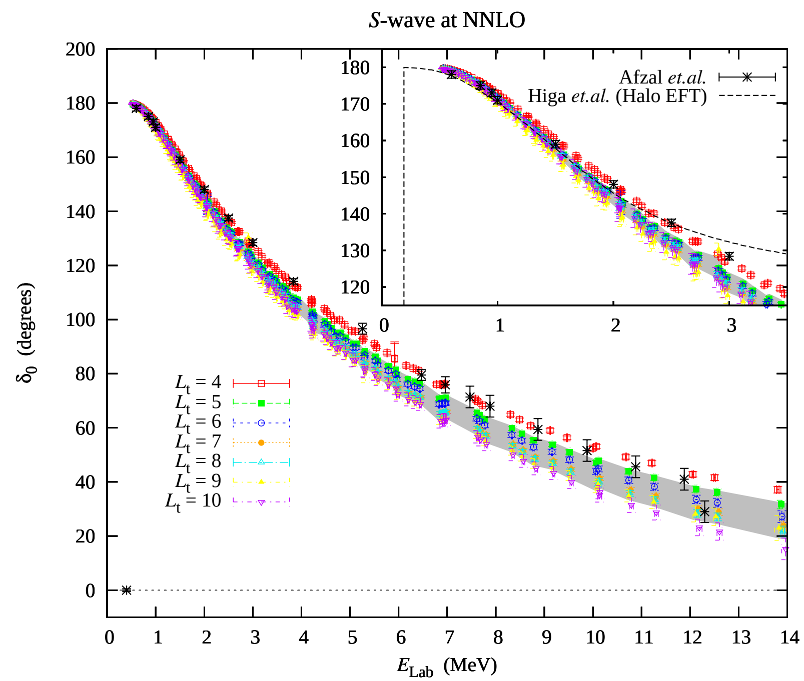
AB INITIO CALCULATION of α-α SCATTERING

- use lattice MC to construct an ab-initio cluster (adiabatic) Hamiltonian
- Use adiabatic Hamiltonian to compute scattering/reaction amplitudes

Elhatisari et al. 2015



$\Rightarrow$



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- D-wave equally well described

ADIABATIC HAMILTONIAN

- Construct the adiabatic Hamiltonian from the dressed cluster states:

$$[H_\tau]_{\vec{R}\vec{R}'} = {}_\tau\langle\vec{R}|H|\vec{R}'\rangle_\tau$$

- States are i.g. not normalized, require *norm matrix*:

$$[N_\tau]_{\vec{R}\vec{R}'} = {}_\tau\langle\vec{R}|\vec{R}'\rangle_\tau$$

- construct the full adiabatic Hamiltonian:

$$\left[H^a_\tau\right]_{\vec{R}\vec{R}'} = \sum_{\vec{R}_n\vec{R}_m} \left[N_\tau^{-1/2}\right]_{\vec{R}\vec{R}_n} \left[H_\tau\right]_{\vec{R}_n\vec{R}_m} \left[N_\tau^{-1/2}\right]_{\vec{R}_m\vec{R}'}$$

→ The structure of the adiabatic Hamiltonian is similar to the Hamiltonian matrix used in recent ab initio NCSM/RGM calculations

Navratil, Quaglioni, Phys. Rev. C 83 (2011) 044609
Navratil, Roth, Quaglioni, Phys. Lett. B 704 (2011) 379
Navratil, Quaglioni, Phys. Rev. Lett. 108 (2012) 042503

TESTING the ADIABATIC HAMILTONIAN

- Consider fermion-dimer scattering:

Microscopic Hamiltonian $\Rightarrow$ Two-cluster adiabatic Hamitltonian

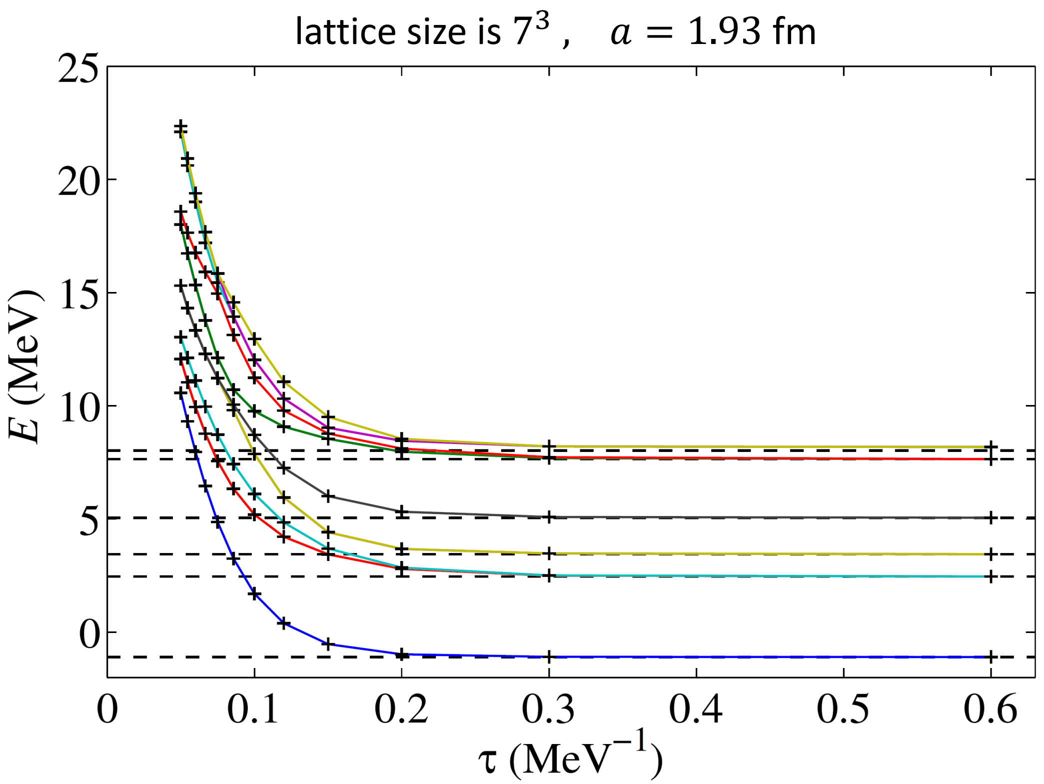
$$L^{3(A-1)} \times L^{3(A-1)} \qquad\qquad L^3 \times L^3$$

- calculation of a $\mathbf{7^3}$ lattice, lattice spacing $\boldsymbol{a = 1.93}$ fm

Pine, Lee, Rupak, EPJA 49 (2013) 151

exact Lanzcos: black dashed lines

adiab. Ham.: solid colored lines

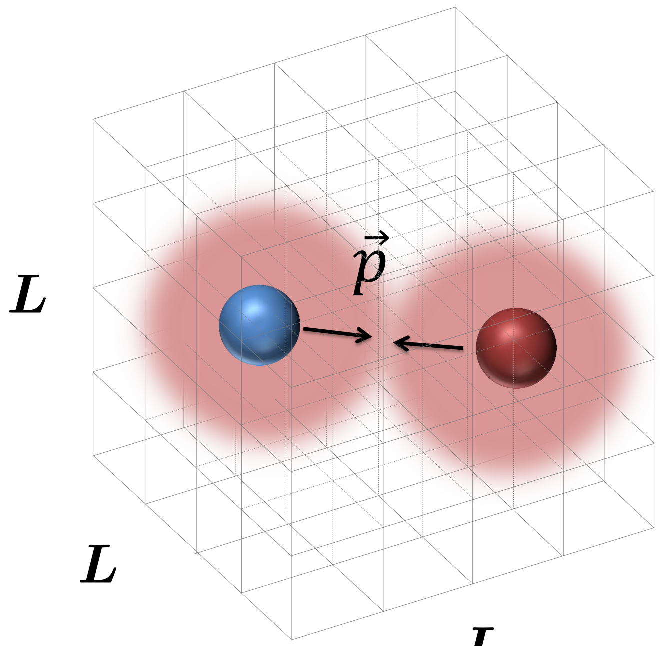


EXTRACTING PHASE SHIFTS on the LATTICE

- Lüscher's method:

  Two-body energy levels below the inelastic threshold on a periodic lattice are related to the phase shifts in the continuum

  Lüscher, Comm. Math. Phys 105 (1986) 153
  Lüscher, Nucl. Phys 354 (1991) 531

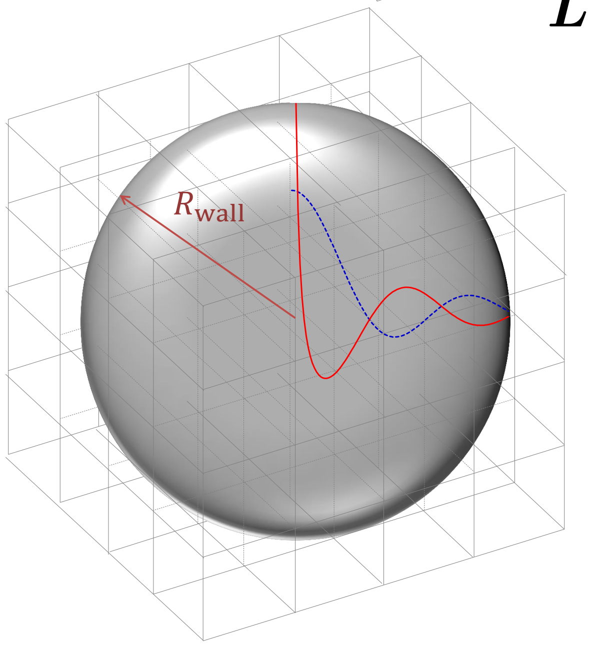


- Spherical wall method:

  Impose a hard wall on the lattice and use the fact that the wave function vanishes for $r = R_{\rm wall}$:

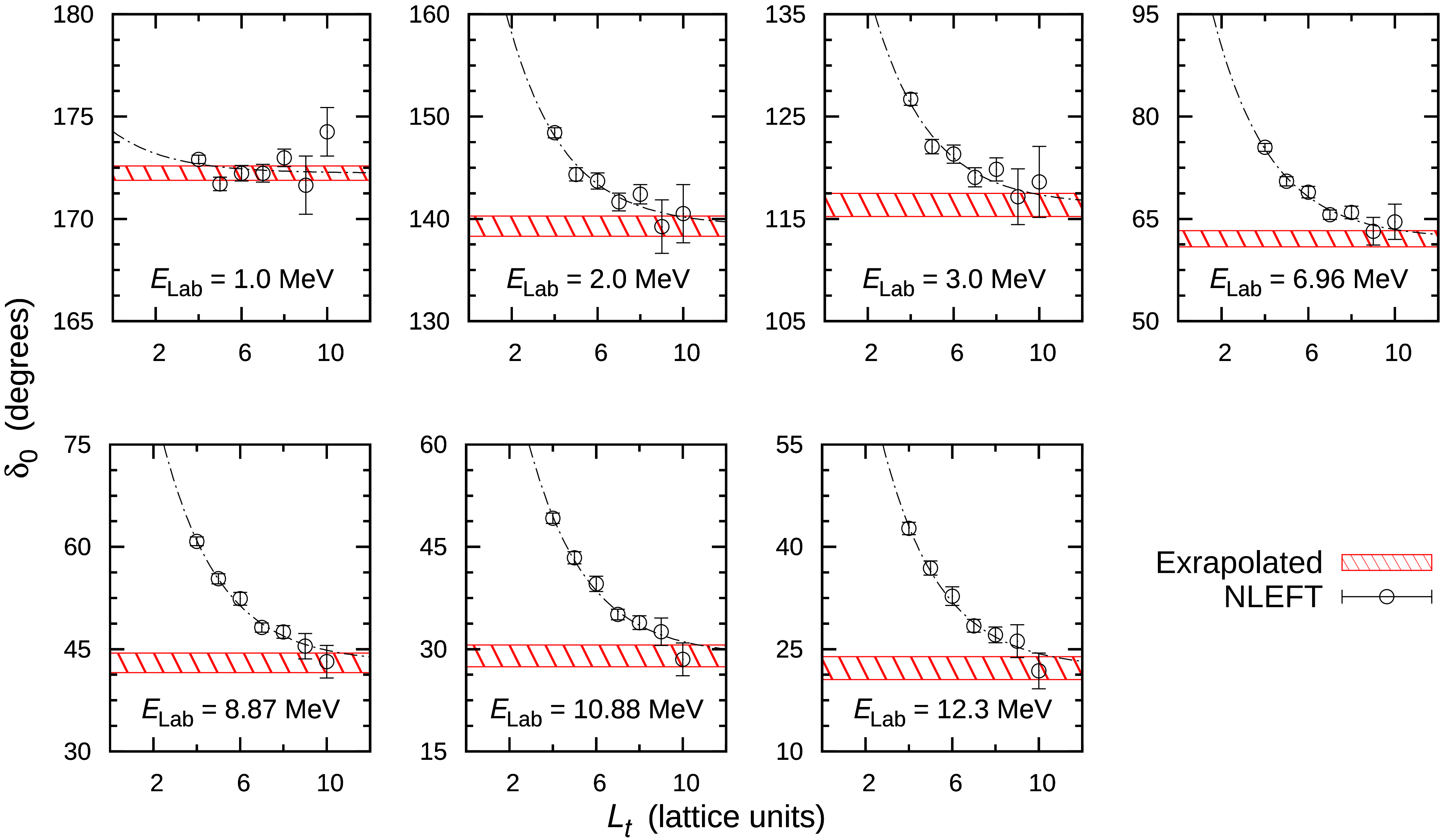
$$\psi_\ell(r) \sim [\cos\delta_\ell(p) F_\ell(pr) + \sin\delta_\ell(p) G_\ell(pr)]$$

  Borasoy, Epelbaum, Krebs, Lee, UGM, EPJA 34 (2007) 185



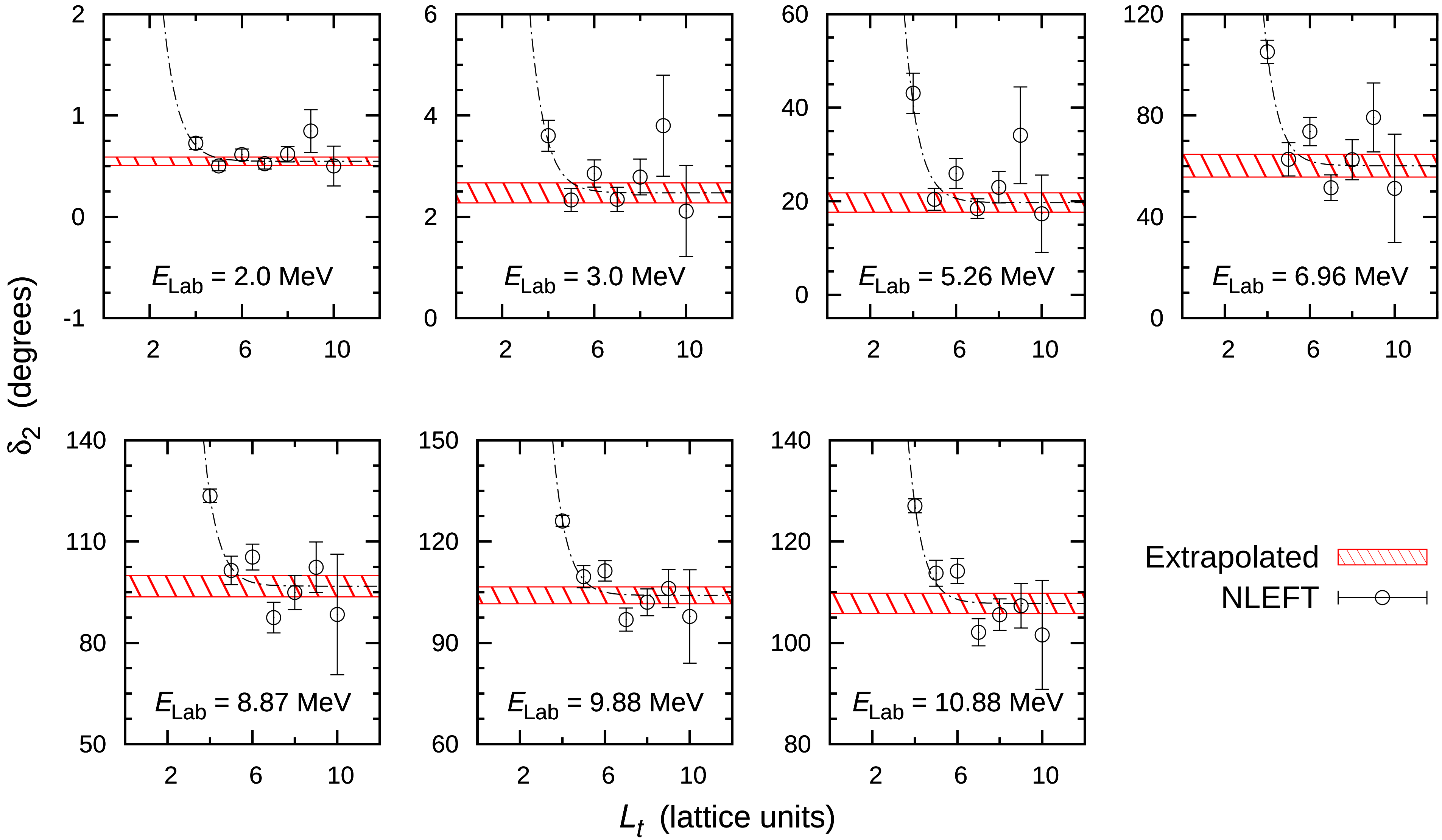
LATTICE DATA I

- Show data for the S-wave:



LATTICE DATA II

- Show data for the D-wave:



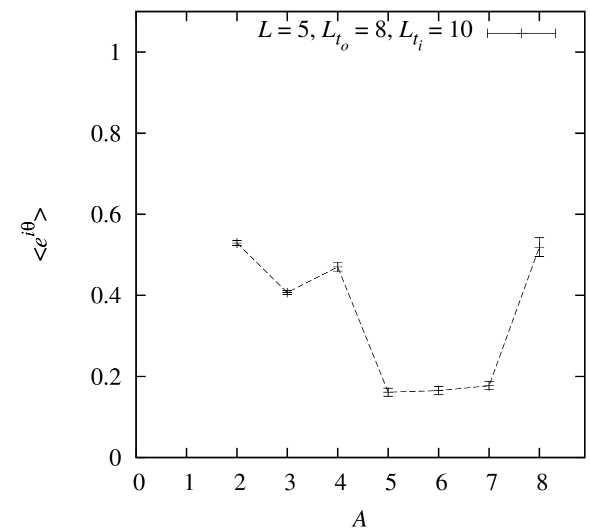
SYMMETRY-SIGN EXTRAPOLATION METHOD

Epelbaum, Krebs, Lähde, Lee, Luu, UGM, Rupak, arXiv:1502.06787

- so far: nuclei with $N = Z$, and $A = 4 \times \mathrm{int}$
  as these have the least sign problem
  due to the approximate SU(4) symmetry

$$\langle \mathrm{sign} \rangle = \langle \exp(i\theta) \rangle = \frac{\det M(t_o, t_i, \ldots)}{|\det M(t_o, t_i, \ldots)|}$$

$M(t_o, t_i, \ldots)$ is the transition matrix



Borasoy et al. (2007)

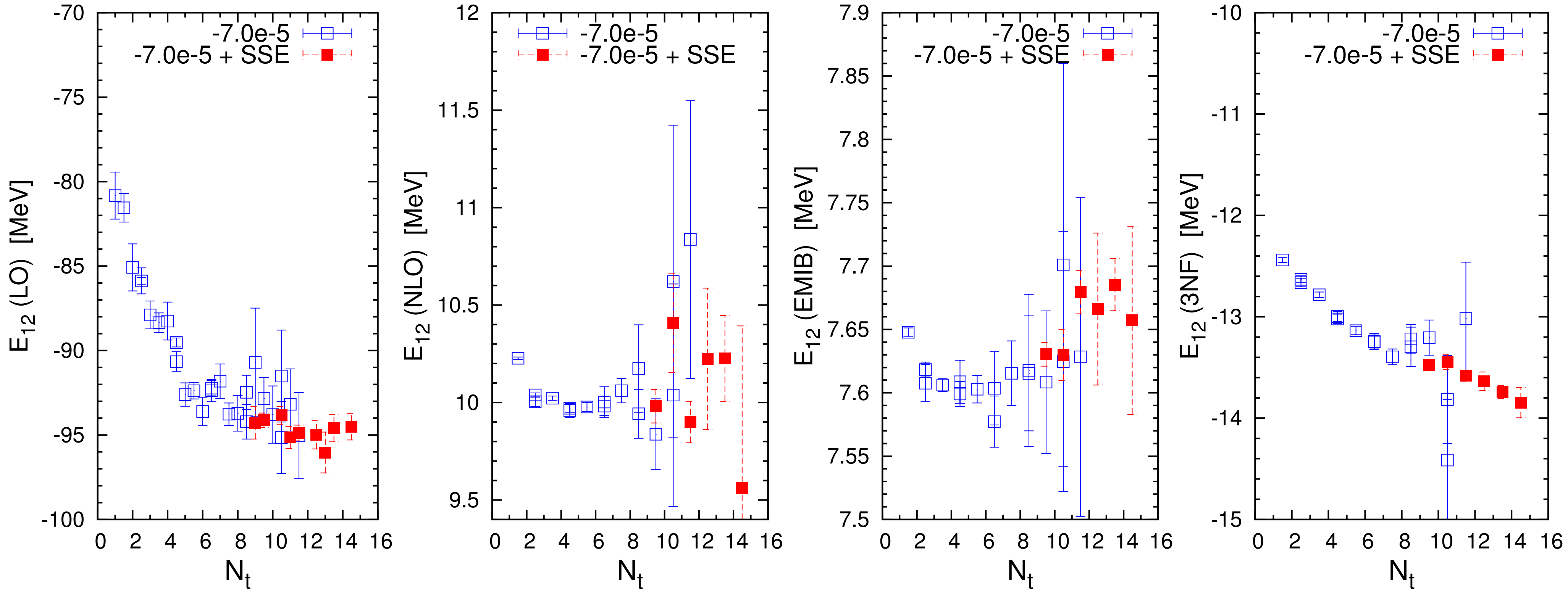
- Symmetry-sign extrapolation (SSE) method: control the sign oscillations

$$H_{d_h} = d_h \cdot H_{\mathrm{phys}} + (1 - d_h) \cdot H_{\mathrm{SU(4)}} \qquad H_{\mathrm{SU(4)}} = \frac{1}{2} C_{\mathrm{SU(4)}} \, (N^\dagger N)^2$$

$\hookrightarrow$ family of solutions for different SU(4) couplings $C_{\mathrm{SU(4)}}$
that converge on the physical value for $d_h = 1$

RESULTS for ^{12}C

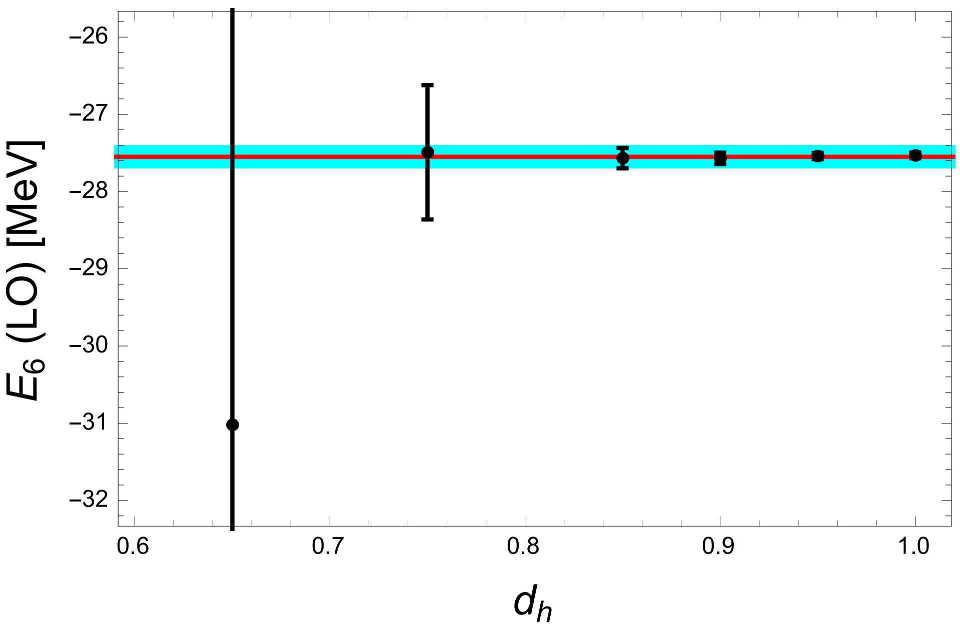
- generate a few more MC data at large N_t using SSE

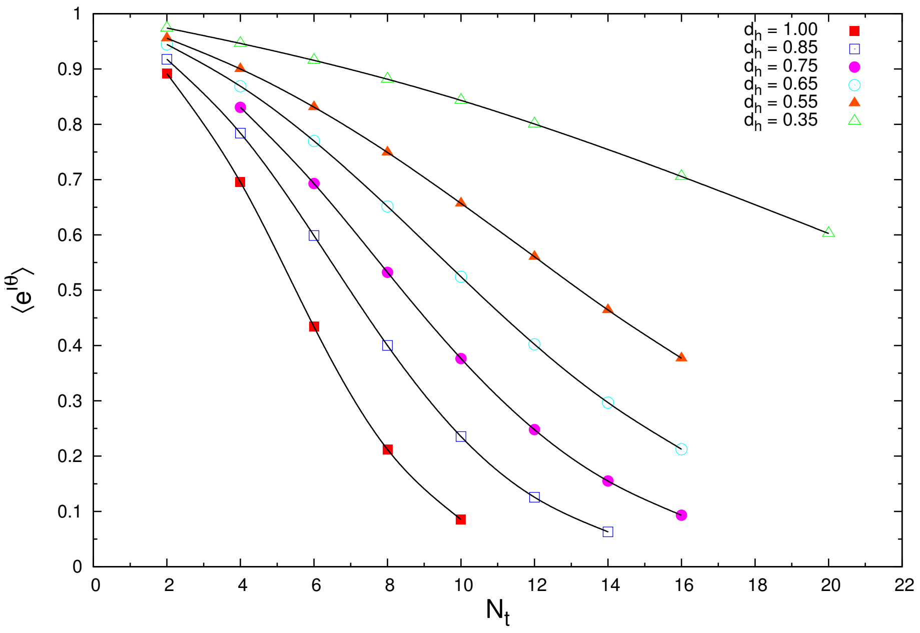


- promising results $\rightarrow$ no more exponential deterioration of the MC data
- results w/ small uncertainties for $d_h \geq 0.8$

RESULTS for $A = 6$

- Simulations for ^{6}He and ^{6}Be





$\Rightarrow$ methods works for nuclei with $A \neq Z$

$\Rightarrow$ neutron-rich nuclei can now be systematically explored (larger volumes)