UNIVERSITÄT BONN

JÜLICH Forschungszentrum

Lectures on
STRONG INTERACTIONS

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STRUCTURE of the LECTURES

I) Short Introduction

II) Effective Field Theories

III) Chiral QCD Dynamics

IV) Testing Chiral Dynamics in Hadron-Hadron Scattering

V) Nuclear Forces from EFT

VI) Chiral Dynamics in Nuclei

- more emphasis on the foundations rather than on specific calculations

Introduction

FORCES in NATURE

type	gauge boson	spin	range	strength
		$[\hbar]$	[m]	@ hadronic scale
gravity	graviton	2	∞	10^{-40}
weak int.	W,Z-bosons	1	10^{-17}	10^{-5}
EM int.	photon	1	∞	$1/137$
strong int.	gluons	1	10^{-15}	~ 1

} SU(3)$_C\times$SU(2)$_L\times$U(1)$_Y$

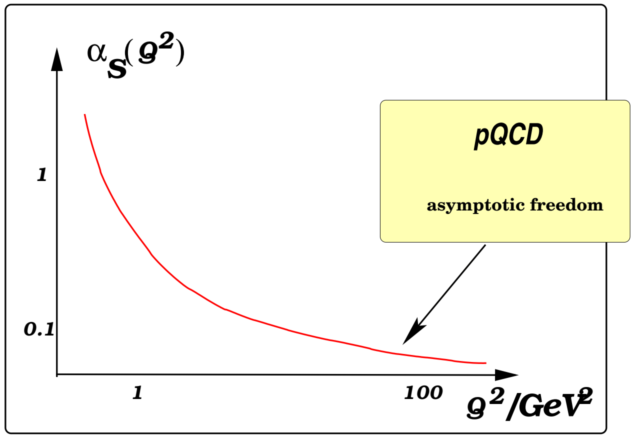
- electro-weak interactions are perturbative at hadronic scales
- strong interactions are really strong $\rightarrow$ non-perturbative

THE CHALLENGE: STRONG QCD

- The Standard Model has two open ends : 1) Physics beyond the SM
  2) strong QCD

Running coupling $\alpha_S(Q^2)$:

Gross, Politzer, Wilczek



- Weak coupling at large momentum transfer (perturbative QCD)
  $\rightarrow$ successfull tests $\surd$
- Grand challenge: STRONG QCD (non-perturbative, $\alpha_s(Q^2) \simeq 1$)

$\Rightarrow$ Effective Field Theories and/or Simulations

QCD LAGRANGIAN

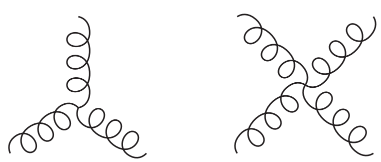
Note: θ-term not discussed here

- $$\mathcal{L}_{\rm QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} + \sum_f \bar{q}_f (i\gamma_\mu D^\mu - \mathcal{M}) q_f + \ldots$$

$$\underbrace{G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g[A^b_\mu, A^c_\nu]}_{\text{gluon field strength tensor}}, \quad \underbrace{D_\mu = \partial_\mu + igA^a_\mu \lambda^a/2}_{\text{covariant derivative}}$$

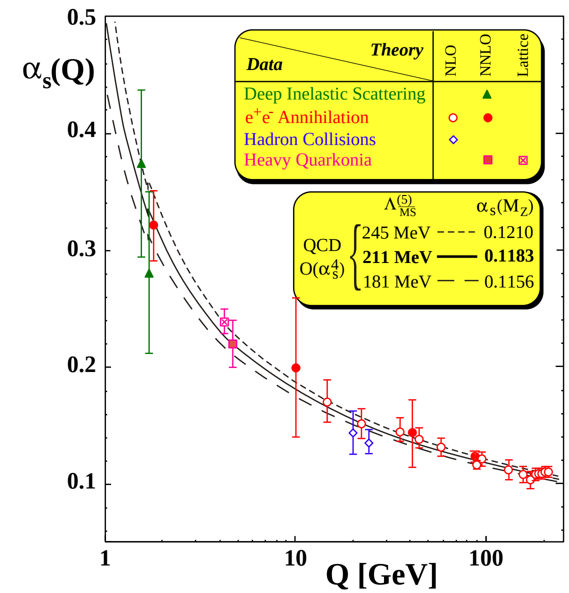
$$\underbrace{f = (u,d,s,c,b,t)}_{\text{flavors}}, \quad \underbrace{\mathcal{M} = \text{diag}(m_u, m_d, m_s, m_c, m_b, m_t)}_{\text{quark mass matrix}}$$

- local color gauge invariance SU(3)$_C$
- non-linear couplings:



- running of $\alpha_s = g^2/4\pi$
- **light** (u,d,s) and heavy (c,b,t) quark flavors:

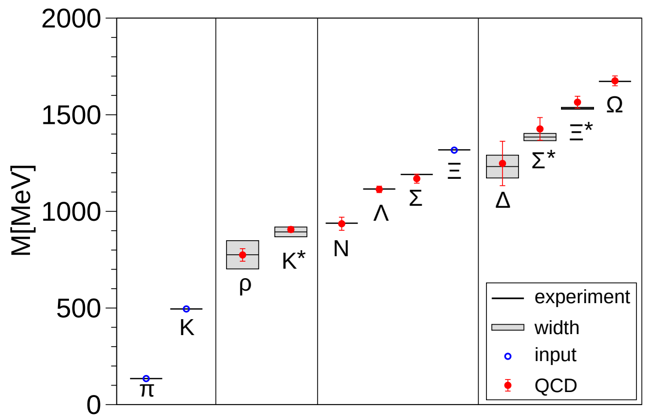
$$m_{\rm light} \ll \Lambda_{\rm QCD},\, m_{\rm heavy} \gg \Lambda_{\rm QCD} \qquad \Lambda_{\rm QCD} \simeq 250 \text{ MeV}$$



- quarks and gluons are **confined** within hadrons & nuclei

$\hookrightarrow$ must study these composites

FACETS of STRONG QCD



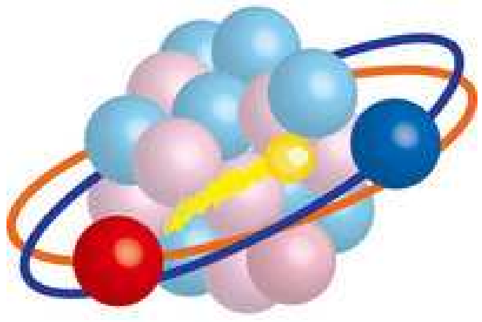
BMW collaboration

- quarks and gluons form hadrons

  ⇒ **hadron physics**

  ⇒ exploring the strong color force

- nucleons and mesons form nuclei



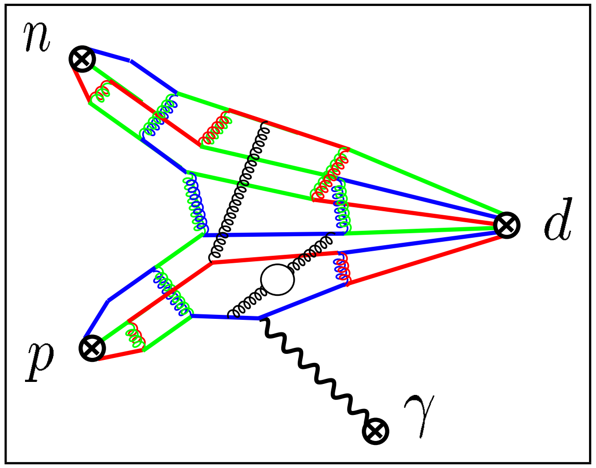
  ⇒ **nuclear physics**

  ⇒ exploring the residual color force

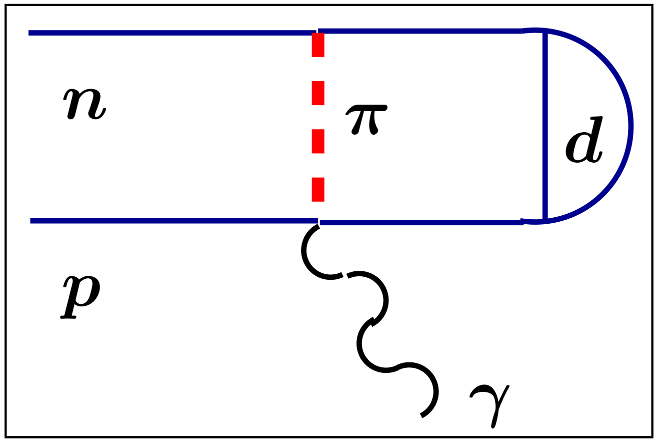
Hadron and nuclear physics ask the same questions:
How do strongly interacting composites emerge?
and what are their properties?

RESIDUAL CHROMODYNAMIC FORCES

- Quarks and gluons are confined within hadrons
- Nuclear forces are the residual forces between colorless objects
- Hadronic energies correspond to a low resolution microscope
- $np \to d\gamma$



$\Rightarrow$



wrong d.o.f.s　　　　right d.o.f.s

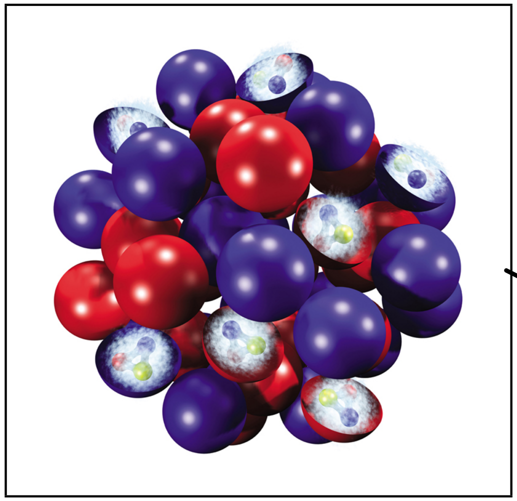
Effective Field Theories

BASIC IDEAS: RESOLUTION MATTERS

- Dynamics at long distances does not depend on what goes on at short distances

- Equivalently, low energy interactions do not care about the details of high energy interactions

- Or: you don't need to understand nuclear physics to build a bridge



BASIC IDEAS: ORGANISATION

- This is quite true, but how to make the idea precise and quantitative?
- necessary & sufficient ingredients to construct an **E**ffective **F**ield **T**heory:

  ⋆ *scale separation* – what is low, what is high?

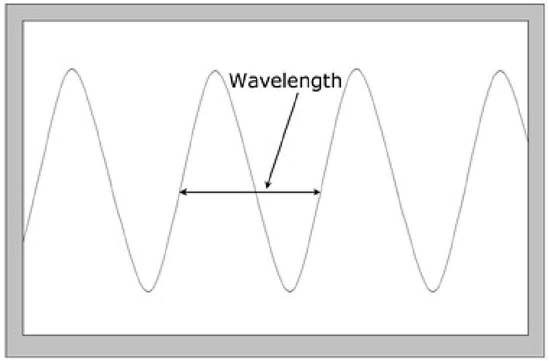
  ⋆ *active degrees of freedom* – what are the building blocks?

  ⋆ *symmetries* – how are the interactions constrained by symmetries?

  ⋆ *power counting* – how to organize the expansion in low over high?

- a note on units for a quantum particle ($\hbar = c = 1$)

$$p \sim \frac{1}{\lambda}, \quad E = p \quad \text{or} \quad E = \frac{p^2}{2m} \quad \text{or} \quad E = \sqrt{p^2 + m^2}$$



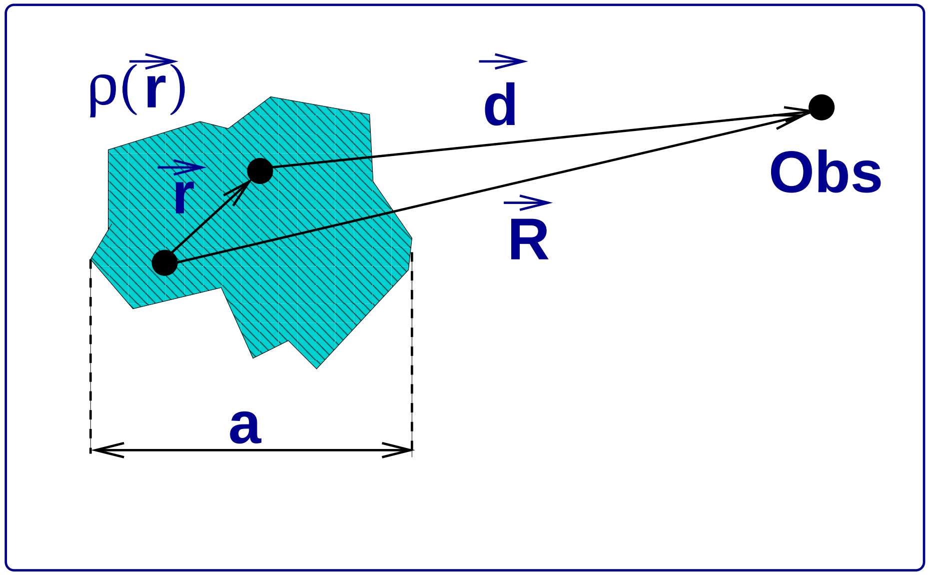
$\rightarrow$ long wavelength $\leftrightarrow$ low momentum

Effective Field Theory: Learning by Example

EXAMPLE 1: MULTIPOLE EXPANSION

- Multipole expansion for electric potentials [not quite a quantum field theory]

$$
\begin{aligned}
V &\approx \int \frac{\rho(\vec{r}\,)}{d} d^3r \\
&= \int \frac{\rho(\vec{r}\,)}{\sqrt{R^2 + 2rR\cos\theta + r^2}} d^3r \\
&= \sum_{n=0}^{\infty} \frac{1}{R^{n+1}} \int r^n P_n(\cos\theta)\rho(\vec{r}\,) d^3r \\
&= q\frac{1}{R} + p\frac{1}{R^2} + Q\frac{1}{R^3} + \ldots
\end{aligned}
$$



- the sum converges quickly for $a \ll R$
- long-distance (low-energy) probes are only sensitive to bulk properties:
  charge q, dipole moment p, ...
- aside: “don’t be a slave of indices, make them your slaves” [Howard Georgi]

EXAMPLE 2: WHY THE SKY IS BLUE

- Light-atom scattering involves very different scales:

$$\lambda_{\rm light} \sim 5000\ \text{Å} \gg a_{\rm atom} \sim \text{ a few Å} \sim \text{ a few } a_0$$

$\Rightarrow$ photons are insensitive to the atomic structure

$$\overset{\text{gauge inv.},P,T}{\Longrightarrow} \quad \boxed{H_{\rm eff} = \chi^\star \left[-\tfrac{1}{2} c_E \vec{E}^2 - \tfrac{1}{2} c_B \vec{B}^2 \right] \chi}$$

(χ = atomic wave function)

- fixing the constants: $\dfrac{\text{field energy}}{\text{volume}} \sim \frac{1}{2}(\vec{E}^2 + \vec{B}^2) \Rightarrow \boxed{c_E = k_E\, a_0^3,\ c_B = k_B\, a_0^3}$

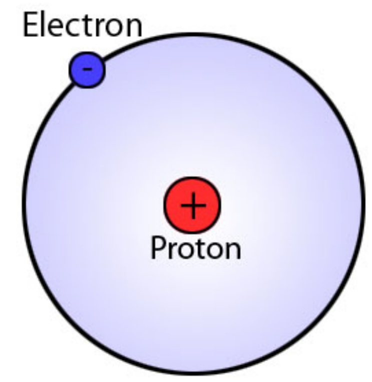
- If k_E and k_B are natural, i.e.of order one, and with $|\vec{E}| \sim \omega$ and $|\vec{B}| \sim |\vec{k}| \sim \omega$:

$$\boxed{\frac{d\sigma}{d\Omega} = |\langle f|H_{\rm eff}|i\rangle|^2 \sim \omega^4\, a_0^6 \left(1 + \frac{\omega^2}{\Delta E^2}\right)}$$

ΔE = corr. due to atomic excitations

EXAMPLE 3: THE HYDROGEN ATOM

- text-book example of a quantum bound state of an electron and a proton



- lowest order: we need the mass & charge of the electron & charge of the proton & the static Coulomb interaction:

$$E = E_0 = -\frac{m_e \alpha^2}{2n^2}\,, \quad \alpha = \frac{e^2}{4\pi}$$

but this is not the *exact* answer, how can we improve on it?

we can get an approximate answer and improve on it $\rightarrow$ difference to maths!

- beyond leading order: $E = E_0 \left[1 + \mathcal{O}\left(\alpha, \frac{m_e}{m_p}\right)\right]$ $\rightarrow$ systematic expansion

  – corrections from the em interaction — fine-structure from $\vec{L}\cdot\vec{S} \sim \alpha^4$ etc.

  – corrections from the proton structure — $m_p \rightarrow$ reduced mass $\mu = \frac{m_e m_p}{m_e + m_p}$

  $\mu_p \rightarrow$ hyperfine interaction

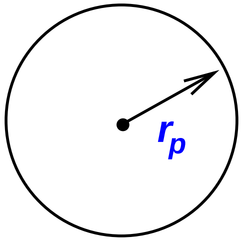
EXAMPLE 3 cont'd: DIMENSIONAL ANALYSIS

- calculate the influence of the proton **size** r_p on the hydrogen energy levels

- natural scales: length $a_0 = 1/(m_e\alpha) \sim 0.5$ Å

  time $1/\text{Ryd} = 2/(m_e\alpha^2)$, $1\ \text{Ryd} = 13.6$ eV

- proton charge radius: $F_1(q^2) = 1 + q^2 F_1'(0) + \ldots$



$$F_1'(0) \simeq \frac{1}{m_p^2}\ , \quad q \sim \frac{1}{a_0} = m_e\alpha \ \rightarrow \ \left(\frac{m_e\alpha}{m_p}\right)^2 \sim 10^{-11}$$

- contribution of $\mathcal{O}(40\,\text{kHz})$ to the energy levels

- $1/m_p = 0.2$ fm. Actual proton size $\simeq 0.85$ fm

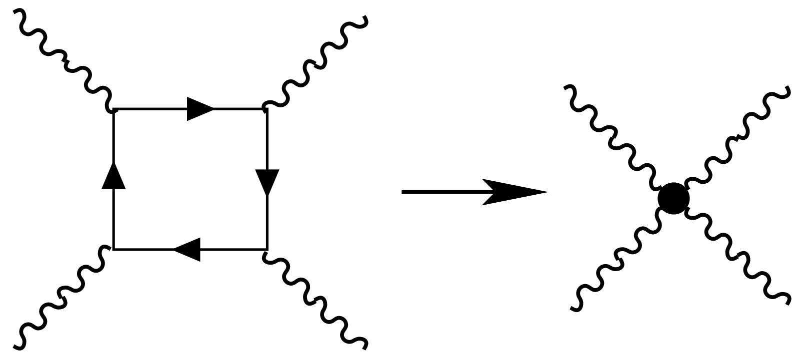
  $\longrightarrow$ net contribution to $1S$ about 1000 kHz

- Proton size measurable in *muonic hydrogen* $(m_\mu/m_e \sim 200)$ Pohl et al. (2010)

what a pleasure: can do calculations without knowing the underlying theory

EXAMPLE 4: LIGHT-BY-LIGHT SCATTERING

Euler, Heisenberg, Kockel 1936



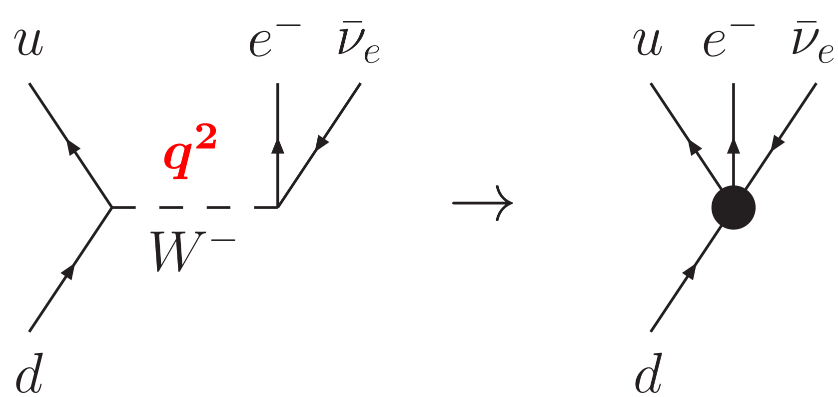
- energy scales: photon energy ω, electron mass m_e
- consider $\omega \ll m_e$
- fermions as massive dofs integrated out: $\mathcal{L}_{\rm QED}[\psi, \bar\psi, A_\mu] \to \mathcal{L}_{\rm eff}[A_\mu]$

$$\mathcal{L}_{\rm eff} = \frac{1}{2}(\vec{E}^2 - \vec{B}^2) + \frac{e^4}{360\pi^2 m_e^4}\left[(\vec{E}^2 - \vec{B}^2)^2 + 7(\vec{E}\cdot\vec{B})^2\right] + \ldots$$

- invariants: $F_{\mu\nu}F^{\mu\nu} \sim \vec{E}^2 - \vec{B}^2$, $F_{\mu\nu}\tilde{F}^{\mu\nu} \sim (\vec{E}\cdot\vec{B})^2$
- energy expansion: $(\omega/m_e)^{2n}$ small parameter
- leads to the Xsection: $\sigma(\omega) = \frac{1}{16\pi^2}\frac{973}{10125\pi}\frac{e^8}{m_e^2}(\omega/m_e)^6$
- seen in heavy-ion collisions ATLAS coll., Nature Physics **13** (2017) 852

EXAMPLE 5: FERMI THEORY

- Weak decays
  - mediated by the charged W bosons, $M_W \simeq 80$ GeV
  - energy release in neutron β-decay $\simeq 1$ MeV $[n \to p e^- \nu_e]$
  - energy release in kaon decays $\simeq$ a few 100 MeV $[K \to \pi\, \ell\, \nu]$



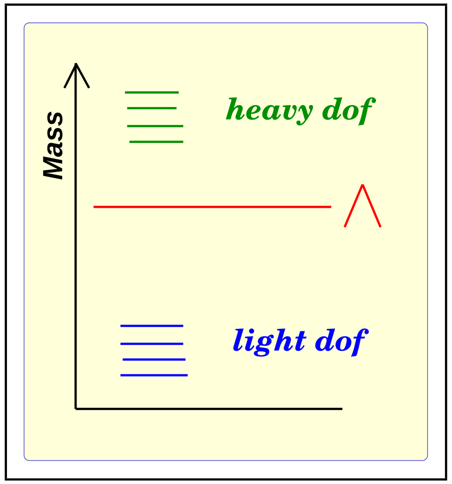
$$\frac{e^2}{8\sin\theta_W} \times \frac{1}{M_W^2 - q^2} \xrightarrow{q^2 \ll M_W^2} \frac{e^2}{8M_W^2 \sin\theta_W}\left\{1 + \frac{q^2}{M_W^2} + \ldots\right\}$$

$$= \frac{G_F}{\sqrt{2}} + \mathcal{O}\left(\frac{q^2}{M_W^2}\right)$$

$\Rightarrow$ Fermi's current-current interaction

BRIEF SUMMARY of EFFECTIVE FIELD THEORY

- Separation of scales: low and high energy dynamics
  - ⋆ low-energy dynamics in terms of relevant dof's
    energies $\sim$ momenta $\sim Q$
  - ⋆ high-energy dynamics not resolved
    $\longrightarrow$ contact interactions



Weinberg 1979

- Small parameter(s) & power counting
  - ⋆ Standard QFT: trees + loops $\longrightarrow$ renormalization
  - ⋆ Expansion in powers of energy/momenta Q over the large scale Λ

$$\mathcal{M} = \sum_\nu \left(\frac{Q}{\Lambda}\right)^\nu f(Q/\mu, g_i)$$

μ – regularization scale
g_i – low–energy constants

- f is a function of $\mathcal{O}(1)$ – "naturalness"
- ν bounded from below

$\Rightarrow$ systematic and controlled expansion

NB: bound states require non-perturbative resummation

The Paradigm Shift in Quantum Field Theory

A NEW LOOK AT RENORMALIZATION

- Renormalization: method to tame the infinities in quantum field theories
- Renormalizable gauge field theories have led to some of the most stunning successes in physics: QED tested to better than 10^{-10}
- It has become clear that no theory works at **all** scales, e.g. the Standard Model must break down at the Plank scale (or even earlier)
- The basic idea about renormalization today is that the influences of higher energy processes are localisable in a few structural properties which can be captured by an adjustment of parameters.

> "In this picture, the presence of infinities in quantum field theory is neither a disaster, nor an asset. It is simply a reminder of a practical limitation – we do not know what happens at distances much smaller than those we can look at directly" (Georgi 1989)

THE PARADIGM OF EFFECTIVE FIELD THEORY

- constructing a **Q**uantum **F**ield **T**heory in 4 steps

1) construct the action $S[\ldots]$, respect *symmetries*

e.g. gauge invariance of QED $\psi \to \psi' = \mathrm{e}^{-i\alpha(x)}\psi,\ A_\mu \to A'_\mu = A_\mu - \partial_\mu \alpha(x)$

2) retain *renormalizable* interactions ($D \leq 4$)

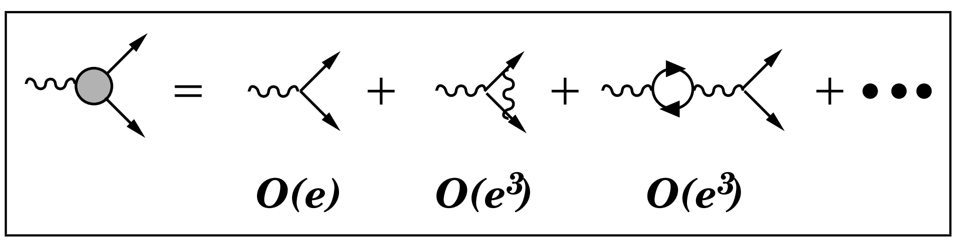
keep $\underbrace{\bar\psi \gamma_\mu \psi A^\mu}_{D=4}, \ \underbrace{F_{\mu\nu}F^{\mu\nu}}_{D=4}, \ldots$ drop $\underbrace{\bar\psi \sigma_{\mu\nu} \psi F^{\mu\nu}}_{D=5}, \ \underbrace{(F_{\mu\nu}F^{\mu\nu})^2}_{D=8}, \ldots$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\ , \quad [F_{\mu\nu}] = 2$

D = canonical dimension, e.g. $[A_\mu] = 1,\ [\psi] = 3/2,\ [\partial_\mu] = 1$

3) *quantize*: calculate scattering processes in perturbation theory:

tree + *loop* graphs



THE PARADIGM OF EFFECTIVE FIELD THEORY cont'd

4) fix the *parameters* from *data*, make *predictions*

e.g. $$\mu_e = -\frac{e g_e \vec{s}_e}{2m_e}\,, \quad g_e = 2\left[1 + \frac{e^2}{8\pi^2} + \mathcal{O}(e^4)\right]$$

- constructing an **E**ffective **F**ield **T**heory

  steps 1,3,4: logically necessary

  step 2: renormalizability = physics at all scales

another consistent & predictive paradigm:

keep rules 1,3,4, but instead use

2*) work at *low* energies & *expand* in powers of the *energy*

– separation of scales

– only a finite number of operators plays a role

– familiar concept → examples just discussed

EFT: FUNDAMENTAL THEOREM

- Weinberg's conjecture: Physica **A96** (1979) 327

> Quantum Field Theory has no content besides unitarity, analyticity, cluster decomposition, and symmetries.

$$\Downarrow$$

To calculate the S-matrix for any theory below some scale, simply use the most general effective Lagrangian consistent with these principles in terms of the appropriate asymptotic states.

Structure of Effective Field Theories

STRUCTURE of EFTs

- Energy expansion [derivative/momentum/. . .]

dimensional analysis:

(a) derivatives $\to$ powers of q [small scale]

(b) be Λ the hard [limiting] scale

$\to$ any derivative $\partial \sim q/\Lambda$

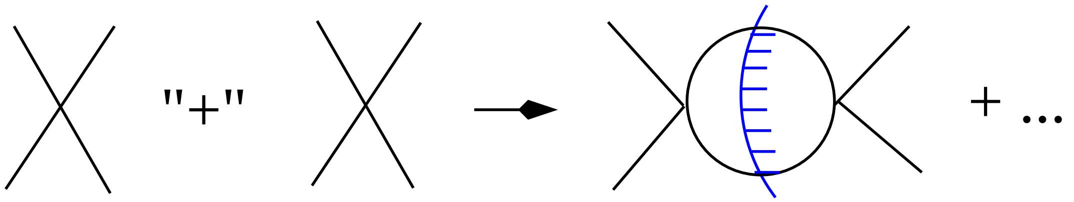
$\to$ N derivative vertex $\sim q^N/\Lambda^N$

$\to$ for $E[q] \ll \Lambda$, terms w/ more derivatives are suppressed

- Energy expansion = Loop expansion

interactions generate loops

loops generate imaginary parts



$\Rightarrow$ all this is contained in the *power counting*, which assigns a dimension [not the canonical one] to each diagram

POWER COUNTING THEOREM

- Consider $\mathcal{L}_{\rm eff} = \sum_d \mathcal{L}^{(d)}$, d bounded from below
- for interacting Goldstone bosons, $d \geq 2$ and $iD(q) = \dfrac{1}{q^2 - M^2}$
- consider an L-loop diagram with I internal lines and V_d vertices of order d

$$Amp \propto \int (d^4q)^L \, \frac{1}{(q^2)^I} \prod_d (q^d)^{V_d}$$

- let $Amp \sim q^\nu \to \nu = 4L - 2I + \sum_d d V_d$
- topology: $L = I - \sum_d V_d + 1$
- eliminate I: $\to$ $\boxed{\nu = 2 + 2L + \sum_d V_d (d-2)}$ $\surd$

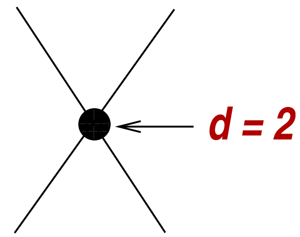
POWER COUNTING for PION-PION SCATTERING

- $\pi(p_1) + \pi(p_2) \to \pi(p_3) + \pi(p_4)$

  leading interaction $\sim \partial\pi\,\partial\pi \Rightarrow d = 2$

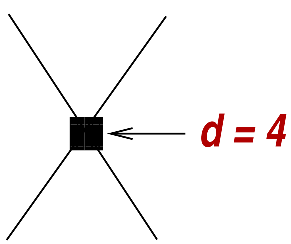
- leading order (LO)

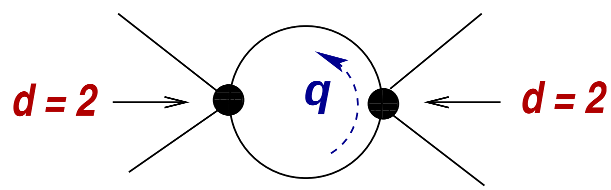
  $d = 2, N_L = 0 \Rightarrow D = 2$



- next-to-leading order (NLO)

  $a)\ d = 4, N_L = 0 \Rightarrow D = 4$ $\qquad$ $b)\ d = 2, N_L = 1 \Rightarrow D = 4$





$$\sim \int d^4q \frac{q_1 \cdot q_2 \;\; q_3 \cdot q_4}{(q^2 - M_\pi^2)(q^2 - M_\pi^2)} \sim \mathcal{O}(q^4)$$

LOW-ENERGY CONSTANTS (LECs)

- consider a covariant & parity-invariant theory of Goldstone bosons parameterized in some matrix-valued field U

$$\mathcal{L}_{\rm eff} = g_2 {\rm Tr}(\partial_\mu U \partial^\mu U^\dagger) + g_4^{(1)} \left[{\rm Tr}(\partial_\mu U \partial^\mu U^\dagger)\right]^2 + g_4^{(2)} {\rm Tr}(\partial_\mu U \partial^\nu U^\dagger) {\rm Tr}(\partial_\nu U \partial^\mu U^\dagger) + \ldots$$

- couplings = **low-energy constants** (LECs)

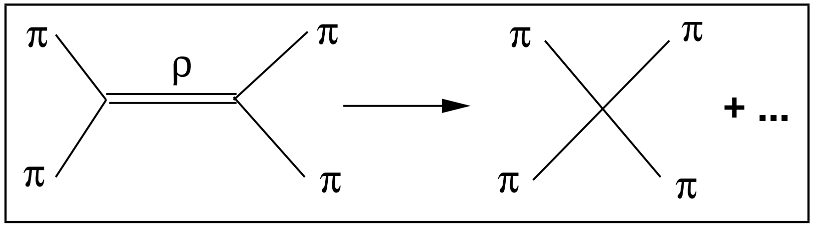
  $g_2 \neq 0$ spontaneous chiral symmetry breaking (cf also the $g^{(i)}_{>2}$)

  $g_4^{(1)}, g_4^{(2)}, \ldots$ must be fixed from data (or calculated from the underlying theory)

- calculations in EFT: fix the LECs from some processes, then make **predictions**

- LECs encode information about the high mass states that are integrated out

$$\frac{g^2_{\rho\pi\pi}}{M_\rho^2 - q^2} \stackrel{q^2 \ll M_\rho^2}{\longrightarrow} \frac{g^2_{\rho\pi\pi}}{M_\rho^2}\left(1 + \frac{q^2}{M_\rho^2} + \ldots\right)$$

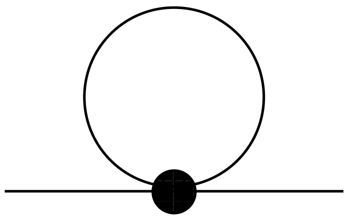


LOOPS and DIVERGENCES

- Loop diagrams generate imag. parts, but are mostly **divergent**

$\Rightarrow$ choose a mass-independent & symmetry-preserving regularization scheme
[like dimensional regularization]

Ex.:



$$= -i\Delta_\pi(0) = \frac{-i}{(2\pi)^d}\int d^dp\,\frac{1}{M^2-p^2-i\varepsilon} \quad [d \text{ space-time dim.}]$$

$$= (2\pi)^{-d}\int d^dk\,\frac{1}{M^2+k^2} \text{ with } p_0 = ik_0\,, \quad -p^2 = k_0^2+\vec{k}^{\,2}$$

$$= (2\pi)^{-d}\int d^dk \int_0^\infty d\lambda\,\exp(-\lambda(M^2+k^2))$$

$$= (2\pi)^{-d}\int_0^\infty d\lambda\,\exp(-\lambda M^2)\underbrace{\int d^dk\,\exp(-\lambda k^2)}_{(\pi/\lambda)^{d/2}}$$

$$= (4\pi)^{-d}\,M^{d-2}\,\Gamma\left(1-\frac{d}{2}\right)$$

has a pole at $d = 4$

$\Rightarrow$ absorb in LECs: $\boxed{\boldsymbol{g_i \to g_i^{\rm ren} + \beta_i \frac{1}{d-4}}}$ always possible!

INTERMEDIATE SUMMARY

- Effective field theories explore scale separation in physical systems

  low-energy physics treated explicitely

  high-energy modes integrated out $\to$ contact interactions

  $\to$ low-energy constants

- Interactions generate loops, loops restore unitarity
- Power counting: systematic ordering of all graphs, loops are suppressed
- Loop graphs are generally divergent $\to$ order-by-order renormalization

FURTHER REMARKS

- Decoupling EFTs: Appelquist, Carrazone (1975)
  - effects of the heavy fields are power-suppressed or appear in the renormalization of the light field couplings
  - as $M_H \to \infty$, heavy fields decouple & shifts become unobservable
  - RGEs / RG flow: powerful tool to analyze decoupling EFTs
  - Examples:
    - QED at $E \ll m_e \to$ Euler-Heisenberg Lagrangian
    - weak int. at $E \ll M_W \to$ Fermi's four-fermion Lagrangian
    - SM at $E \ll 1$ TeV $\to \mathcal{L}_{\rm eff} = SU(3)_C \times SU(2)_L \times U(1)_Y$

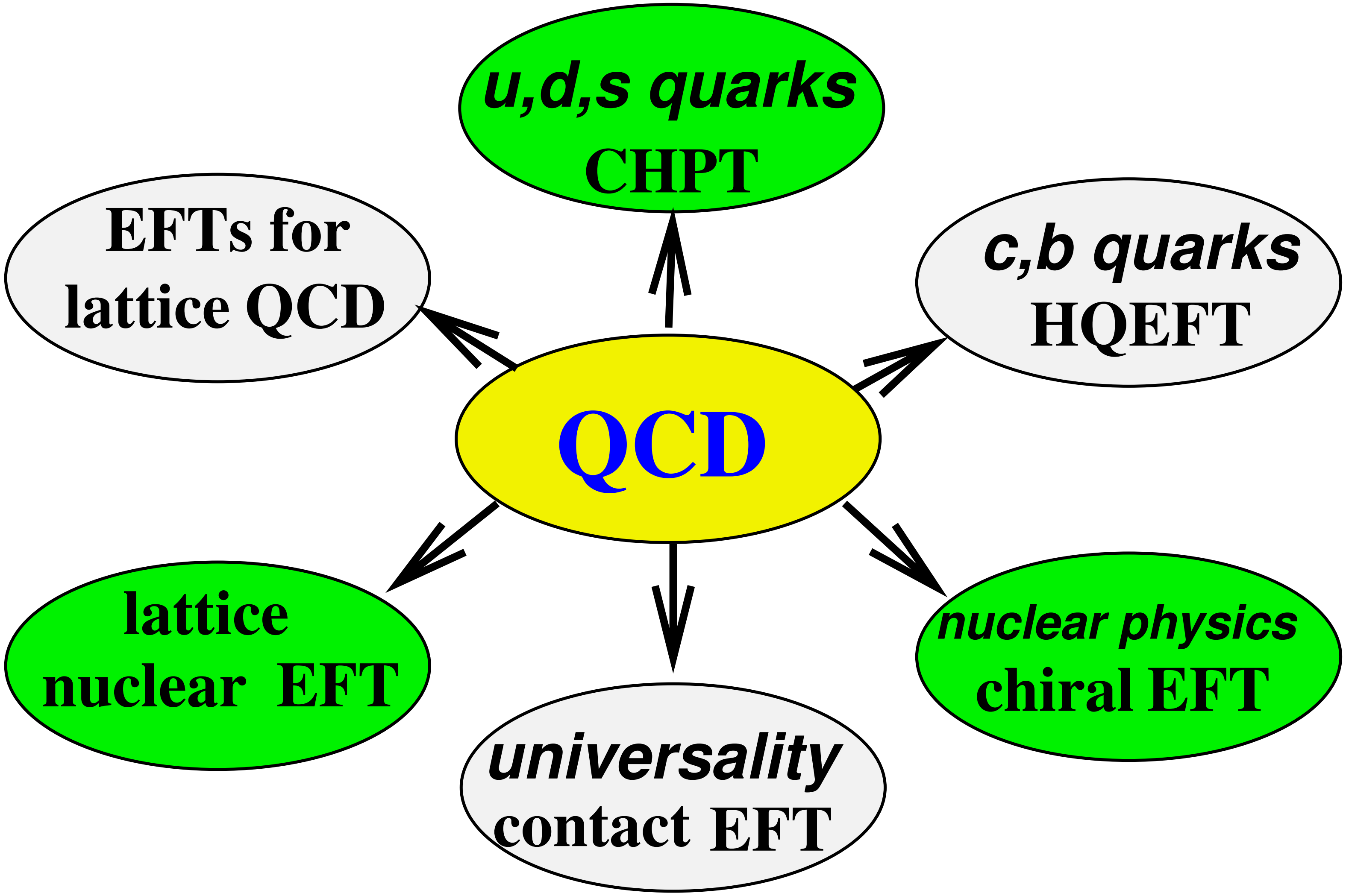
FURTHER REMARKS cont'd

- Non-decoupling EFTs:
  - during the transition $\mathcal{L} \to \mathcal{L}_{\text{eff}}$, phase transition via spontaneous symmetry breaking w/ generation of (pseudo-) Goldstone bosons with masses $M_{\text{GB}} \ll \Lambda_{\text{SSB}}$
  - SSB entails relations between MEs w/ different no. of GBs
    $\to D < 4$ or $D \geq 4$ becomes meaningless
    $\to \mathcal{L}_{\text{eff}}$ is intrinsically non-renormalizable
  - Examples:
    - SM w/o Higgs $\to$ GBs = longitudinal comp. of the V-bosons
    - SM below $\Lambda_{\chi SB} \simeq 1$ GeV $\to$ QCD chiral dynamics

FINAL SUMMARY on EFTs

- Basic ideas underlying EFT:
  Separate different scales, identify proper degrees of freedom
- Implement the consequences of symmetries
- EFT allows you to compute using dimensional analysis
  (even if you don't know the theory)
- EFT is very useful way of thinking about problems
- All quantum field theories are EFTs

EFTs of the STRONG INTERACTIONS



- strongly intertwined
- these lectures

QCD chiral dynamics

INTRO: CHIRAL SYMMETRY

- Massless fermions exhibit **chiral symmetry**:

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial^\mu\psi$$

- left/right-decomposition:

$$\psi = \frac{1}{2}(1-\gamma_5)\psi + \frac{1}{2}(1+\gamma_5)\psi = P_L\psi + P_R\psi = \psi_L + \psi_R$$

- projectors:

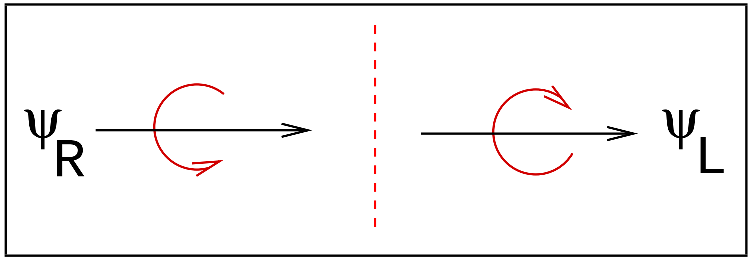
$$P_L^2 = P_L,\ P_R^2 = P_R,\ P_L \cdot P_R = 0,\ P_L + P_R = \mathbb{1}$$

- helicity eigenstates:

$$\frac{1}{2}\hat{h}\psi_{L,R} = \pm\frac{1}{2}\psi_{L,R} \quad \hat{h} = \vec{\sigma}\cdot\vec{p}/|\vec{p}\,|$$

- L/R fields do **not** interact $\rightarrow$ conserved L/R currents

$$\mathcal{L} = i\bar{\psi}_L\gamma_\mu\partial^\mu\psi_L + i\bar{\psi}_R\gamma_\mu\partial^\mu\psi_R$$



- mass terms break chiral symmetry: $\bar{\psi}\mathcal{M}\psi = \bar{\psi}_R\mathcal{M}\psi_L + \bar{\psi}_L\mathcal{M}\psi_R$

CHIRAL SYMMETRY of QCD

- Three flavor QCD:

$$\boxed{\mathcal{L}_{\rm QCD} = \mathcal{L}^0_{\rm QCD} - \bar{q}\mathcal{M}q}\ , \ q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}\ , \quad \mathcal{M} = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$$

- $\mathcal{L}^0_{\rm QCD}$ is invariant under **chiral** $SU(3)_L \times SU(3)_R$ (split off U(1)'s)

$$\mathcal{L}^0_{\rm QCD}(G_{\mu\nu}, q', D_\mu q') = \mathcal{L}^0_{\rm QCD}(G_{\mu\nu}, q, D_\mu q)$$

$$q' = RP_R q + LP_L q = Rq_R + Lq_L \quad R, L \in SU(3)_{R,L}$$

- conserved L/R-handed [vector/axial-vector] Noether currents:

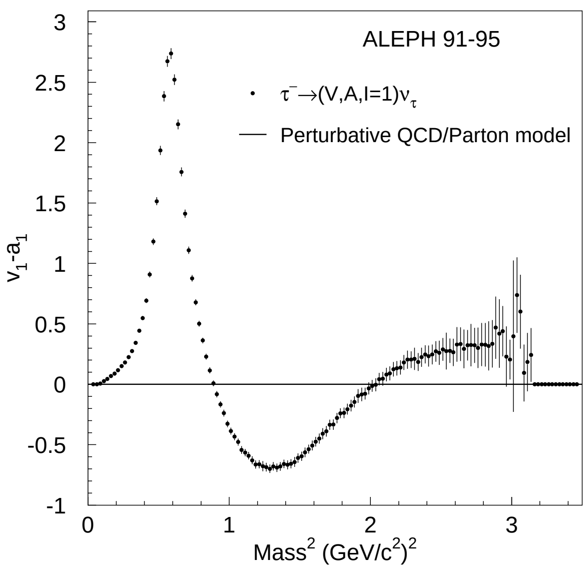
$$J^{\mu,a}_{L,R} = \bar{q}_{L,R}\gamma^\mu \frac{\lambda^a}{2} q_{L,R}\ , \quad a = 1, \ldots, 8$$

$$\partial_\mu J^{\mu,a}_{L,R} = 0 \quad [\text{or} \ \ V^\mu = J^\mu_L + J^\mu_R\ , \ \ A^\mu = J^\mu_L - J^\mu_R]$$

- Is this symmety reflected in the vacuum structure/hadron spectrum?

THE FATE of QCD's CHIRAL SYMMETRY

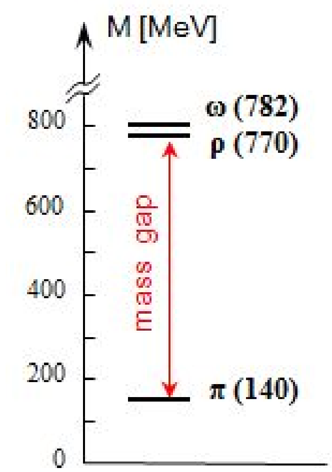
- the chiral symmetry is not "visible" (spontaneously broken)
  - no parity doublets
  - $\langle 0|AA|0\rangle \neq \langle 0|VV|0\rangle$
  - scalar condensate $\bar{q}q$ acquires v.e.v.
  - Vafa-Witten theorem [NPB 234 (1984) 173]
  - (almost) massless pseudoscalar bosons



- the chiral symmetry is realized in the Nambu-Goldstone mode
  - weakly interacting massless pseudoscalar excitations
  - approximate symmetry (small quark masses)

    $\to \pi, K, \eta$ as Pseudo-Goldstone Bosons
  - calls for an effective field theory

  $\Rightarrow$ Chiral Perturbation Theory



THE FATE of QCD's CHIRAL SYMMETRY II

- Wigner mode $Q_5^a|0\rangle = Q^a|0\rangle = 0$ $(a = 1, \ldots, 8)$?

- parity doublets: $dQ_5^a/dt = 0 \to [H, Q_5^a] = 0$

  single particle state: $H|\psi_p\rangle = E_p|\psi_p\rangle$

  axial rotation: $H(\mathrm{e}^{iQ_5^a}|\psi_p\rangle) = \mathrm{e}^{iQ_5^a}H|\psi_p\rangle = \underbrace{E_p(\mathrm{e}^{iQ_5^a}|\psi_p\rangle)}$

  same mass but opposite parity

- VV and AA spectral functions (without pion pole):

$$\begin{array}{c}
\langle 0|VV|0\rangle = \langle 0|(L+R)(L+R)|0\rangle = \langle 0|L^2+R^2+2LR|0\rangle = \langle 0|L^2+R^2|0\rangle \\
\| \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \| \\
\langle 0|AA|0\rangle = \langle 0|(L-R)(L-R)|0\rangle = \langle 0|L^2+R^2-2LR|0\rangle = \langle 0|L^2+R^2|0\rangle
\end{array}$$

since L and R are orthogonal

PROPERTIES of GOLDSTONE BOSONS

- GBs are massless [no explicit symmetry breaking]

consider a broken generator $[Q, H] = 0$ but $Q|0\rangle \neq 0$

define $|\psi\rangle \equiv Q|0\rangle$

$$\rightarrow H|\psi\rangle = HQ|0\rangle = QH|0\rangle = 0$$

$\rightarrow$ not only G.S. $|0\rangle$ has $E = 0$

There exist massless excitations, non-interacting as $E, p \rightarrow 0$

[NB: proper argumentation requires more precise use of the infinite volume]

- explicit symmetry breaking, perturbative [small parameter ε]

Goldstone bosons acquire a small mass $M^2_{\rm GB} \sim \varepsilon$

In QCD, this symmetry breaking is given in terms of the light quark masses

$$\Rightarrow M_\pi^2 \sim (m_u + m_d)$$

CHIRAL EFT of QCD

Gasser, Leutwyler, Weinberg, van Kolck, Kaplan, Savage, Wise, Bernard, Kaiser, M., . . .

- Starting point: CHIRAL LAGRANGIAN (two flavors)

$$\mathcal{L}_{\rm QCD} \to \mathcal{L}_{\rm EFF} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \ldots$$

- dofs: quark & gluon fields $\to$ pions, nucleons, external sources
- Spontaneous chiral symmetry breaking of QCD $\to$ pions are Goldstone bosons
- Systematic expansion in powers of q/Λ_χ & M_π/Λ_χ, with $\Lambda_\chi \simeq 1$ GeV
- pion and pion-nucleon sectors are perturbative in q $\to$ chiral perturbation theory
- Parameters in $\mathcal{L}_{\pi\pi}$ and $\mathcal{L}_{\pi N}$ known from CHPT studies $\to$ low-energy constants
- $\mathcal{L}_{NN}$ collects short-distance contact terms, to be fitted
- NN interaction requires non-perturbative resummation

$\to$ chirally expand $V_{\rm NN}$, use in regularized LS equation

CHIRAL PERTURBATION THEORY

- Consider first the mesonic chiral effective Lagrangian

$$\mathcal{L}_{\rm eff} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \ldots$$

$$\mathcal{L}^{(2)} = \frac{F_\pi^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U\rangle \qquad [U^\dagger U = U U^\dagger = 1, U \to L U R^\dagger]$$

$$U = \exp(i\Phi/F_\pi)\ , \quad \Phi = \sqrt{2}\begin{pmatrix} \pi^0/\sqrt{2}+\eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2}+\eta/\sqrt{6} & K^0 \\ K^- & \bar{K^0} & -2\eta/\sqrt{6} \end{pmatrix}$$

$\chi = 2B\mathcal{M} + \ldots,\ B = |\langle 0|\bar{q}q|0\rangle|/F_\pi^2 \leftarrow$ scalar quark condensate

- Two parameters:

$F_\pi \simeq 92$ MeV = pion decay constant (GB coupling to the vacuum)

$B \simeq 2$ GeV = normalized vacuum condensate

- Goldstone boson masses: $M^2_{\pi^+} = (m_u + m_d)B\ ,\ \ M^2_{K^+} = (m_d + m_s)B\ , \ldots$

- has been extended to two loops $\mathcal{O}(q^6)$ in many cases

FROM QUARK to MESON MASSES

- symmetry breaking Lagrangian: $\mathcal{L}_{\rm SB} = \mathcal{M} \times f(U, \partial_\mu U, \ldots)$, $\mathcal{M} = \mathrm{diag}(m_u, m_d)$ (with ∂_μ crossed out)
- LO invariants: $\mathrm{Tr}(\mathcal{M}U^\dagger)$, $\mathrm{Tr}(U\mathcal{M}^\dagger)$

$\Rightarrow \mathcal{L}_{\rm SB} = \frac{1}{2}F_\pi^2 \left\{B\,\mathrm{Tr}(\mathcal{M}U^\dagger + U\mathcal{M}^\dagger)\right\}$ B is a real constant if CP is conserved

$= (m_u + m_d)\, B \left[F_\pi^2 - \frac{1}{2}\pi^2 + \frac{\pi^4}{24F_\pi^2} + \ldots\right]$ [expand $U = \exp(i\vec{\tau}\cdot\vec{\pi}/F_\pi)$]

First term (vacuum): $\left.\dfrac{\partial\mathcal{L}_{\rm QCD}}{\partial m_q}\right|_{m_q=0} = -\bar{q}q$

$$\Rightarrow \langle 0|\bar{u}u|0\rangle = \langle 0|\bar{d}d|0\rangle = -BF_\pi^2\,(1 + \mathcal{O}(\mathcal{M}))$$

Second term (pion mass): $-\frac{1}{2}M_\pi^2\pi^2 \Rightarrow M_\pi^2 = (m_u + m_d)B$

combined: $\boxed{M_\pi^2 = -(m_u + m_d)\,\langle 0|\bar{u}u|0\rangle / F_\pi^2}$ Gell-Mann–Oakes–Renner rel.

repeat for SU(3) $\Rightarrow 3M_\eta^2 = 4M_K^2 - M_\pi^2$ Gell-Mann–Okubo relation

MESON MASSES $\to$ QUARK MASS RATIOS

- lowest order:

$$M^2_{\pi^+} = (m_u + m_d)B \simeq (0.140\,\mathrm{GeV})^2$$
$$M^2_{K^0} = (m_u + m_s)B \simeq (0.494\,\mathrm{GeV})^2$$
$$M^2_{K^+} = (m_d + m_s)B \simeq (0.497\,\mathrm{GeV})^2$$

$$\xrightarrow{\text{ratios}} \quad \frac{m_u}{m_d} = 0.66\,, \quad \frac{m_s}{m_d} = 20.1\,, \quad \frac{\hat m}{m_s} = \frac{1}{24.2} \quad [\hat m = \tfrac{1}{2}(m_u + m_d)]$$

- corrections: next-to-leading order and beyond

electromagnetism — Weinberg, Gasser, Leutwyler, ...

$$\longrightarrow \quad \boxed{\frac{m_u}{m_d} = 0.553 \pm 0.043\,, \quad \frac{m_s}{m_d} = 18.9 \pm 0.8\,, \quad \frac{\hat m}{m_s} = \frac{1}{24.4 \pm 1.5}}$$

absolute values: sum rules or lattice QCD $\to$ exercise: calc. ratios from PDG

no large isospin violation since $m_u - m_d$ so small vs hadronic scale

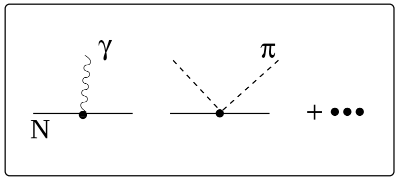
CHIRAL EFECTIVE PION-NUCLEON THEORY

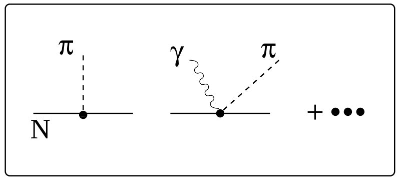
- view nucleon as matter fields [from now on, SU(2) only]
- chiral symmetry *dictates* the couplings to pions & external sources

$\downarrow$ a few steps well documented in the literature

$$\mathcal{L} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)} + \ldots$$

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\psi} \left(i \not{D} - m_N + \tfrac{1}{2} g_A \gamma_\mu \gamma_5 u^\mu \right) \psi$$





- tree calculations: tree graphs w/ insertions from $\mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)}$
- one-loop calculations: tree graphs w/ insertions from $\mathcal{L}_{\pi N}^{(1)} + \ldots + \mathcal{L}_{\pi N}^{(4)}$

  plus loop graphs w/ (one) insertion(s) from $\mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)}$

EFFECTIVE LAGRANGIAN AT ONE LOOP

- Pion-nucleon Lagrangian: $\boxed{\mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)}}$

[$^{(n)}$ = chiral dimension]

with

$$\mathcal{L}_{\pi N}^{(1)} = \bar\Psi\left(i\,\not{D} - m_N + \tfrac{1}{2} g_A\,\not{u}\,\gamma_5\right)\Psi$$

[$u_\mu \sim \partial_\mu \phi$]

$$\mathcal{L}_{\pi N}^{(2)} = \sum_{i=1}^{7} c_i \bar\Psi O_i^{(2)} \Psi = \bar\Psi\Big(c_1 \langle \chi_+\rangle + c_2\big(-\tfrac{1}{8m_N^2}\langle u_\mu u_\nu\rangle D^{\mu\nu} + \text{h.c.}\big) + c_3\,\tfrac{1}{2}\langle u\cdot u\rangle$$

$$+ c_4\,\tfrac{i}{4}\,[u_\mu, u_\nu]\,\sigma^{\mu\nu} + c_5\,\widetilde{\chi}_+ + c_6\,\tfrac{1}{8m_N} F_{\mu\nu}^+ \sigma^{\mu\nu} + c_7\,\tfrac{1}{8m_N}\langle F_{\mu\nu}^+\rangle\,\sigma^{\mu\nu}\Big)\Psi$$

- dynamical LECs $g_A \sim \partial_\mu \phi$, and $c_2, c_3, c_4 \sim \partial_\mu^2 \phi, \partial_\mu \partial_\nu \phi$
- symmetry breaking LECs $c_1 \sim m_u + m_d$, $c_5 \sim m_u - m_d$
- external probe LECs $c_6, c_7 \sim eQ\mathcal{A}_\mu$

$$\mathcal{L}_{\pi N}^{(3)} = \sum_{i=1}^{23} d_i\,\bar\Psi\,O_i^{(3)}\,\Psi\,, \qquad \mathcal{L}_{\pi N}^{(4)} = \sum_{i=1}^{118} e_i\,\bar\Psi\,O_i^{(4)}\,\Psi$$

for details, see Fettes et al., Ann. Phys. 283 (2000) 273 [hep-ph/0001308]

POWER-COUNTING in the PION-NUCLEON THEORY

- nucleon mass $m_N \sim 1\,\text{GeV} \rightarrow$ only three-momenta can be soft
  $\rightarrow$ complicates the power counting (see fig.) Gasser, Sainio, Svarc, Nucl. Phys. B **307** (1988) 779

- solutions:

  (1) Heavy-baryon approach

  Jenkins, Manohar; Bernard, Kaiser, M., . . .

  $1/m_N$ expansion a la Foldy-Wouthuysen of the Lagrangian

  m_N only appears in vertices, no longer in the propagator

  (2) Infrared Regularization [or variants thereof like EOMS]

  Becher, Leutwyler; Kubis, M.; Gegelia, Scherer, . . .

  extraction of the soft parts from the loop integrals

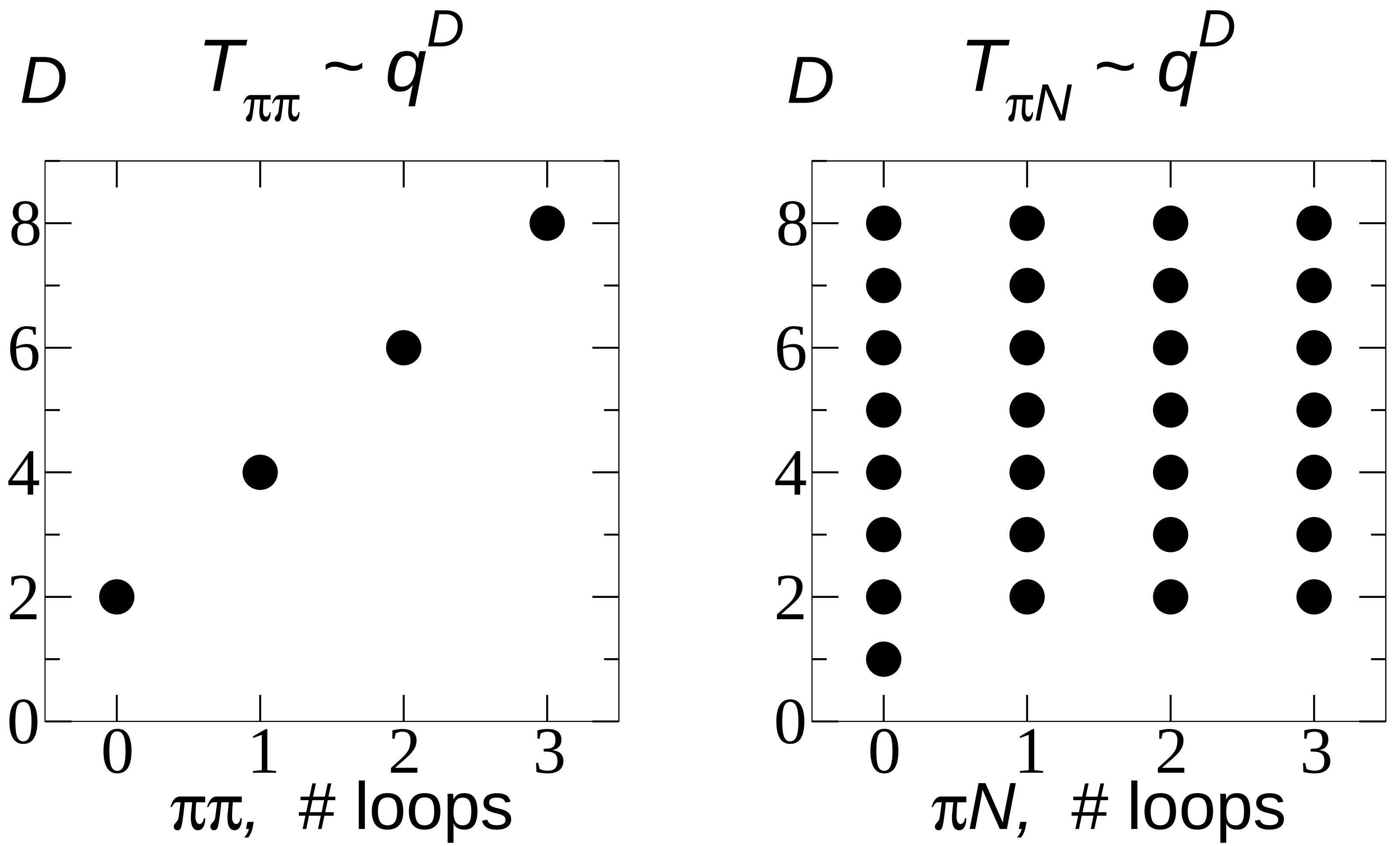
  easier to retain proper analytic structure

- most calculations at one loop, only two at two loop accuracy (g_A, m_N)

  Bernard, M.; Schindler, Scherer, Gegelia

FAILURE of the POWER-COUNTING

- naive extension of loop graphs from the pion to the pion-nucleon sector



HEAVY BARYON APPROACH I

- consider the nucleon as a static, heavy source → four-velocity v_μ:

Jenkins, Manohar 1991

$$\boxed{p_\mu = m_N v_\mu + \ell_\mu}\,, \quad v^2 = 1, \quad p^2 = m_N^2, \quad v\cdot\ell \ll m_N$$

- velocity-projection: $\Psi(x) = \exp(-i m_N v\cdot x)\,[H(x) + h(x)]$

with $\quad \not{v} H = H,\ \not{v} h = -h \quad$ [“large/small” components]

- H- and h-components decouple, separated by large mas gap $2m_N$:

$$\mathcal{L}_{\pi N}^{(1)} = \bar\Psi\left(i\not{D} - m_N + \tfrac{1}{2} g_A \not{u}\gamma_5\right)\Psi$$

$$u_\mu = i u^\dagger \nabla_\mu U u^\dagger, U = u^2$$

$$\nabla_\mu U = \partial_\mu U - ieA_\mu[Q,U], Q = \mathrm{diag}(1,0)$$

$$D_\mu\Psi = \partial_\mu\Psi + \tfrac{1}{2}(u^\dagger(\partial_\mu - ieA_\mu Q)u + u(\partial_\mu - ieA_\mu Q)u^\dagger)\Psi$$

$$\to \boxed{\mathcal{L}_{\pi N}^{(1)} = \bar H\,(iv\cdot D + g_A S\cdot u)\,H + \mathcal{O}\left(\frac{1}{m_N}\right)}$$

HEAVY BARYON APPROACH II

- covariant spin-vector à la Pauli-Lubanski:

$$S_\mu = \tfrac{i}{2}\gamma_5\sigma_{\mu\nu}v^\nu, \quad S\cdot v = 0, \quad \{S_\mu, S_\nu\} = \tfrac{1}{2}(v_\mu v_\nu - g_{\mu\nu}), \quad S^2 = \frac{1-d}{4}$$

- the Dirac algebra simplifies considerably (only v_μ and S_μ):

$$\bar{H}\gamma_\mu H = v_\mu \bar{H}H, \; \bar{H}\gamma_5 H = \mathcal{O}(\tfrac{1}{m_N}), \; \bar{H}\gamma_\mu\gamma_5 H = 2\bar{H}S_\mu H, \ldots$$

- propagator:

$$S(\omega) = \frac{i}{\omega + i\eta}, \quad \omega = v\cdot\ell, \quad \eta \to 0^+$$

- mass scale moved from the propagator to $1/m_N$ suppressed vertices
  $\to$ power counting

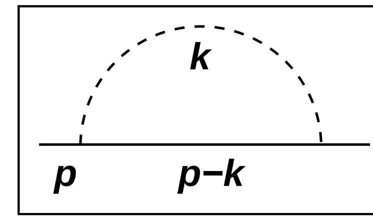
- can be systematically extended to arbitrary orders in $1/m_N$
  Bernard, Kaiser, Kambor, M., 1992

INFRARED REGULARIZATION I

- relativistic calculation of the nucleon self-energy:

Gasser, Sainio, Švarč, 1988, Becher, Leutwyler 1999

$$H(p^2) = \frac{1}{i}\int \frac{d^dk}{(2\pi)^d}\frac{1}{M_\pi^2 - k^2}\frac{1}{m_N - (p-k)^2}$$



$$\to H(s_0) = c(d)\frac{M_\pi^{d-3} + m_N^{d-3}}{M_\pi + m_N} = I + R\,, \quad s_0 = (M_\pi + m_N)^2$$

infrared singular piece I: generated by momenta of the order M_π
contains the chiral physics like chiral logs etc.

infrared regular piece R: generated by momenta of the order m_N
leads to the violation of the power counting
polynomial in external momenta and quark masses
$\to$ can be absorbed in the LECs of the eff. Lagr.

INFRARED REGULARIZATION II

Becher, Leutwyler 1999

- this symmetry-preserving splitting can be *uniquely* defined for any one-loop graph

- method to separate the infrared singular and regular parts (end-point singularity at $z = 1$):

$$H = \int \frac{d^dk}{(2\pi)^d} \frac{1}{AB} = \int_0^1 dz \int \frac{d^dk}{(2\pi)^d} \frac{1}{[(1-z)A + zB]^2}$$

$$= \left\{ \int_0^\infty - \int_1^\infty \right\} dz \int \frac{d^dk}{(2\pi)^d} \frac{1}{[(1-z)A + zB]^2} = I + R$$

$$A = M_\pi^2 - k^2 - i\eta\,, \quad B = m^2 - (p-k)^2 - i\eta\,, \quad \eta \to 0^+$$

- preserves the low-energy analytic structure of any one-loop graph

- extension to higher loop graphs difficult but doable

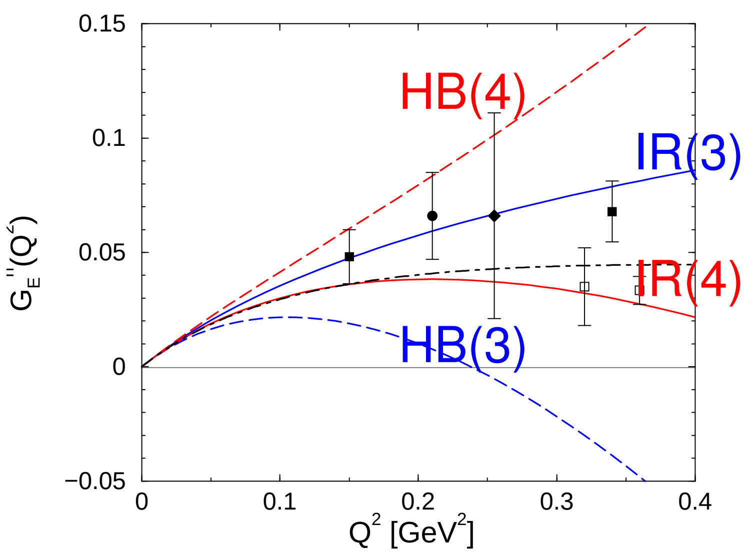
Lehmann, Prezeau, 2002

HEAVY BARYON vs INFRARED REGULARIZATION

- Heavy baryon (HB) is algebraically much simpler than infrared regularization (IR)
- HB can be extended to higher loop orders (IR requires modifications)
- Strict HB approach sometimes at odds with the analytic structure, IR not,
  e.g. anomalous threshold in triangle diagram (isovector em form factors)

$$t_c = 4M_\pi^2 - M_\pi^4/m_N^2 \stackrel{HB}{=} 4M_\pi^2 + \mathcal{O}(1/m_N^2)$$

- IR resums kinetic energy insertions
  $\longrightarrow$ sometimes improves convergence
  e.g. neutron electric ff $G_E^n(Q^2)$
  Kubis, M., 2001



- for a detailed discussion, see the review
  Bernard, Prog. Nucl. Part. Phys. **60** (2008) 82

EOMS REGULARIZATION I

Fuchs, Gegelia, Japaridze, Scherer 2003

- Extended-on-mass-shell scheme (EOMS), consider the chiral limit $M = 0$:

$$H(p^2, m_N^2, 0; d) = \frac{1}{i}\int \frac{d^dk}{(2\pi)^d} \frac{1}{[k^2 + i\varepsilon]} \frac{1}{[(p-k)^2 - m_N^2 + i\varepsilon]}$$

$\longrightarrow$ modify the integrand by subtracting suitable counterterms:

$$\sum_{\ell=0}^{\infty} \frac{p^2 - m_N^2}{\ell!} \left[\left(\frac{1}{p^2} p_\mu \frac{\partial}{\partial p_\mu} \right)^\ell \frac{1}{[k^2 + i\varepsilon]} \frac{1}{[((p^2 - m_N^2) + k^2 - 2k\cdot p + i\varepsilon]} \right]_{p^2 = m_N^2}$$

$$= \frac{1}{(k^2 + i\varepsilon)(k^2 - 2k\cdot p + i\varepsilon)}\bigg|_{p^2 = m_N^2}$$

$$+ (p^2 - m_N^2) \left[\frac{1}{2m_N^2} \frac{1}{(k^2 - 2k\cdot p + i\varepsilon)^2} - \frac{1}{2m_N^2} \frac{1}{(k^2 + i\epsilon)(k^2 - 2k\cdot p + i\varepsilon)} \right.$$

$$\left. - \frac{1}{(k^2 + i\epsilon)(k^2 - 2k\cdot p + i\varepsilon)^2} \right] + (p^2 - m_N^2)^2 \times \ldots$$

EOMS REGULARIZATION II

Fuchs, Gegelia, Japaridze, Scherer 2003

- Formal definition of the EOMS scheme:

$\rightarrow$ subtract from the integrand of those terms of the series which violate the p. c.

$\hookrightarrow$ These terms are always analytic in the small parameter

$\hookrightarrow$ They do not not contain infrared singularities

$\hookrightarrow$ EOMS acts on the integrand, not on the integration boundaries

- Nucleon self-energy: only subtract the first term on the r.h.s.

e.g. the second term (last summand) is IR singular as $\boldsymbol{k^3/k^4}$

- Can be formulated more elegantly using the generating functional and utilizing heat kernel regularization

Du, Guo, UGM, 2016

- consider the nucleon mass being eliminated, e.g. in the heavy baryon scheme $S(q) \sim 1/(v \cdot q)$ and vertices with $d \geq 1$

- Goldstone bosons as before, $d \geq 2$ and $D(q) \sim 1/(q^2 - M^2)$

- consider an L-loop diagram with I_B internal baryon lines, I_M internal meson lines, V_d^M mesonic vertices and V_d^{MB} meson-nucleon vertices of order d

$$Amp \propto \int (d^4q)^L \frac{1}{(q^2)^{I_M}} \frac{1}{(q)^{I_B}} \prod_d (q^d)^{(V_d^M + V_d^{MB})}$$

- let $Amp \sim q^\nu \to \nu = 4L - 2I_M + I_B + \sum_d d(V_d^M + V_d^{MB})$

- topology: $L = I_M + I_B - \sum_d (V_d^m + V_d^{MB}) + 1$
  and **one** baryon line through the diagram: $\sum_d V_d^{MB} = I_B + 1$

- eliminate I_M: $\boxed{\nu = 1 + 2L + \sum_d V_d^m (d-2) + \sum_d (d-1) V_d^{MB}}$ $\boxed{\to \nu \geq 1}$

STRUCTURE of the PION-NUCLEON INTERACTION

- Pion-nucleon scattering in chiral pertubation theory

Leading order (LO) ($\nu = 1$):

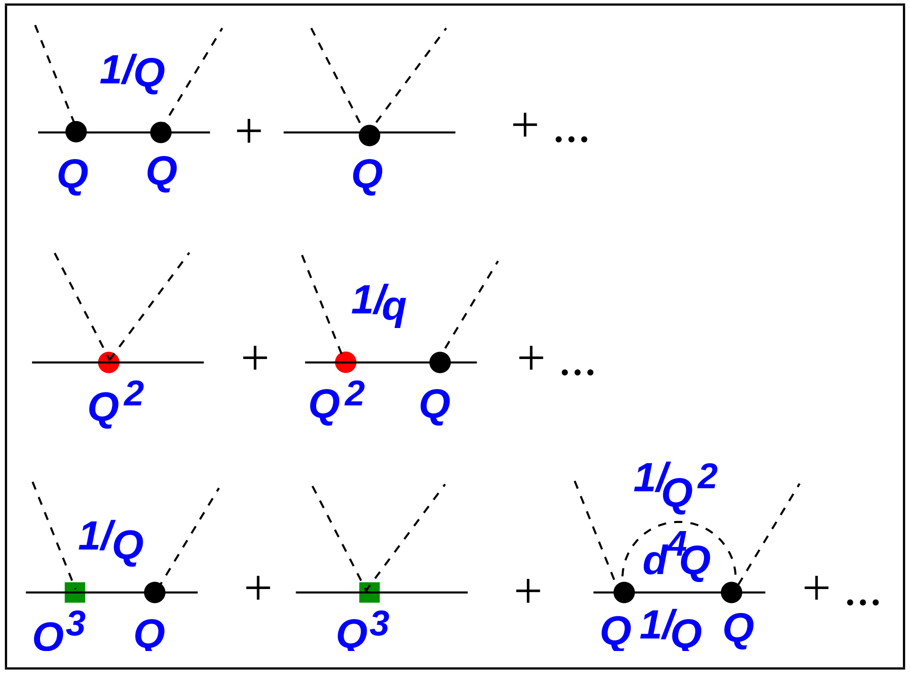
tree graphs w/ insertions with $d = 1$

Next-to-leading order (NLO) ($\nu = 2$):

tree graphs w/ insertions with $d = 1, 2$

Next-to-next-to-leading order (NNLO) ($\nu = 3$):

tree graphs w/ insertions with $d = 1, 2, 3$
and one-loop graphs w/insertion with $d = 1$



- calculations have been performed up to $\nu = 4$ (NNNLO = complete one-loop):

  heavy-baryon scheme Fettes, M., Nucl. Phys. A **676** (2000) 311, Krebs et al., Phys. Rev. C**85** (2012) 054006

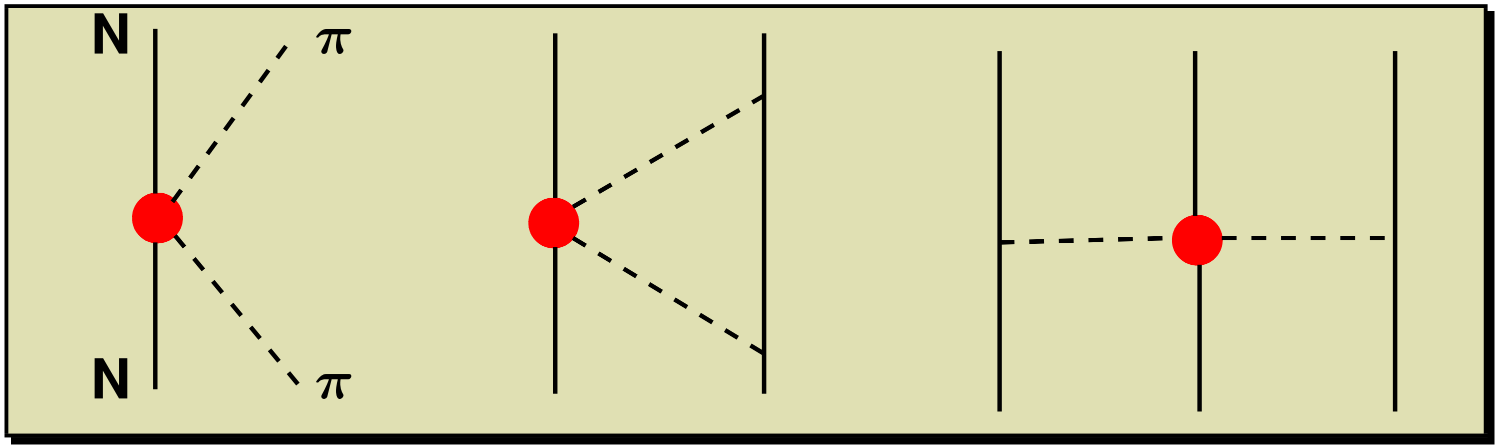
  infrared-regularization scheme Becher, Leutwyler, JHEP **06** (2001) 017

  covariant EOMS scheme Alarcon et al., Phys. Rev. C**83** (2011) 055205; Siemens et al., Phys.Rev. C**94** (2016) 014620

APPLICATION: DIMENSION-TWO LECS

- Low-energy constants (LECs) relate ***many*** processes

- e.g. the dimension-two LECs c_i in πN, NN, NNN, $\ldots$



● = operator from $\mathcal{L}^{(2)}_{\pi N} \propto c_i \quad (i = 1, 2, 3, 4)$

Here:
- determine the c_i from the purest process $\pi N \to \pi N$
- later use in the calculation of nuclear forces

DETERMINATION OF THE LECs

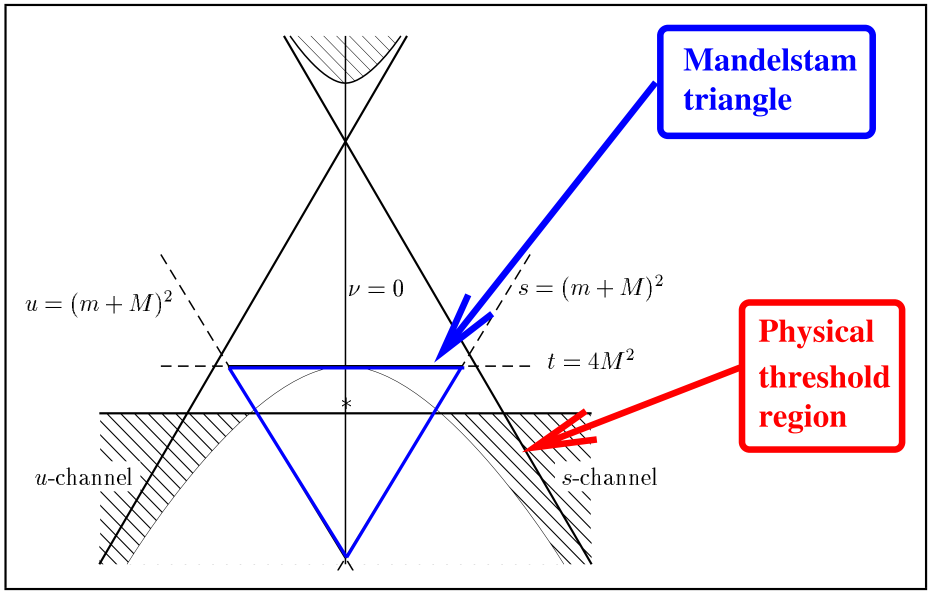
- πN scattering data can be explored in different ways (CHPT or disp. rel.):

- πN scattering inside the Mandelstam triangle:

$\rightarrow$ best convergence, relies on dispersive analysis

$\rightarrow$ not sensitive to all LECs, esp. c_2

Büttiker, M., Nucl. Phys. A 668 (2000) 97 [hep-ph/9908247]



- πN scattering in the threshold region:

$\rightarrow$ large data basis, not all consistent

$\rightarrow$ use threshold parameters and global fits

$\rightarrow$ sizeable uncertainties remain in some LECs

Fettes, M., Steininger, Nucl. Phys. A 640 (1998) 119 [hep-ph/9803266]

Fettes, M., Nucl. Phys. A 676 (2000) 311 [hep-ph/0002182]

Becher, Leutwyler, JHEP 0106 (2001) 017 [arXiv:hep-ph/0103263]

- πN scattering from Roy-Steiner equations:

$\rightarrow$ hyperbolic partial-wave dispersion relations (unitarity & analyticity & crossing symmetry)

$\rightarrow$ most accurate representation of the πN amplitudes

Hoferichter, Ruiz de Elvira, Kubis, UGM, Phys. Rev. Lett. **115** (2015) 092301; Phys. Rev. Lett. **115** (2015) 192301; Phys. Rept. **625** (2016) 1

RESULTS for the LECs

- Chiral expansion expected to work best at the subthreshold point (polynomial, maximal distance to singularities)

- Express subthreshold parameters in terms of LECs $\to$ invert system

- LECs c_i of the dimension two chiral effective πN Lagrangian:

LEC	RS	KGE 2012	UGM 2005
c_1 [GeV^{-1}]	-1.11 ± 0.03	$-1.13 \ldots -0.75$	$-0.9^{+0.2}_{-0.5}$
c_2 [GeV^{-1}]	3.13 ± 0.03	$3.49 \ldots 3.69$	3.3 ± 0.2
c_3 [GeV^{-1}]	-5.61 ± 0.06	$-5.51 \ldots -4.77$	$-4.7^{+1.2}_{-1.0}$
c_4 [GeV^{-1}]	4.26 ± 0.04	$3.34 \ldots 3.71$	$-3.5^{+0.5}_{-0.2}$

Krebs, Gasparyan, Epelbaum, Phys. Rev. C85 (2012) 054006
UGM, PoS LAT2005 (2006) 009

- also results for pertinent dimension three and four LECs

INTERMEDIATE SUMMARY

- QCD has a chiral symmetry in the light quark sector (neglecting quark masses)
- Chiral symmetry is *spontaneously* and *explicitely* broken
  appearance of almost massless Goldstone bosons (π, K, η)
  Goldstone boson interactions vanish as $E, p \to 0$
- Chiral perturbation theory is the EFT of QCD that explores chiral symmetry
- Meson sector: only even powers in small momenta, many successes
- Single nucleon sector: odd & even powers, quite a few successes
- Low-energy constants relate many processes (in particular the c_i)
- Isospin-breaking can be systematically incorporated $\to$ spares
- NREFT can be set up for hadronic atoms $\to$ extraction of scattering lengths

Testing chiral dynamics in hadron-hadron scattering

WHY HADRON-HADRON SCATTERING?

- Weinberg's 1966 paper "Pion scattering lengths"

Weinberg, Phys. Rev. Lett. **17** (1966) 616

– pion scattering on a target with mass m_t and isospin T_t:

$$a_T = -\frac{L}{1 + M_\pi/m_t}\,[T(T+1) - T_t(T_t+1) - 2]$$

– pion scattering on a pion ["the more complicated case"]:

$$a_0 = \frac{7}{4}L\,, \quad a_2 = -\frac{1}{2}L$$

$$\boxed{L = \frac{g_V^2 M_\pi}{8\pi F_\pi^2} \simeq 0.1\, M_\pi^{-1}}$$

[$F_\pi = 92.1$ MeV]

- amazing predictions - witness to the power of chiral symmetry
- what have we learned since then?

Example 1

ELASTIC PION-PION SCATTERING

- Purest process in two-flavor chiral dynamics (really light quarks)
- scattering amplitude at threshold: two numbers (a_0, a_2)
- History of the prediction for a_0:

LO (tree):	$a_0 = 0.16$	Weinberg 1966
NLO (1-loop):	$a_0 = 0.20 \pm 0.01$	Gasser, Leutwyler 1983
NNLO (2-loop):	$a_0 = 0.217 \pm 0.009$	Bijnens et al. 1996

- even better: match 2-loop representation to Roy equation solution

Roy + 2-loop:	$a_0 = 0.220 \pm 0.005$	Colangelo et al. 2000

$\Rightarrow$ this is an *amazing* prediction!

- same precision for a_2, but corrections very small . . .

HOW ABOUT EXPERIMENT?

- Kaon decays (K_{e4} and $K^0 \to 3\pi^0$): most precise
- Lifetime of pionium: experimentally more difficult

Kaon decays:

$$a_0^0 = 0.2210 \pm 0.0047_{\rm stat} \pm 0.0040_{\rm sys}$$

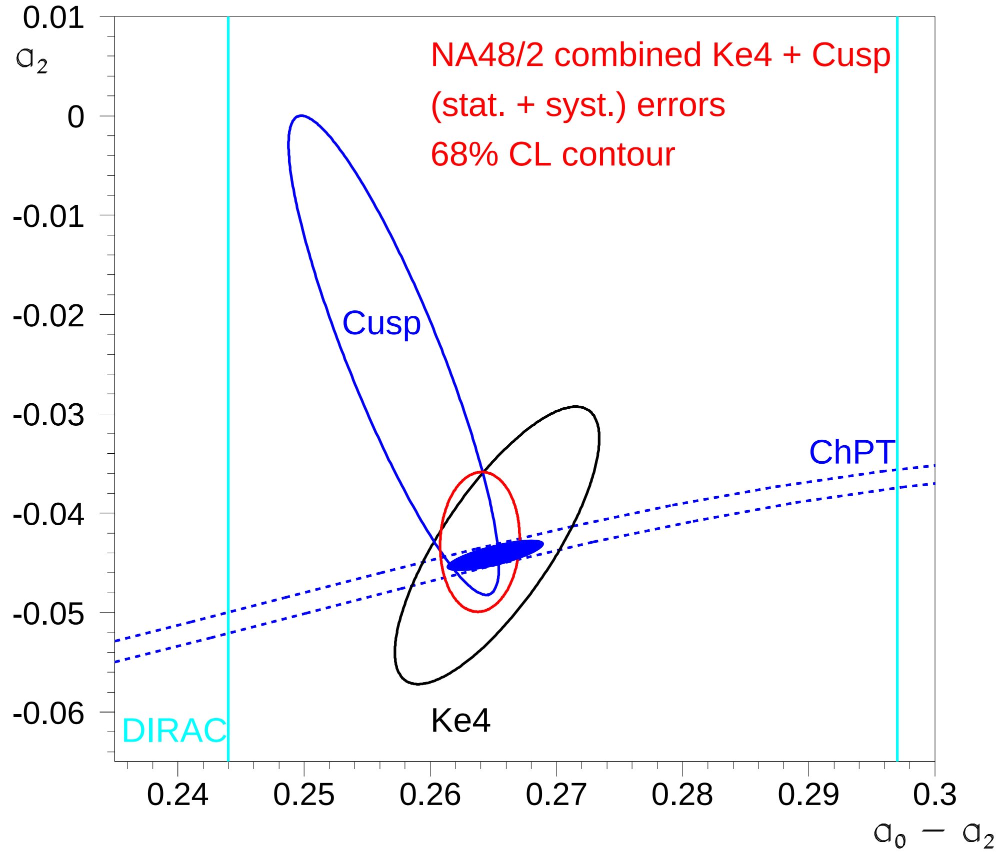
$$a_0^2 = -0.0429 \pm 0.0044_{\rm stat} \pm 0.0028_{\rm sys}$$

J. R. Batley et al. [NA48/2 Coll.] EPJ C 79 (2010) 635

Pionium lifetime:

$$|a_0^0 - a_0^2| = 0.264^{+0.033}_{-0.020}$$

B. Adeva et al. [DIRAC Coll.] PL B 619 (2005) 50

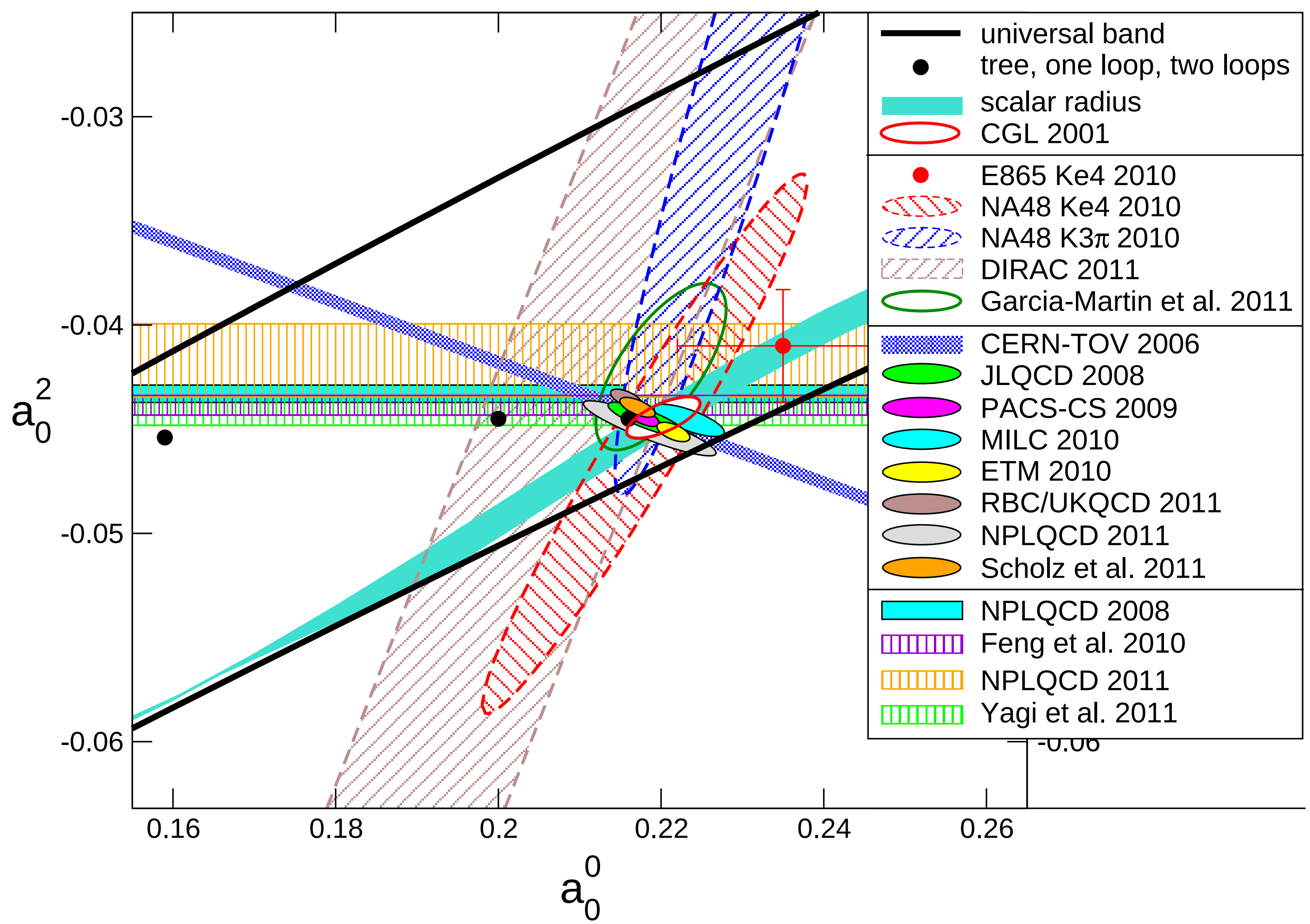


- and how about the lattice?

$\Rightarrow$ direct and indirect determinations of the scattering lengths

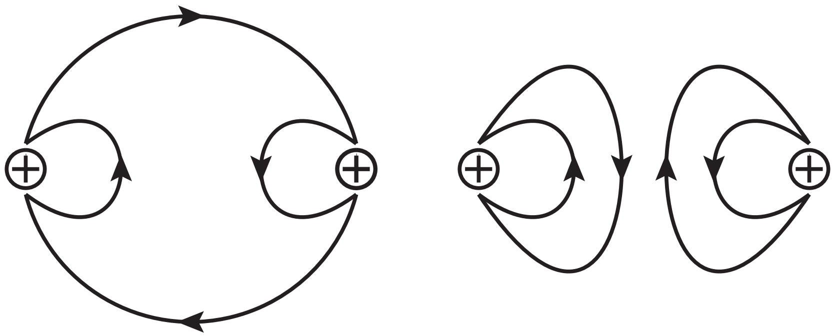
THE GRAND PICTURE

Fig. courtesy Heiri Leutwyler 2012



• one of the finest tests of the Standard Model (what about lattice a_0 calcs?)

ELASTIC PION-PION SCATTERING – LATTICE a_0



- Only a few lattice determinations of a_0
  - $\hookrightarrow$ disconnected diagrams difficult
  - $\hookrightarrow$ quantum numbers of the vacuum

- only a few results:

Author(s)	a_0	Fermions	Pion mass range
Fu	0.214(4)(7)	asqtad staggered	240 - 430 MeV
Liu et al.	0.198(9)(6)	twisted mass	250 - 320 MeV

Fu, PRD87 (2013) 074501; Liu et al., PRD96 (2017) 054516

$\longrightarrow$ use EFT of PQQCD to investigate these contributions

Acharya, Guo, UGM, Seng, Nucl.Phys. B922 (2017) 480

$\longrightarrow$ more work needed!

Example 2

STRANGE QUARK MYSTERIES

- Is the strange quark really light? $\boxed{m_s \sim \Lambda_{\rm QCD}}$

$\to$ expansion parameter: $\xi_s = \frac{M_K^2}{(4\pi F_\pi)^2} \simeq 0.18$ [SU(2): $\xi = \frac{M_\pi^2}{(4\pi F_\pi)^2} \simeq 0.014$]

- many predictions of SU(3) CHPT work quite well, but:

$\hookrightarrow$ indications of bad convergence in some recent lattice calculations:

$\star$ masses and decay constants — Allton et al. 2008

$\star$ $K_{\ell 3}$-decays — Boyle et al. 2008

$\hookrightarrow$ suppression of the three-flavor condensate?

$\star$ sum rule: $\Sigma(3) = \Sigma(2)[1 - 0.54 \pm 0.27]$ — Moussallam 2000

$\star$ lattice: $\Sigma(3) = \Sigma(2)[1 - 0.23 \pm 0.39]$ — Fukuya et al. 2011

ELASTIC PION-KAON SCATTERING

- Purest process in three-flavor chiral dynamics
- scattering amplitude at threshold: two numbers $(a_0^{1/2}, a_0^{3/2})$
- History of the chiral predictions:

	CA [1]	1-loop [2]	2-loop [3]
$a_0^{1/2}$	0.14	0.18 ± 0.03	0.220 [0.17 . . . 0.225]
$a_0^{3/2}$	-0.07	-0.05 ± 0.02	$-0.047[-0.075 \ldots -0.04]$

[1] Weinberg 1966, Griffith 1969 [2] Bernard, Kaiser, UGM 1990 [3] Bijnens, Dhonte, Talavera 2004

- match 1-loop representation to Roy-Steiner equation solution

$$a_0^{1/2} = 0.224 \pm 0.022\,, \quad a_0^{3/2} = -0.0448 \pm 0.0077$$

Büttiker et al. 2003

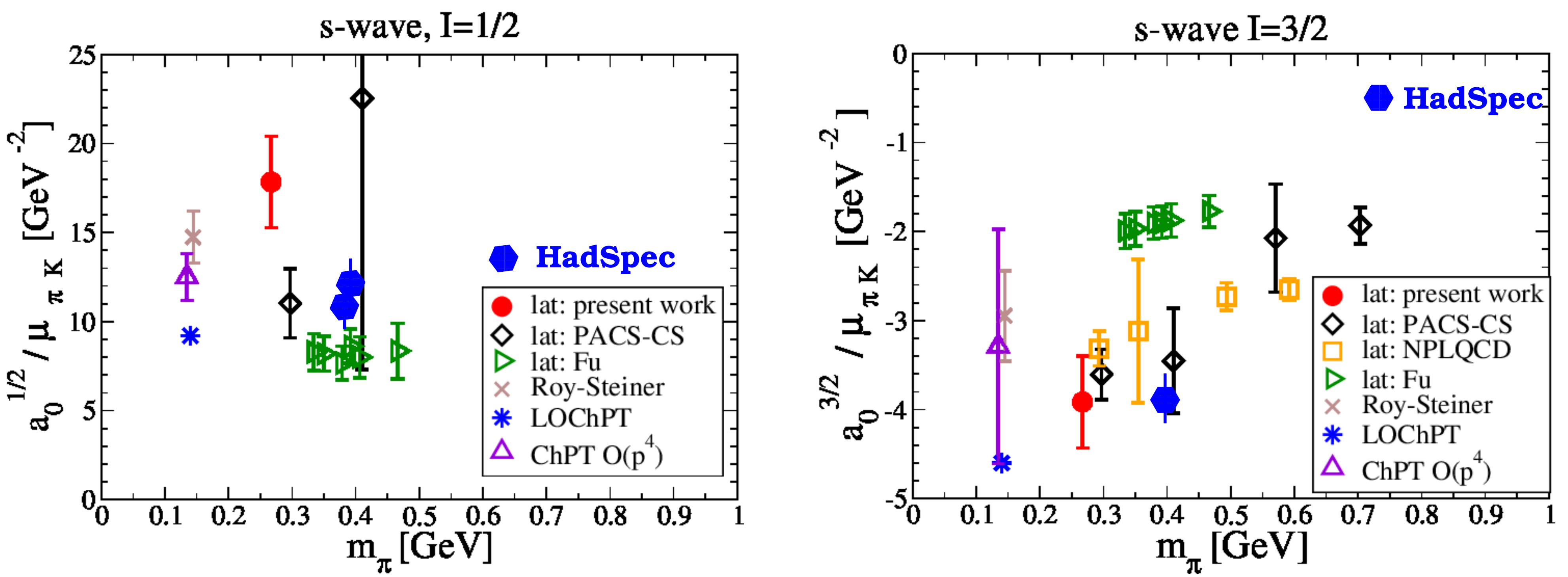
- constrained forward dispersion relations:

$$a_0^{1/2} = 0.22 \pm 0.01\,, \quad a_0^{3/2} = -0.054^{+0.010}_{-0.014}$$

Pelaez, Rodas 2016

THE GRAND PICTURE

Fig. from Lang et al., Phys.Rev. D **86** (2012) 054508 [1207.3204]
updated incl. Wilson et al., Phys.Rev. D **91** (2015) 054008 [1411.2004]



- tension between lattice results and/or Roy-Steiner
- need improved lattice results (more direct calculations) $\Rightarrow$ work required
- see also Pion-Kaon Interactions Workshop at JLab website

https://www.jlab.org/conferences/pki2018/program.html [arXiv:1804.06528]

Example 3

PION-NUCLEON SCATTERING

- simplest scattering process involving nucleons
- intriguing LO prediction for isoscalar/isovector scattering length:

$$a^+_{\rm CA} = 0, \quad a^-_{\rm CA} = \frac{1}{1+M_\pi/m_p}\frac{M_\pi^2}{8\pi F_\pi^2} = 79.5\cdot 10^{-3}/M_\pi,$$

- chiral corrections:
  - — chiral expansion for a^- converges fast — Bernard, Kaiser, UGM 1995
  - — large cancellations in a^+, even sign not known from scattering data

	$\mathcal{O}(q)$	$\mathcal{O}(q^2)$	$\mathcal{O}(q^3)$	$\mathcal{O}(q^4)$
fit to KA85	0.0	0.46	-1.00	-0.96
fit to EM98	0.0	0.24	0.49	0.45
fit to SP98	0.0	1.01	0.14	0.27

Fettes, UGM 2000

A WONDERFUL ALTERNATIVE: HADRONIC ATOMS

- Hadronic atoms: bound by the static Coulomb force (QED)

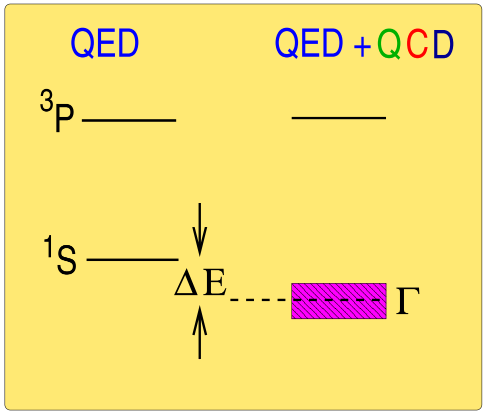
- Many species: $\pi^+\pi^-$, $\pi^\pm K^\mp$, $\pi^- p$, $\pi^- d$, $K^- p$, $K^- d$, ...

- Observable effects of QCD: strong interactions as **small** perturbations

  $\star$ energy shift ΔE

  $\star$ deacy width Γ

  $\Rightarrow$ access to scattering at zero energy!
  = S-wave scattering lengths



- can be analyzed in suitable NREFTs

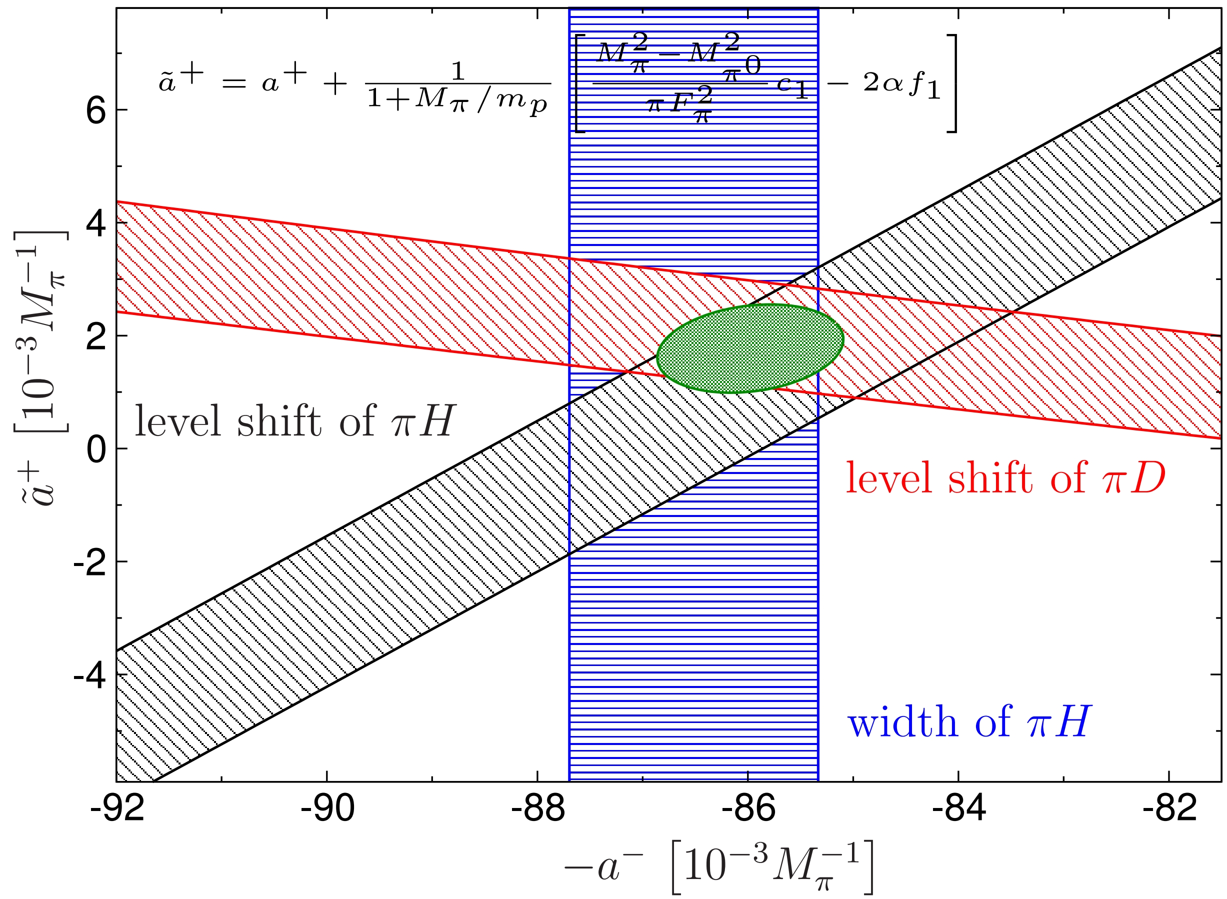
  Pionic hydrogen — Gasser, Rusetsky, . . . 2002

  Pionic deuterium — Baru, Hoferichter, Kubis . . . 2011

PION–NUCLEON SCATTERING LENGTHS

- Superb experiments performed at PSI — Gotta et al.
- Hadronic atom theory (Bern, Bonn, Jülich) — Gasser et al., Baru et al.

Baru, Hoferichter, Hanhart, Kubis, Nogga, Phillips, Nucl. Phys. A **872** (2011) 69



- πH level shift $\Rightarrow \pi^- p \to \pi^- p$
- πD level shift $\Rightarrow$ isoscalar $\pi^- N \to \pi^- N$
- πH width $\Rightarrow \pi^- p \to \pi^0 n$

$\downarrow$

$$a^+ = (7.6 \pm 3.1) \cdot 10^{-3}/M_\pi$$

$$a^- = (86.1 \pm 0.9) \cdot 10^{-3}/M_\pi$$

$\Rightarrow$ very precise value for a^- & first time definite sign for a^+

ROLE of the PION-NUCLEON σ-TERM

- Scalar couplings of the nucleon:

$$\langle N|m_q\bar{q}q|N\rangle = f_q^N\, m_N \qquad (N = p, n)$$
$$(q = u, d, s)$$

$\hookrightarrow$ Dark Matter detection

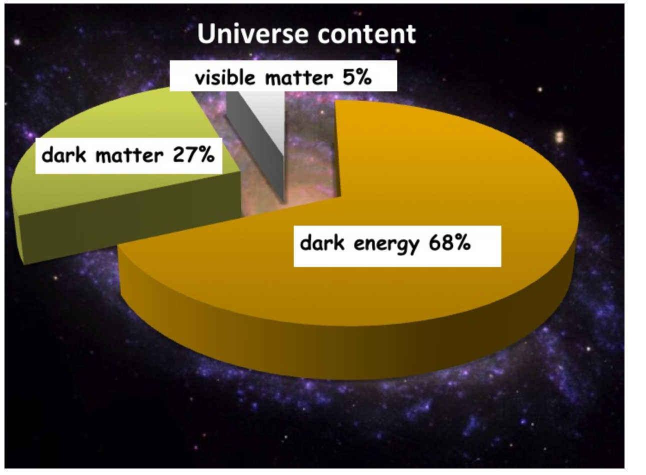
$\hookrightarrow$ $\mu \to e$ conversion in nuclei

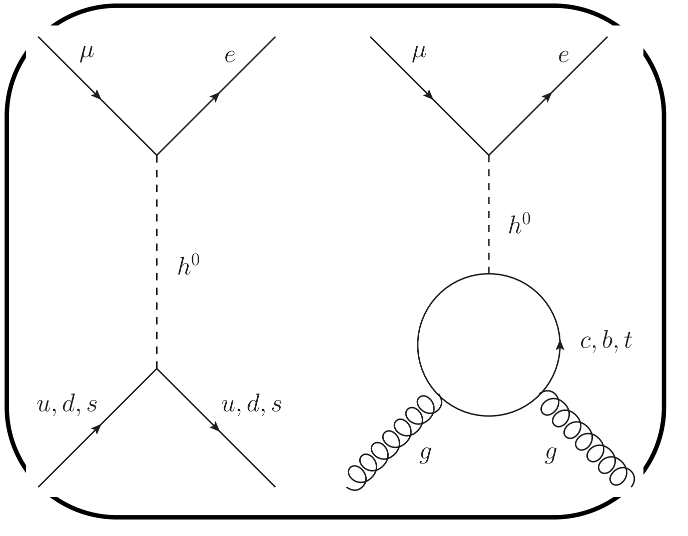
- Condensates in nuclear matter

$$\frac{\langle\bar{q}q\rangle(\rho)}{\langle 0|\bar{q}q|0\rangle} = 1 - \frac{\rho\,\sigma_{\pi N}}{F_\pi^2 M_\pi^2} + \ldots$$

- CP-violating πN couplings

$\hookrightarrow$ hadronic EDMs (nucleon, nuclei)





Crivellin, Hoferichter, Procura

RESULTS for the SIGMA-TERM

- Basic formula: $$\sigma_{\pi N} = F_\pi^2(d_{00}^+ + 2M_\pi^2 d_{01}^+) + \Delta_D - \Delta_\sigma - \Delta_R$$

- Subthreshold parameters output of the RS equations:

$$d_{00}^+ = -1.36(3) M_\pi^{-1} \qquad \text{[KH: } -1.46(10) M_\pi^{-1}\text{]}$$

$$d_{01}^+ = 1.16(3) M_\pi^{-3} \qquad \text{[KH: } 1.14(2)\ M_\pi^{-3}\text{]}$$

- $\Delta_D - \Delta_\sigma = (1.8 \pm 0.2)$ MeV — Hoferichter, Ditsche, Kubis, UGM (2012)
- $\Delta_R \lesssim 2$ MeV — Bernard, Kaiser, UGM (1996)
- Isospin breaking in the CD theorem shifts $\sigma_{\pi N}$ by $+3.0$ MeV

$\Rightarrow$ Final result:

$$\sigma_{\pi N} = (59.1 \pm 1.9_{\rm RS} \pm 3.0_{\rm LET})\ {\rm MeV} = (59.1 \pm 3.5)\ {\rm MeV}$$

- consistent with scattering data analysis: $\sigma_{\pi N} = 58 \pm 5$ MeV
  Ruiz de Elvira, Hoferichter, Kubis, UGM (2018)
- recover $\sigma_{\pi N} = 45$ MeV if KH80 scattering lengths are used

RESULTS for the SIGMA-TERM

- Recent results from various LQCD collaborations:

collaboration	$\sigma_{\pi N}$ [MeV]	reference	tension to RS
BMW	$38(3)(3)$	Dürr et al. (2015)	3.8σ
χQCD	$45.9(7.4)(2.8)$	Yang et al. (2015)	1.5σ
ETMC	$37.22(2.57)\left(^{+0.99}_{-0.63}\right)$	Abdel-Rehim et al. (2016)	4.9σ
CRC 55	$35(6)$	Bali et al. (2016)	4.0σ

- We seem to have a problem - do we? [we = RS folks]

- Robust prediction of the RS analysis:

$$\sigma_{\pi N} = (59.1 \pm 3.1)\,\mathrm{MeV} + \sum_{I_s} c_{I_s}\left(a^{I_s} - \bar{a}^{I_s}\right) \quad \left(I_s = \tfrac{1}{2}, \tfrac{3}{2}\right)$$

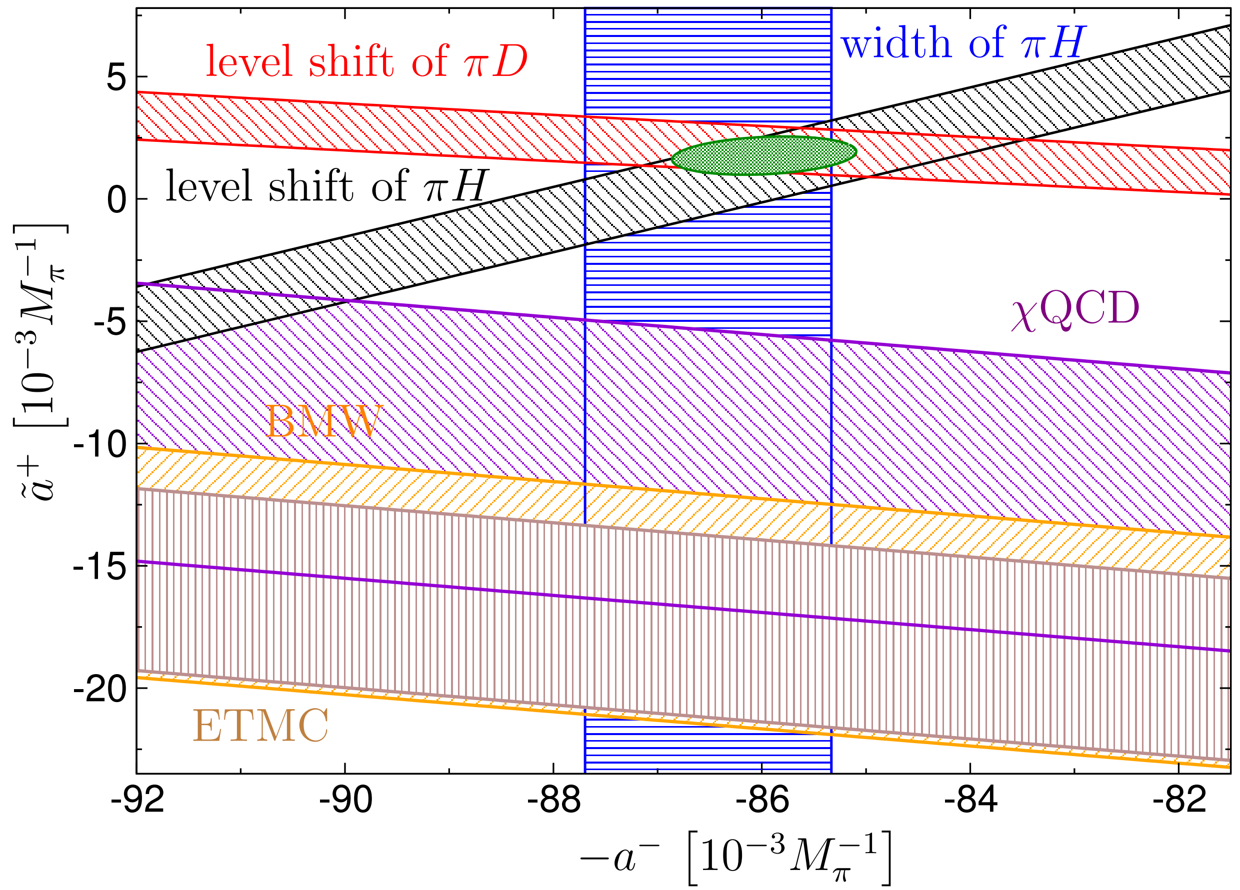
$$c_{1/2} = 0.242\,\mathrm{MeV} \times 10^3 M_\pi, \quad c_{3/2} = 0.874\,\mathrm{MeV} \times 10^3 M_\pi$$

$$\bar{a}^{1/2} = (169.8 \pm 2.0) \times 10^{-3} M_\pi^{-1}, \quad \bar{a}^{3/2} = (-86.3 \pm 1.8) \times 10^{-3} M_\pi^{-1}$$

$\to$ expansion around the reference values from πH and πD

RESULTS for the SIGMA-TERM

- Apply this linear expansion to the lattice data:



$\Rightarrow$ Lattice results clearly at odds with empirical information on the scattering lengths!

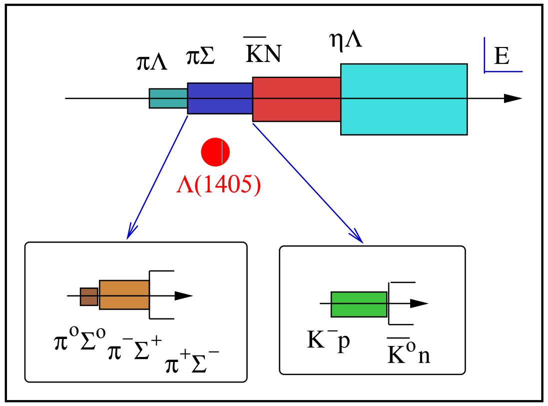
$\Rightarrow$ scattering lengths to $[\mathbf{5\ldots10}]\%$ $\to$ $\boldsymbol{\delta\sigma_{\pi N} = [5.0\ldots8.5]}$ MeV

Example 4

ANTIKAON-NUCLEON SCATTERING

- $K^- p \to K^- p$: fundamental scattering process with strange quarks

- coupled channel dynamics



- dynamic generation of the $\Lambda(1405)$

  Dalitz, Tuan 1960

- major playground of **unitarized CHPT**

- chiral Lagrangian + unitarization leads to generation of certain resonances

  like e.g. the $\Lambda(1405)$, $S_{11}(1535)$, $S_{11}(1650)$, $\ldots$

  Kaiser, Siegel, Weise, Oset, Ramos, Oller, UGM, Lutz, $\ldots$

- two-pole scenario of the $\Lambda(1405)$ emerges Oller, UGM 2001

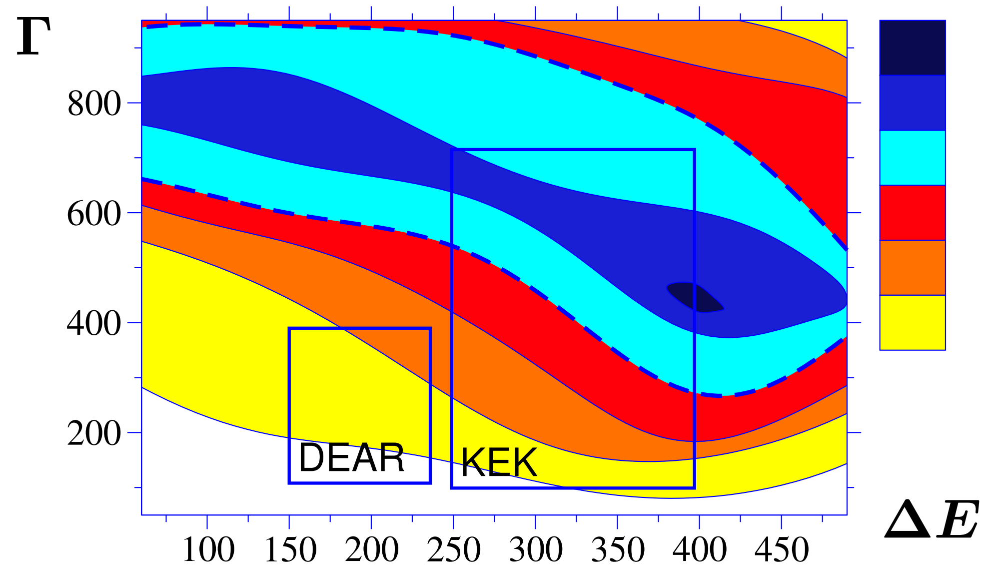
- loopholes: convergence a posteriori, crossing symmetry, on-shell approximation, unphysical poles, $\ldots$

A PUZZLE RESOLVED

- DEAR data inconsistent with scattering data

UGM, Raha, Rusetsky 2004

$\Rightarrow$ vaste number of papers ...



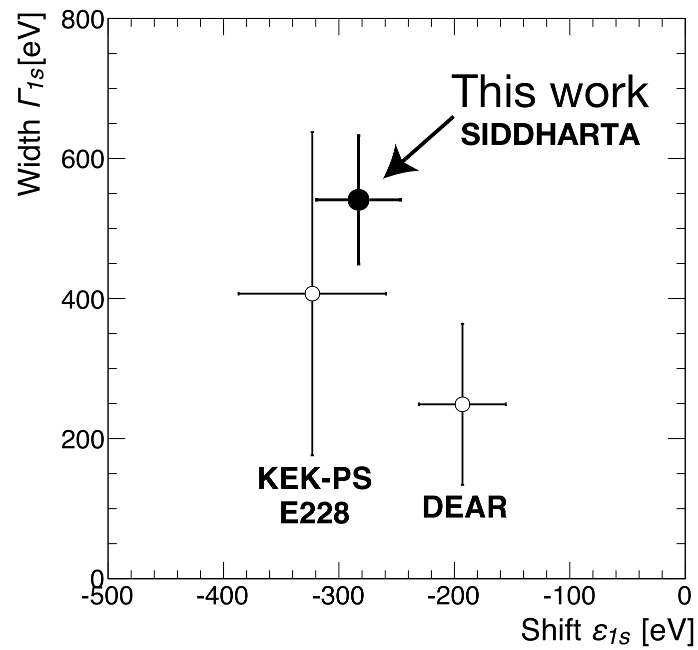
- SIDDHARTA to the rescue

Bazzi et al. 2011

$\Rightarrow$ more precise, consistent with KpX

$$\epsilon_{1s} = -283 \pm 36(\mathrm{stat}) \pm 6(\mathrm{syst})\ \mathrm{eV}$$

$$\Gamma_{1s} = 541 \pm 89(\mathrm{stat}) \pm 22(\mathrm{syst})\ \mathrm{eV}$$



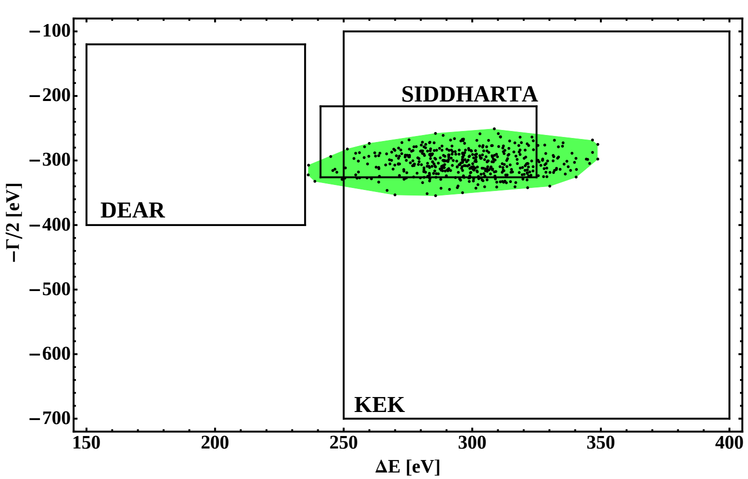
CONSISTENT ANALYSIS

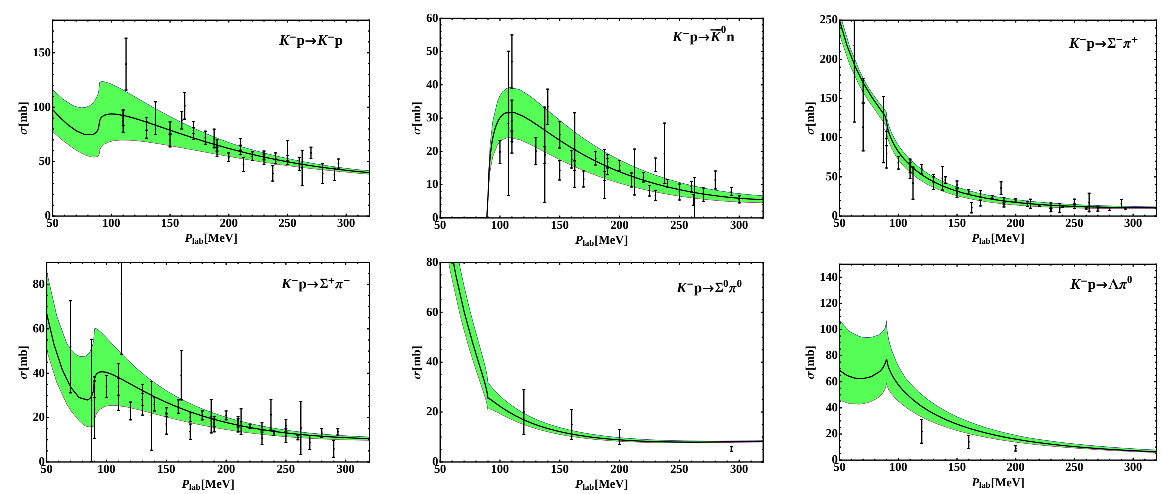
- kaonic hydrogen + scattering data can now be analyzed consistently
- use the chiral Lagrangian at NLO, three groups (different schemes)

Ikeda, Hyodo, Weise 2011; UGM, Mai 2012, Guo, Oller 2012

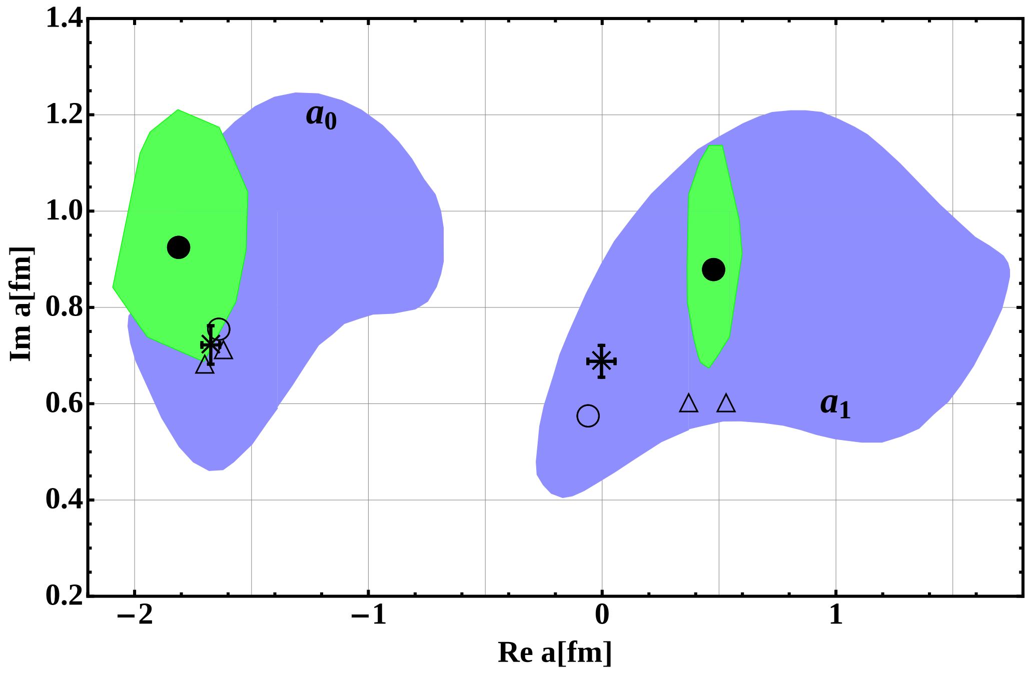
- 14 LECs and 3 subtraction constants to fit

$\Rightarrow$ simultaneous description of the SIDDHARTA and the scattering data





KAON–NUCLEON SCATTERING LENGTHS



$$a_0 = -1.81^{+0.30}_{-0.28} + i\,0.92^{+0.29}_{-0.23}\ \mathrm{fm}$$

$$a_1 = +0.48^{+0.12}_{-0.11} + i\,0.87^{+0.26}_{-0.20}\ \mathrm{fm}$$

$$a_{K^-p} = -0.68^{+0.18}_{-0.17} + i\,0.90^{+0.13}_{-0.13}\ \mathrm{fm}$$

SIDDHARTA only:

$$a_{K^-p} = -0.65^{+0.15}_{-0.15} + i\,0.81^{+0.18}_{-0.18}\ \mathrm{fm}$$

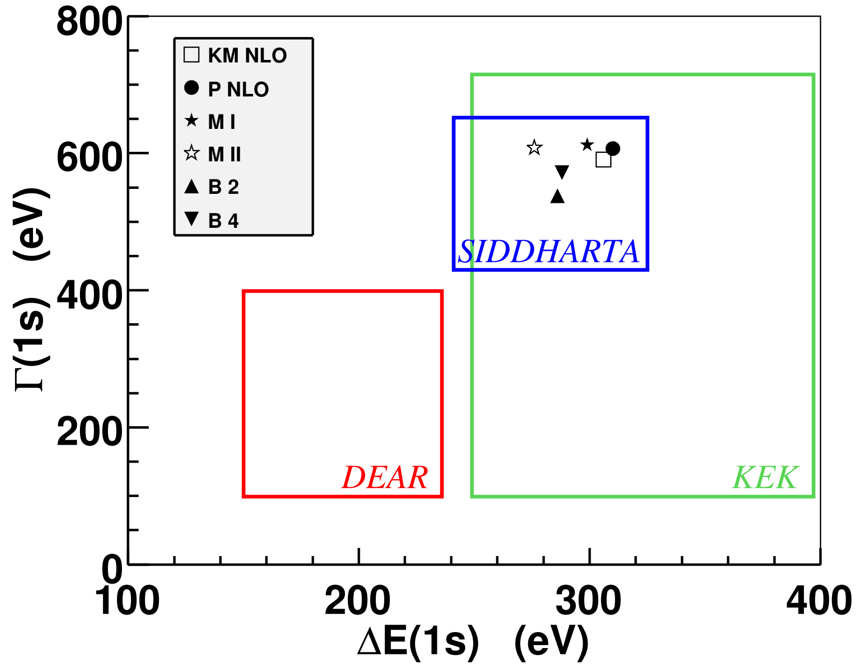
- clear improvement compared to scattering data only

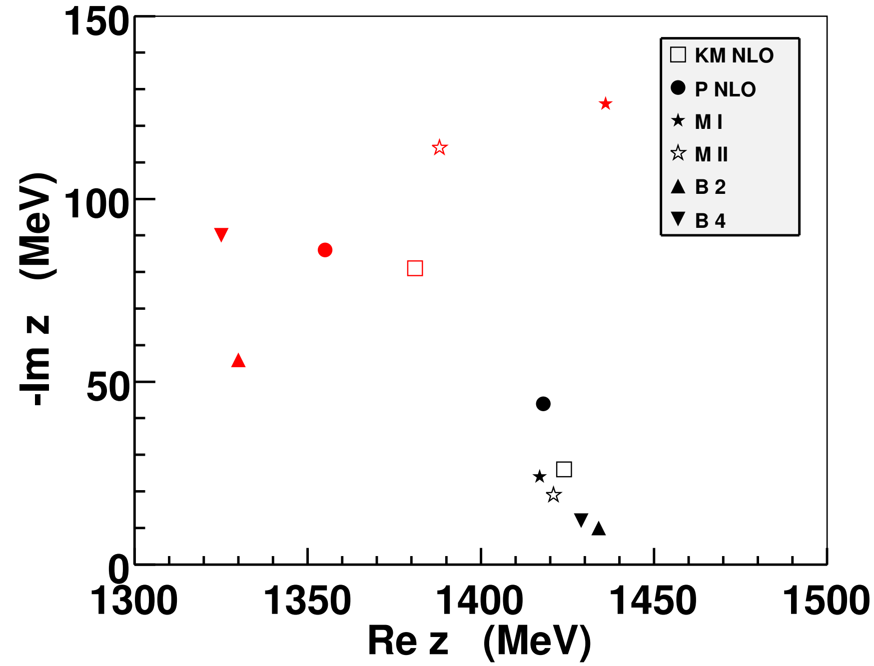
$\Rightarrow$ fundamental parameters to within about 15% accuracy

COMPARISON of VAROIUS APPROACHES

- systematic study of the two-pole scenario of the $\Lambda(1405)$ using various approaches

Cieply, Mai, UGM, Smejkal, Nucl. Phys. A954 (2016) 17





$\hookrightarrow$ higher/lower pole well/not well determined

$\hookrightarrow$ some solutions also include precise data on $\gamma p \to \Sigma K \pi$ from JLab

$\hookrightarrow$ need more data on the $\pi\Sigma$ mass distribution from various reactions

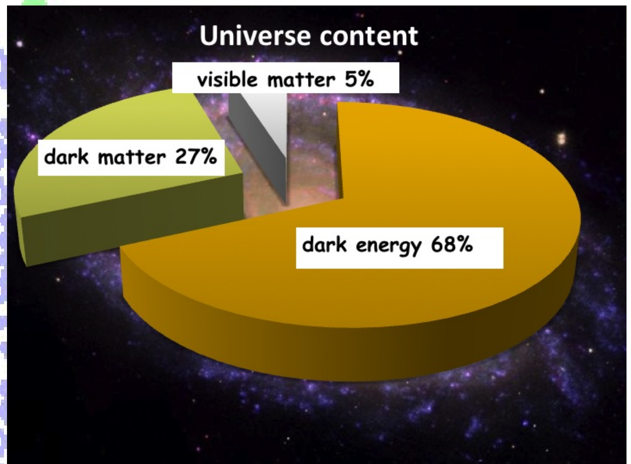
INTERMEDIATE SUMMARY

- Hadron-hadron scattering: important role of chiral symmetry (CHPT)
  $\to$ combine with dispersion relations, unitarization, lattice
- Pion-pion scattering
  $\to$ a fine test of the Standard Model
- Pion-kaon scattering
  $\to$ tension between lattice and Roy-Steiner solution
- Pion-nucleon scattering
  $\to$ superbe accuracy from EFTs for pionic hydrogen/deuterium
- Antikaon-nucleon scattering
  $\to$ consistent determination of the scattering lengths possible
- same methods: Goldstone-boson scattering off $D, D^\star$–mesons
  $\to$ lattice test of molecular states possible

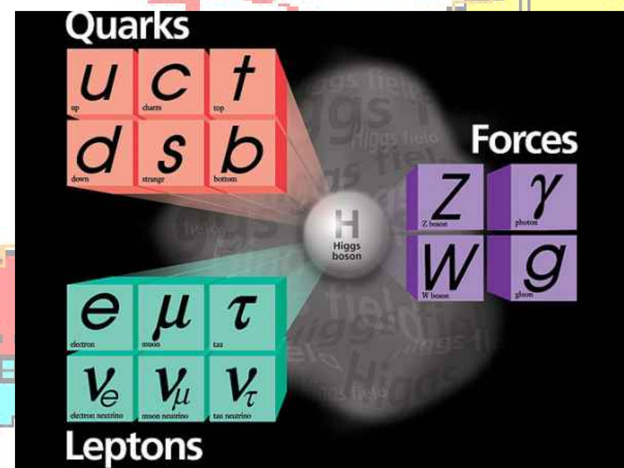
Nuclear Forces from EFT

WHY NUCLEAR PHYSICS?

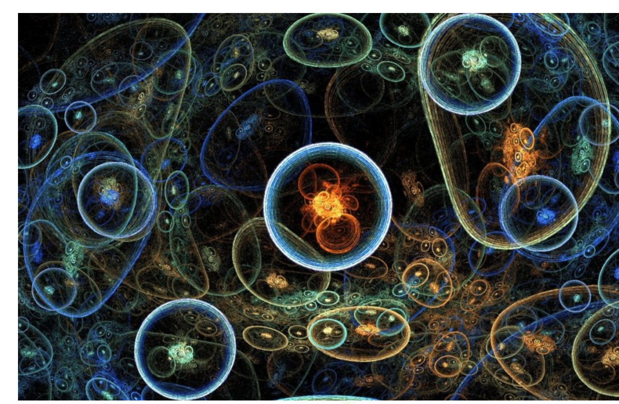
- **The matter we are made off**



- **The last frontier of the SM**



- **Access to the Multiverse**



WHAT DO WE KNOW ABOUT NUCLEAR FORCES?

- At nuclear lengths scales, hadrons are the relevant degrees of freedom
- Nuclei are made of protons and neutrons & virtual mesons
- Nuclear binding energies $\ll$ nuclear masses $\rightarrow$ non-relativistic problem

$\rightarrow$ can solve the nuclear A-body problem w/ the Schrödinger equation

$$
\begin{aligned}
H\Psi_A &= E_A\Psi_A \\
H &= T + V = \sum_A \frac{p_A^2}{2m_N} + V \\
V &= V_{\rm NN} + V_{\rm 3N} + V_{\rm 4N} + \ldots
\end{aligned}
$$

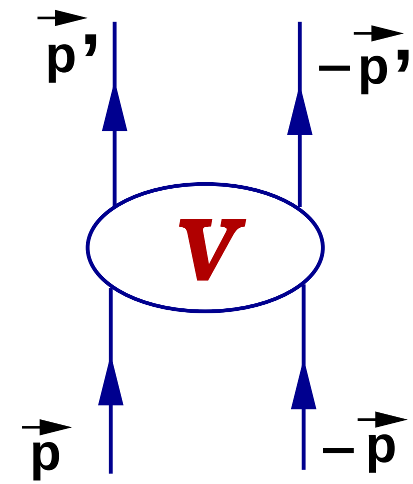
Inputs: $V_{\rm NN}$ from pp and np phase shift analysis
$\rightarrow$ high precision nucleon-nucleon potentials (CD-Bonn, Nijm I,II, AV18, ...)

$V_{\rm 3N}$ small, from phenomenological fits/models

- Ab initio (MC) calculations based on this are astonishingly precise

Carlson, Phandaripande, Pieper, Wiringa, . . .

THE TWO-NUCLEON FORCE: FUNDAMENTALS



- One-pion exchange as the longest range interaction
  (Yukawa 1935)

$$V_{1\pi}(\vec{q}\,) \propto \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^{\,2} + M_\pi^2}\,, \quad \vec{q} = \vec{p}\,' - \vec{p}$$

- Parameterize the shorter-range terms in the most general way
  available vectors $\vec{\sigma}_1, \vec{\sigma}_2, \vec{q}, \vec{k} = \vec{p} + \vec{p}\,'$ and isovectors $\vec{\tau}_1, \vec{\tau}_2$

$\rightarrow$ hermiticity, isospin conservation, invariance under rotations, space reflection and time reversal yields 10 structures

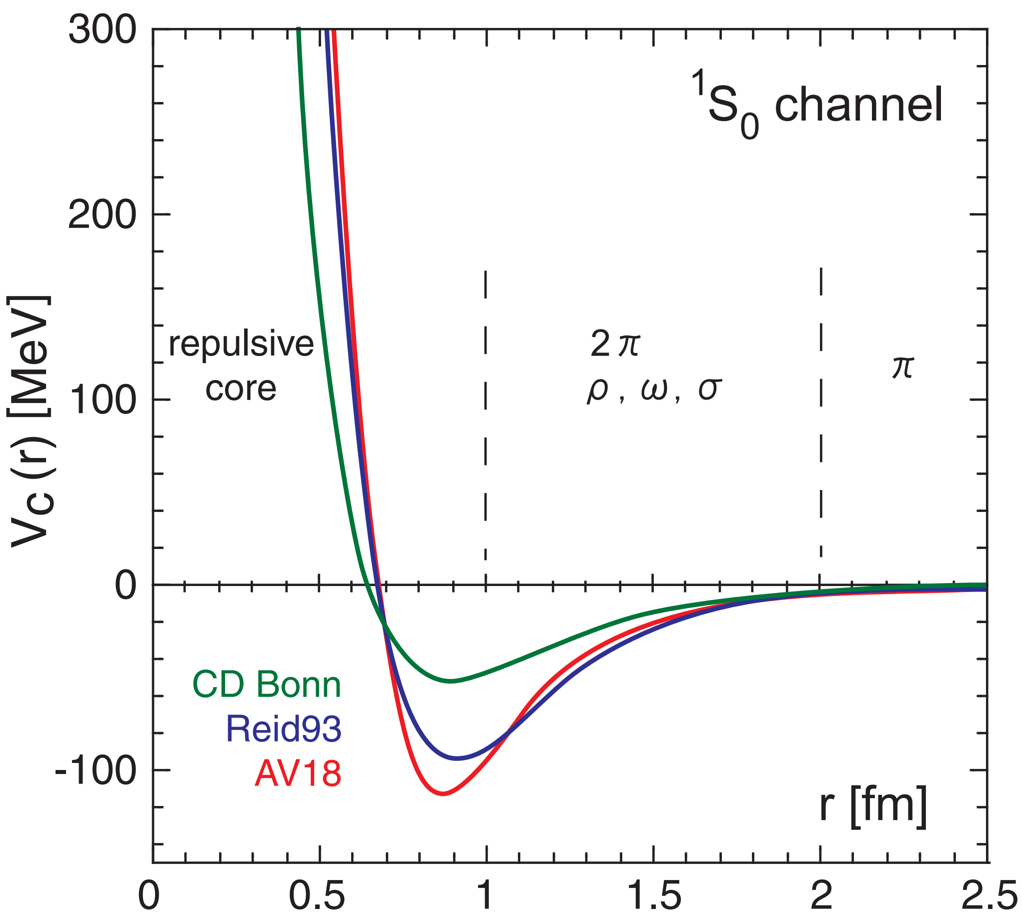
$$\{1,\ \ \vec{\sigma}_1\cdot\vec{\sigma}_2,\ \ i(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\vec{q}\times\vec{k},\ \ \vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q},\ \ \vec{\sigma}_1\cdot\vec{k}\,\vec{\sigma}_2\cdot\vec{k}\,\} \otimes \{1, \vec{\tau}_1\cdot\vec{\tau}_2\}$$

times scalar functions, to be obtained from a fit to data

- so-called “high-precision” potentials (AV18, CD Bonn, NijmI/II, Reid93)
  - nearly perfect description of pp and np data below ~ 350 MeV
  - need typically about 40 -50 parameters

THE CENTRAL NN POTENTIAL

- consider the central potential ($\mathbf{1} \otimes \mathbf{1}$) in the spin-singlet, S-wave $^1S_0$



- universal features:

  long-range one-pion exchange

  intermediate-range attraction

  short-range repulsion

- note, however:

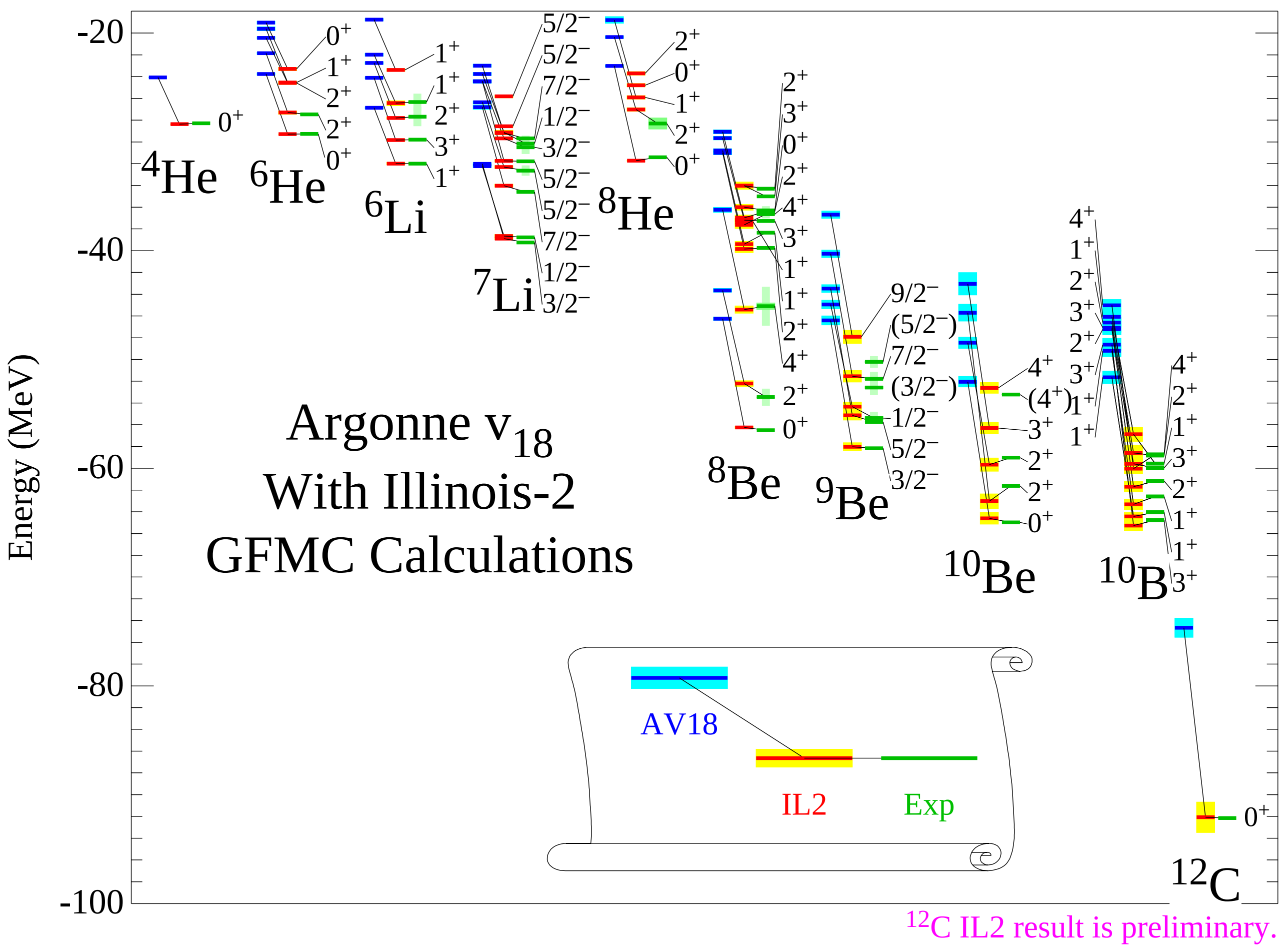
  potential is **not an observable**

  short-range physics is representation dependent

QUANTUM MC CALCULATIONS OF NUCLEI

S. Pieper, Nucl. Phys. A751 (2005) 516, Nollett, Pieper, Wiringa, Phys. Rev. Lett. 99 (2007) 022502

- large numerical effort (^{12}C costed 75000 CPU hrs on a HPC)



⇒ a small three-nucleon force is needed!

OPEN ENDS

- Why is there this hierarchy $\boxed{V_{2N} \gg V_{3N} \gg V_{4N}}$?
- Gauge and chiral symmetries difficult to include (meson-exchange currents)

Brown, Riska, Gari, ...

- Connection to QCD ?

most models have one-pion-exchange, but not necessarily respect chiral symmetry

some models have two-pion exchange reconstructed via dispersion relations from $\pi N \to \pi N$

$\Rightarrow$ We want an approach that

- — is linked to QCD via its symmetries
- — allows for systematic calc's with a controlled theoretical error
- — explains the observed hierarchy of the nuclear forces
- — matches nucleon structure to nuclear dynamics
- — allows for a lattice formulation / chiral extrapolations
- — puts nuclear physics on a sound basis

THE NUCLEAR LANDSCAPE: AIMS & METHODS

- Theoretical methods:

– Lattice QCD: *A = 0,1,2, . . .*

– NCSM, Faddeev-Yakubowsky, GFMC, ... : *A = 3 - 16*

– coupled cluster, . . .: *A = 16 -100*

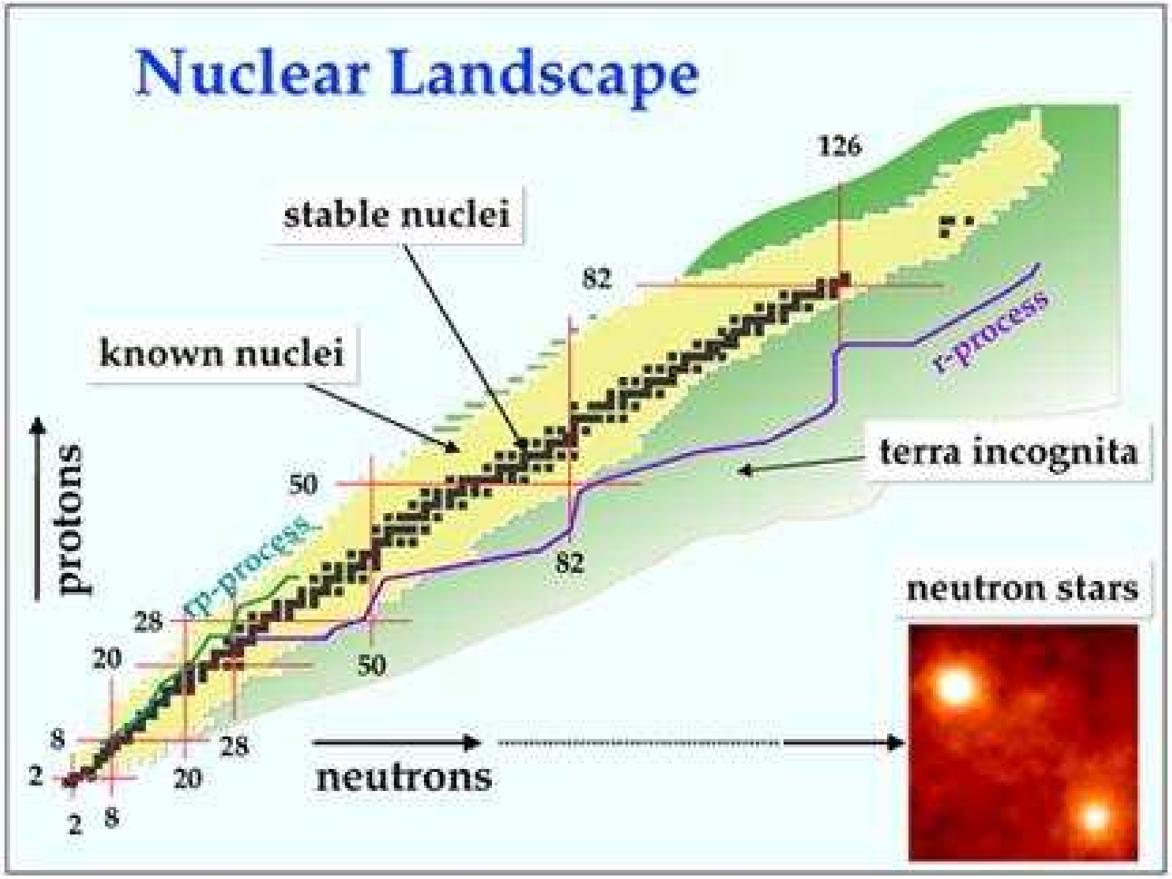
– density functional theory, . . .: $A \geq 100$

- Chiral EFT:

– provides accurate NN and 3N forces

– successfully applied in light nuclei with *A = 2, 3, 4*

– combine with simulations to get to larger A



⇒ Nuclear Forces from Chiral Effective Field Theory

A toy model for NN scattering

A TOY MODEL

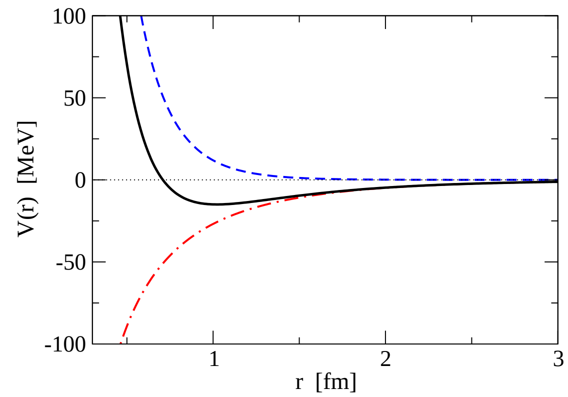
- Consider a toy model with light & heavy boson exchanges

$$V(\vec{q}) = \frac{\alpha_l}{\vec{q}^{\,2} + M_l^2} + \frac{\alpha_h}{\vec{q}^{\,2} + M_h^2} \to V(\vec{r}) = \underbrace{\frac{\alpha_l}{4\pi r} \mathrm{e}^{-M_l r}}_{\text{long-range}} + \underbrace{\frac{\alpha_h}{4\pi r} \mathrm{e}^{-M_h r}}_{\text{short-range}}$$

– $M_l = 200$ MeV , $M_h = 750$ MeV

– $\alpha_l = -1.5$, $\alpha_h = 10.81$ [attractive, repulsive]

$\to$ S-wave bound state: $E_B = 2.2229$ MeV



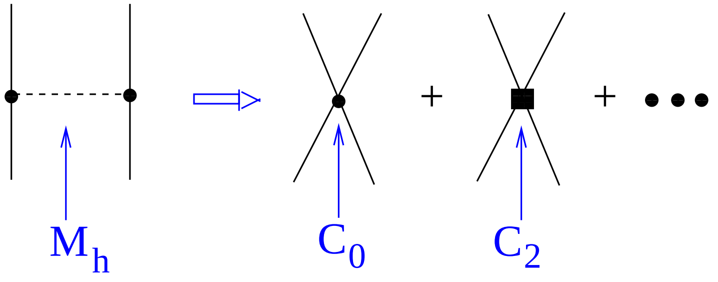
- Effective theory

– at low energy $q \sim M_l \ll M_h$, structure of short-distance potential irrelevant

– represent short-range potential by a series of contact interactions

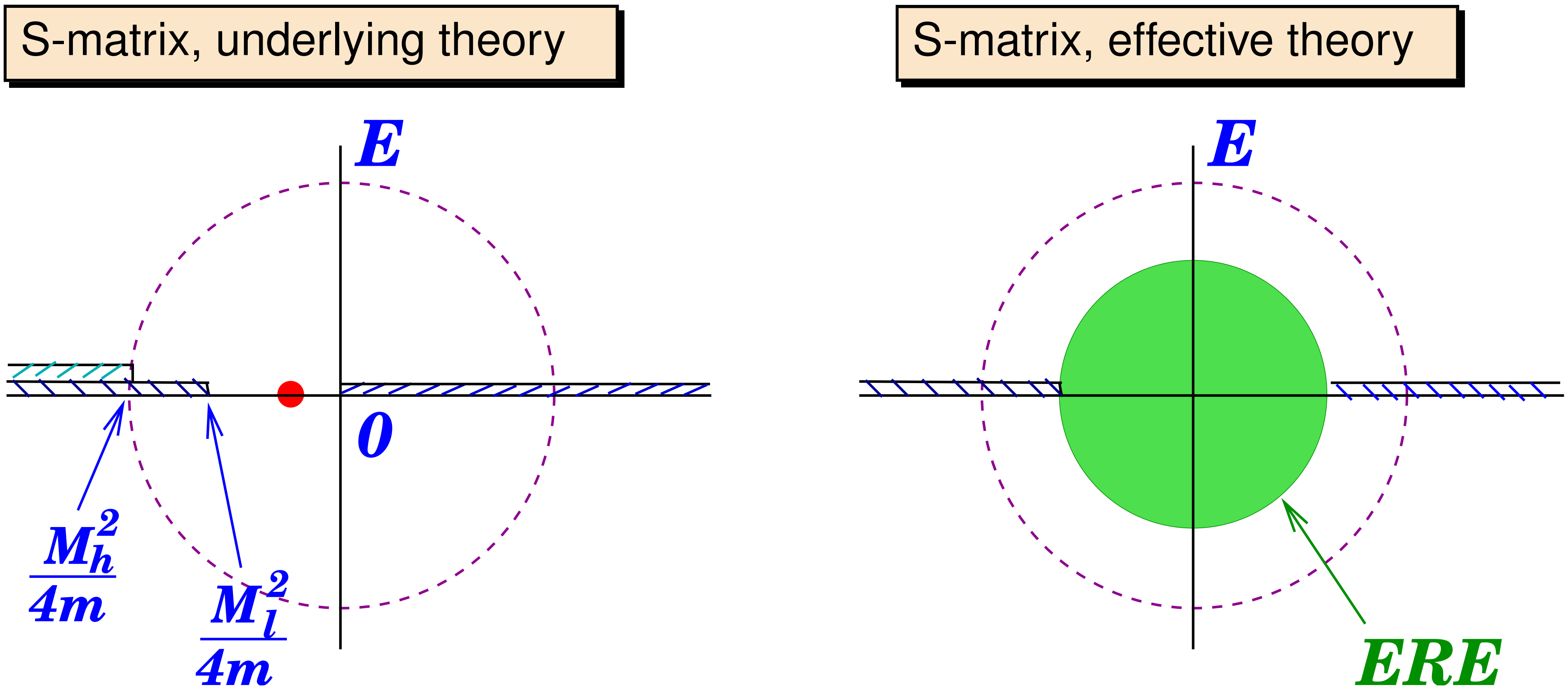
$$\to \boxed{V_{\rm eff} = V_{\rm long-range} + C_0 + C_2(\vec{p}^{\,2} + \vec{p}^{\,\prime 2}) + \ldots}$$

corresponding to $\frac{1}{M_h^2 - q^2} = \frac{1}{M_h^2} - \frac{q^2}{M_h^4} + \ldots$



TOY MODEL cont'd

- Expectations:



- should work for momenta $|k| \leq \frac{M_h}{2} = 375$ MeV (or $E_{\rm lab} \leq \frac{M_h^2}{2m} \sim 300$ MeV)

- should go beyond the ERE, converges for $|k| \leq \frac{M_l}{2} = 100$ MeV (or $E_{\rm lab} \leq \frac{M_l^2}{2m} \sim 20$ MeV)

[ERE = effective range expansion]

TOY MODEL cont'd

- T-matrix of the effective theory:

weak interaction $|\alpha_{l,h}| \ll 1: \quad \langle f|T|i\rangle \simeq \langle f|V_{\rm eff}|i\rangle$

strong interaction $|\alpha_{l,h}| \geq 1: \quad \langle f|T|i\rangle = \langle f|V_{\rm eff}|i\rangle + \sum_n \frac{\langle f|V_{\rm eff}|n\rangle\langle n|V_{\rm eff}|i\rangle}{E_i - E_n + i\epsilon} + \ldots$

sum diverges, high-momentum physics $\rightarrow$ introduce UV cutoff Λ: $M_l \ll \Lambda \sim M_h$

- Fix the $C_i(\Lambda)$ from some low-energy data $\rightarrow$ make predictions
- use e.g. the ERE: $k\cot\delta = -\frac{1}{a} + \frac{1}{2} r k^2 + v_2 k^4 + v_3 k^6 + \ldots$

LO: $V_{\rm eff} = V_{\rm long} + C_0 f_\Lambda(p,p') \longrightarrow C_0 \text{ from } a \quad [f_\Lambda(p,p') = \exp(-(p^2+p'^2)/\Lambda^2)]$

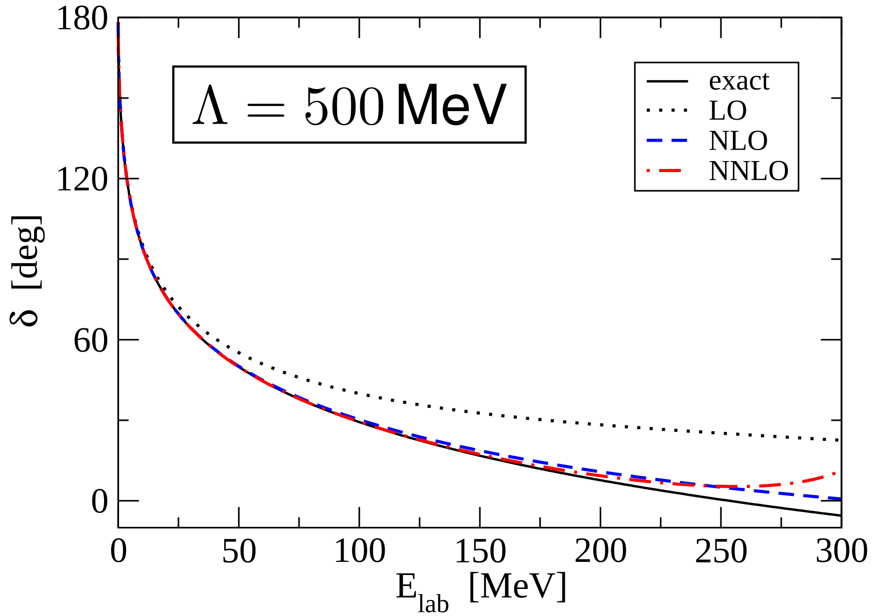
NLO: $V_{\rm eff} = V_{\rm long} + \left[C_0 + C_2(p^2+p'^2)\right] f_\Lambda(p,p') \longrightarrow C_0, C_2 \text{ from } a, r$

NNLO: $V_{\rm eff} = V_{\rm long} + \left[C_0 + C_2(p^2+p'^2) + C_4 p^2 p'^2\right] f_\Lambda(p,p')$

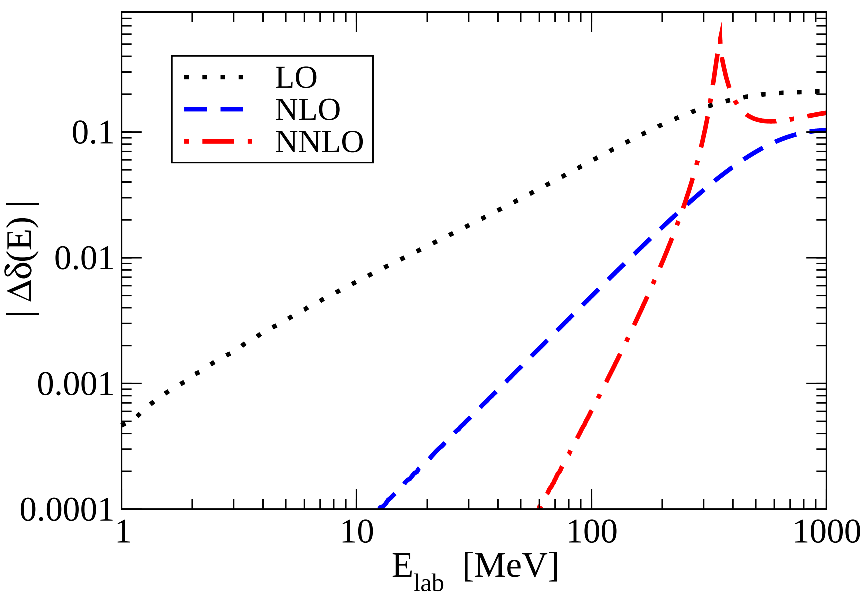
$\longrightarrow C_0, C_2, C_4 \text{ from } a, r, v_2$

TOY MODEL: RESULTS

- Phase shift



- relative error



- error at order n: $\Delta\delta(k) \sim (k/\tilde{\Lambda})^{2n}$, $\tilde{\Lambda} \sim 400$ MeV

  agrees with $\tilde{\Lambda} \sim M_h/2$ [breakdown scale]

- results for the bound state: $E_B = \underbrace{2.1594}_{\rm LO} + \underbrace{0.0638}_{\rm NLO} - \underbrace{0.0003}_{\rm NNLO} = 2.2229$ MeV

TOY MODEL: LESSONS

- Incorporate the *correct long-range force*
- Represent short-range physics by local contact interactions in V_{eff}, respect symmetries
- Introduce an UV cut-off Λ (large enough but not neccessarily ∞)
- Fix LECs from some (low-energy) data and make predictions

$\Rightarrow$ At low energies model-independent and systematically improvable!

- for more details see:

  G.P. Lepage, "How to renormalize the Schrödinger equation", nucl-th/9706029

Nuclear forces from chiral EFT

SCALES IN NUCLEAR PHYSICS

- Natural scales (Yukawa, 1935, QCD)

  Long-range one-pion-exchange interaction: $\boxed{\lambda_\pi = 1/M_\pi \simeq 1.5\,\text{fm}}$

  Intermediate range attraction (mostly 2π exchange)

  Nucleons do not like to overlap, short-distance repulsion

- But: nuclei exhibit UNNATURAL scales

  Large S-wave scattering lengths:

  $$\boxed{a_{np}(^1S_0) = -23.8\,\text{fm}\ ,\ \ a_{np}(^3S_1) = 5.4\,\text{fm} \gg 1/M_\pi}$$

  NB: effective ranges are of natural size

  Shallow nuclear binding:

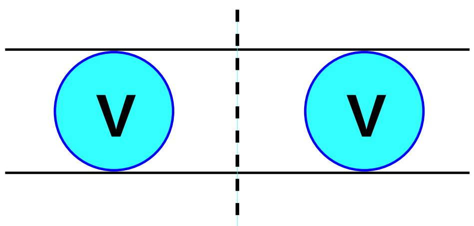
  $$\boxed{\gamma = \sqrt{E_D m_N} = 45\,\text{MeV} \ll M_\pi} \quad (E_D = 2.22\ldots\,\text{MeV})$$

$\Rightarrow$ the corresponding EFT requires a non-perturbative resummation

CALCULATIONAL SCHEME

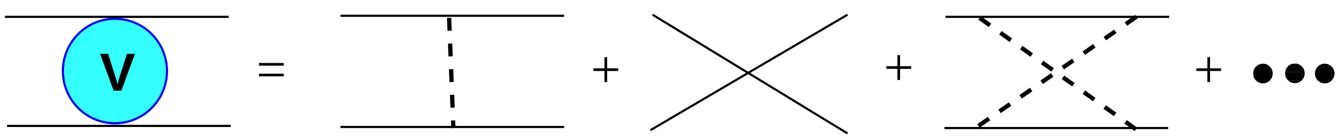
S. Weinberg, Nucl. Phys. **B 363** (1991) 3

- No perturbative description for bound states



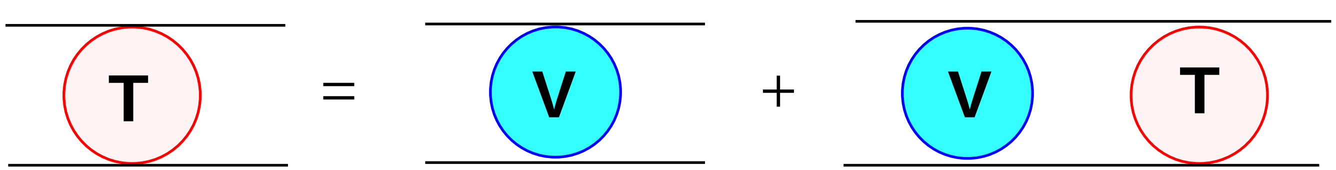
$\Rightarrow$ NN cuts violate perturbative power counting
$\hookrightarrow$ next slide

- Effective potential can be constructed **perturbatively** from chiral EFT



- Solve **non-perturbative** Lippmann-Schwinger/Schrödinger equation

  (requires regularization)

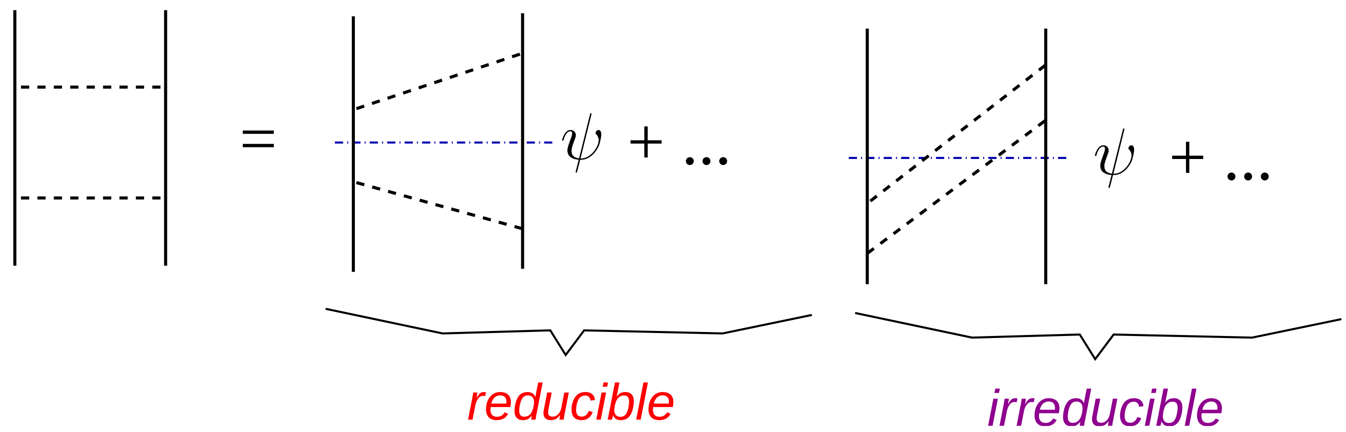


- check convergence for observables *a posteriori*

FAILURE of PERTURBATION THEORY

- Enhancement caused by reducible diagrams (IR divergent in the static limit)
- consider time-ordered perturbation theory

$$Amp = \langle NN|H_I|NN\rangle + \sum_\psi \frac{\langle NN|H_I|\psi\rangle\langle\psi|H_I|NN\rangle}{E_{NN} - E_\psi} + \ldots$$



$$\frac{1}{E_{NN} - E_\psi} = \frac{2m_N}{\vec{p}^{\,2} - \vec{q}^{\,2}} \sim \frac{m_N}{q^2} \gg \frac{1}{Q}\,, \quad \frac{1}{E_{NN} - E_\psi} \sim \frac{1}{M_\pi} \sim \frac{1}{Q}$$

[Q = small parameter (mass, momentum)]

LIPPMANN-SCHWINGER EQUATION

- compact operator form

$$\boxed{T = V + VG_0T}$$ G_0 = free two-nucleon propagator

- partial wave representation = projection onto states with orbital angular momentum L, total spin S and total angular momentum J $\rightarrow$ next slide

$$T^{SJ}_{L',L}(p',p) = V^{SJ}_{L',L}(p',p) + \sum_{L''} \int_0^\infty \frac{dp''(p'')^2}{(2\pi)^3} V^{SJ}_{L',L''}(p',p'') \frac{2\mu}{p^2 - p''^2 + i\eta} T^{SJ}_{L'',L}(p'',p)$$

  - sometimes also relativistic kinematics used (for comparison w/ PWA)
  - potential also projected on the partial waves
  - potential requires UV regularization (r-space preferred)

$$\boxed{V^{\rm reg}_{\rm long-range}(\vec{r}\,) = V_{\rm long-range}(\vec{r}\,)f(r/R)}$$

    - typical regulator function: $f(r/R) = \left[1 - \exp(-r^2/R^2)\right]^n, n \geq 4$
    - R in the range from 0.9 to 1.2 fm (part of the error budget)

REMINDER of NN PHASE SHIFTS

- nucleons have spin 1/2 and isospin 1/2

↪ Pauli principle couples spin & isospin

isospin	spin	bound state
$I = 0$ antisymm.	$S = 1$ symmetric	yes deuteron
$I = 1$ symmetric	$S = 0$ antisymm.	no virtual pp

- partial waves in spectroscopic notation: $^{2S+1}L_J$ S total spin, L orbital ang. mom., J total ang. mom.
  - in the np channel we have $I = 0$ and $I = 1$
  - in the pp and nn channels we have only $I = 1$
  - $I = 0$ antisymm. $\rightarrow\, ^3S_1,\, ^3D_{1,2,3}, \ldots,\, ^1P_1,\, ^1F_3, \ldots$
  - $I = 1$ symmetric $\rightarrow\, ^1S_0,\, ^1D_2, \ldots,\, ^3P_{0,1,2},\, ^3F_{2,3,4}, \ldots$
  - tensor force mixes states with equal J and $\Delta L = 2$, e.g. $^3S_1$-$^3D_1$

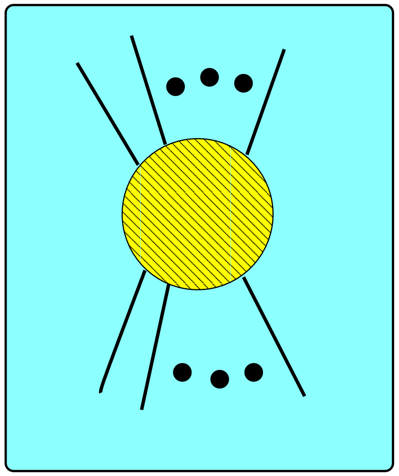
    ↪ parameterized by mixing angles, e.g. ε_1
  - need polarization observables to separate the partial waves

POWER COUNTING for the EFECTIVE POTENTIAL

Weinberg, Rho, van Kolck, . . .,

- N-nucleon interactions receives contributions $\sim (Q/\Lambda)^\nu$: (with Q the small momentum/mass)

$$\nu = -2 + 2N + 2(L - C) + \sum_i V_i \Delta_i$$



– N = number of nucleon fields (in- & out-states)

– L = number of pion loops

– C = number of connected pieces

– V_i = number of vertices with the vertex dimension $\Delta_i = d_i + \frac{1}{2} n_i - 2$

  – d_i = number of derivatives or pion mass insertions at the vertex i

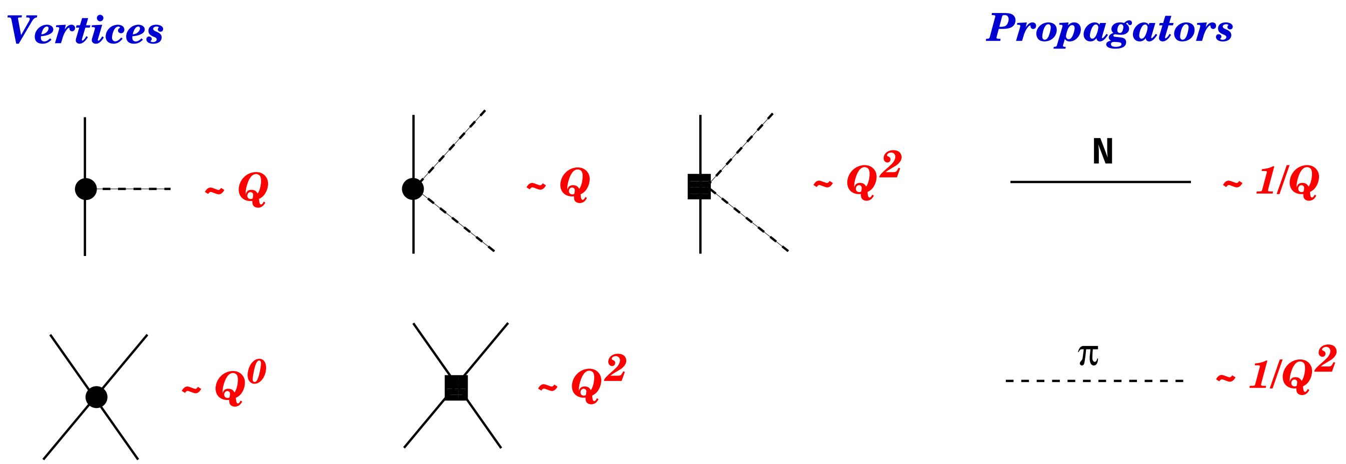
  – n_i = number of nucleon fields at the vertex i

  – external sources & virtual photons can easily be included

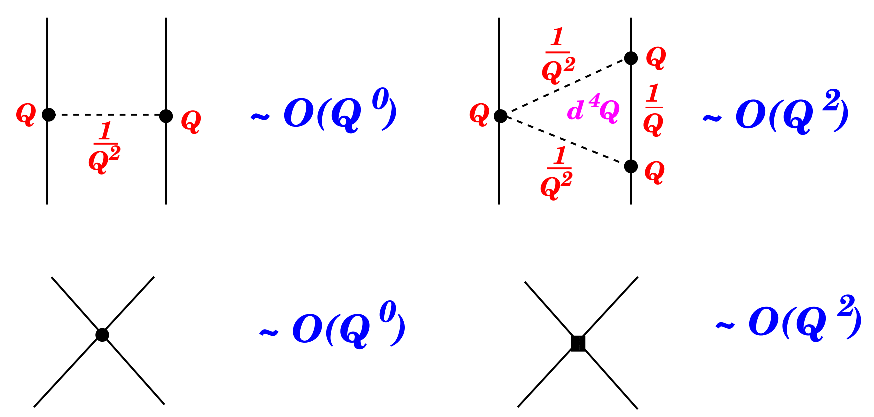
- central observation: Δ_i (ν) is bounded from below because of chiral symmetry

- LO vertices have $\Delta_i = 0 \Rightarrow \nu_{\rm min} = 0$ [NB: state normalization]

POWER COUNTING: EXAMPLES

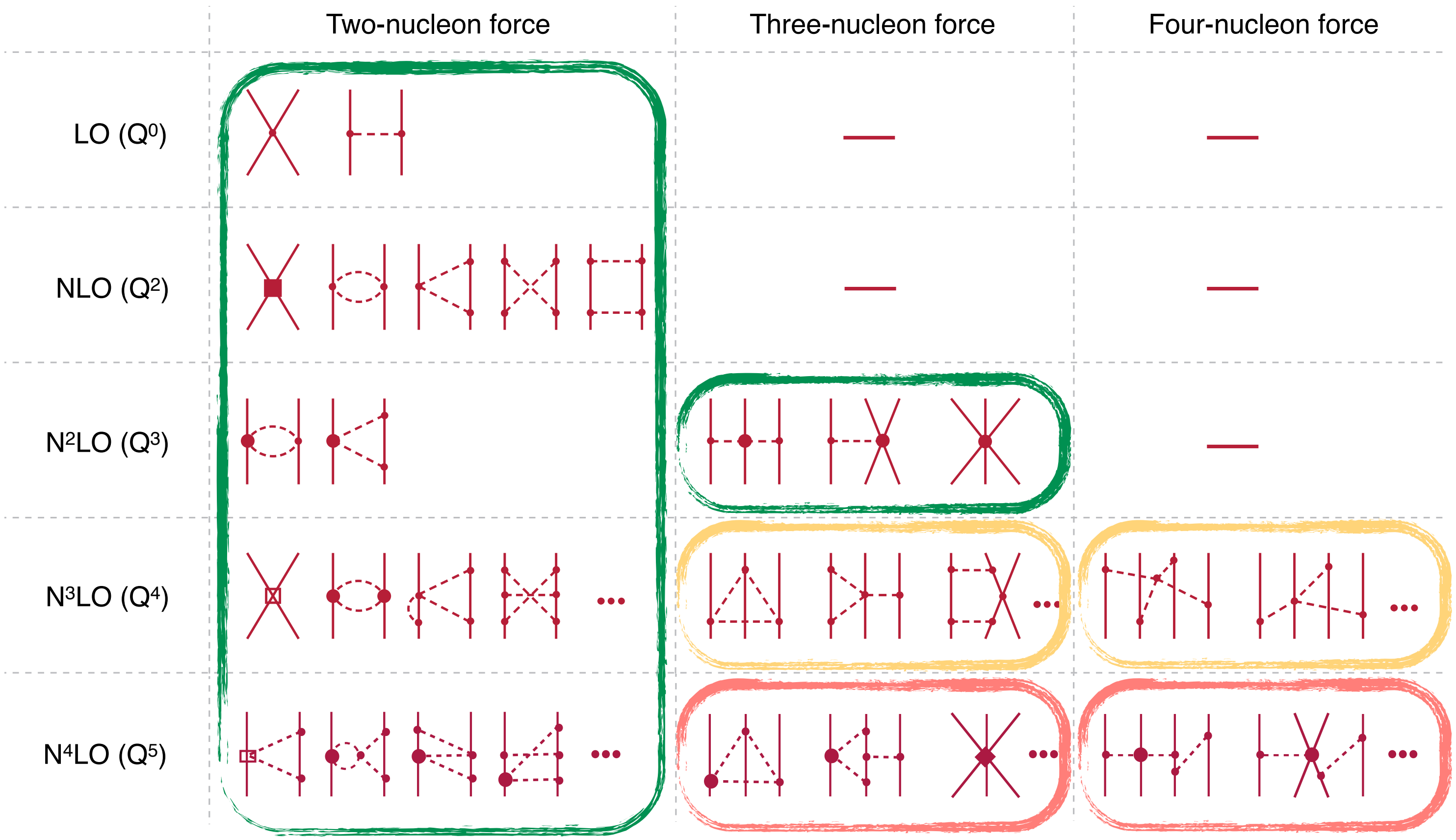


- Examples



NUCLEAR FORCES in CHIRAL NUCLEAR EFT

- expansion of the potential in powers of Q [small parameter]
- explains observed hierarchy of the nuclear forces



worked out and applied   worked out and to be applied   calculations in progress

NUCLEAR POTENTIAL from CHIRAL EFT I

- various methods considered to derive the nuclear forces from the chiral Lagrangian:
  - ⋆ Time-ordered perturbation theory (TOPT)
    Weinberg 1990, 1991; Ordonez et al. 1992, 1994; van Kolck 1994
  - ⋆ S-matrix based appraoch:
    V from perturbative matching to the scattering amplitude
    Robilotta, da Rocha, 1997: Kaiser et al., 1997-2001; Higa et al., 2003, 2004
  - ⋆ Method of unitary transformation
    Epelbaum, Glöckle, Krebs, M., 1998, 2000, 2005, 2015

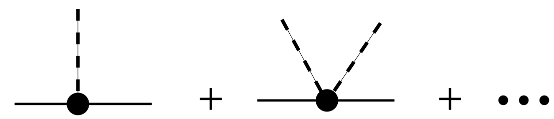
- all standard methods adapted to the problem
- lead to the same results $\surd$ (if energy-dependence is taken care of)
- concentrate here on the method of unitary transformation (and a bit TOPT)

NUCLEAR POTENTIAL from CHIRAL EFT II

- consider mesons interacting with non-relativistic nucleons:

$$H = H_0 + H_I$$

H_I =



- decompose the Fock space as: $|\Psi\rangle = |\phi\rangle + |\psi\rangle$

$$|\phi\rangle \equiv |N\rangle + |NN\rangle + |NNN\rangle + \ldots$$ ← no mesons

$$|\psi\rangle \equiv |N\pi\rangle + |N\pi\pi\rangle + \ldots + |NN\pi\rangle + \ldots$$ ← at least one meson

- Schrödinger equation:

$$\begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} \begin{pmatrix} |\phi\rangle \\ |\psi\rangle \end{pmatrix} = E \begin{pmatrix} |\phi\rangle \\ |\psi\rangle \end{pmatrix}$$

  – η, λ are projection operators on the $|\phi\rangle, |\psi\rangle$ subspaces

  – infinite-dimensional equation due to the πN coupling

  – how to reduce to an effective eq. for $|\phi\rangle$ that can be solved?

TAMM-DANCOFF METHOD

Tamm 1945, Dancoff 1950

- Use the Schrödinger eq. to project out the unwanted component $|\psi\rangle$:

$$\begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} \begin{pmatrix} |\phi\rangle \\ |\psi\rangle \end{pmatrix} = E \begin{pmatrix} |\phi\rangle \\ |\psi\rangle \end{pmatrix} \Longrightarrow |\psi\rangle = \frac{1}{E - \lambda H \lambda} H |\phi\rangle$$

$$\Longrightarrow (H_0 + V_{\rm eff}^{\rm TD}) |\phi\rangle = E |\phi\rangle$$

with the effective potential $V_{\rm eff}^{\rm TD} = \eta H_I \eta + \eta H_I \lambda \frac{1}{E - \lambda H \lambda} \lambda H_I \eta$

- remarks:
  - the potential depends on the energy E
  - $|\phi\rangle$ not orthonormal: $\langle \phi_i | \phi_j \rangle = \delta_{ij} - \langle \phi_i | H_I (\frac{1}{E - \lambda H \lambda})^2 H_I | \phi_j \rangle$
  - reduces to time-ordered perturbation theory:

$$V_{\rm eff}^{\rm TD} = \eta H_I \eta + \eta H_I \frac{\lambda}{E - H_0} H_I \eta + \eta H_I \frac{\lambda}{E - H_0} H_I \frac{\lambda}{E - H_0} H_I \eta + \ldots$$

METHOD of UNITARY TRANSFORMATION I

Fukuda, Sawada, Taketani 1954, Okubo 1954

- Use unitary transformation U to decouple the $|\psi\rangle$ and $|\phi\rangle$ spaces:

$$H = \begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} \Longrightarrow \tilde{H} \equiv U^\dagger H U = \begin{pmatrix} \eta \tilde{H} \eta & 0 \\ 0 & \lambda \tilde{H} \lambda \end{pmatrix}$$

- Advantages:
  - no dependence on the energy (per construction)
  - unitary transformation preserves the norm of $|\phi\rangle$

- How to compute U? Parameterize U in terms of the operator $A = \lambda A \eta$:

$$U = \begin{pmatrix} \eta(1 + A^\dagger A)^{-1/2} & -A^\dagger(1 + A^\dagger A)^{-1/2} \\ A(1 + A^\dagger A)^{-1/2} & \lambda(1 + A^\dagger A)^{-1/2} \end{pmatrix}$$

require that: $\eta \tilde{H} \lambda = \lambda \tilde{H} \eta = 0 \Longrightarrow$ $\boxed{\lambda(H - [A, H] - AHA)\eta = 0}$

- the major problem is to solve the non-linear decoupling equation

METHOD of UNITARY TRANSFORMATION II

- Ex. for solving the decoupling equation - expand in the coupling constant:

$$H_I \propto g \Longrightarrow \text{ansatz:} \quad A = A^{(1)} + A^{(2)} + A^{(3)} + \ldots$$

- recursive solution of the decoupling equation:

$$g^1 : \quad \lambda(H_I - [A^{(1)}, H_0])\eta = 0 \qquad \Longrightarrow A^{(1)} = -\lambda \frac{H_I}{E_\eta - E_\lambda}\eta$$

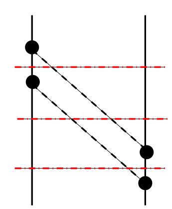
$$g^2 : \quad \lambda(H_I A^{(1)} - [A^{(2)}, H_0])\eta = 0 \Longrightarrow A^{(2)} = -\lambda \frac{H_I A^{(1)}}{E_\eta - E_\lambda}\eta$$

$$\ldots$$

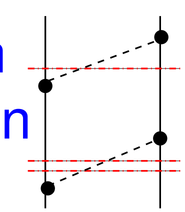
- in the static approximation ($m \to \infty$) we have $E_\eta - E_\lambda \sim E_\pi$, so that:

$$V_{\rm eff} = -\eta H_I \frac{\lambda}{E_\pi} H_I \eta - \underbrace{\eta H_I \frac{\lambda}{E_\pi} H_I \frac{\lambda}{E_\pi} H_I \frac{\lambda}{E_\pi} H_I \eta}_{} + \underbrace{\frac{1}{2}\eta H_I \frac{\lambda}{E_\pi} H_I \eta H_I \frac{\lambda}{E_\pi^2} H_I \eta}_{} + \ldots$$

same as TOPT



wave-function renormalization



Nucleon-Nucleon Potential

STRUCTURE of the NN POTENTIAL

- LO: one-pion-exchange (OPE) plus contact interactions w/o derivatives **2 LECs**

$$V^{(0)} = -\left(\frac{g_A}{2F_\pi}\right)^2 \vec{\tau}_1 \cdot \vec{\tau}_2 \, \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{q^2 + M_\pi^2} + C_S + C_T \, \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

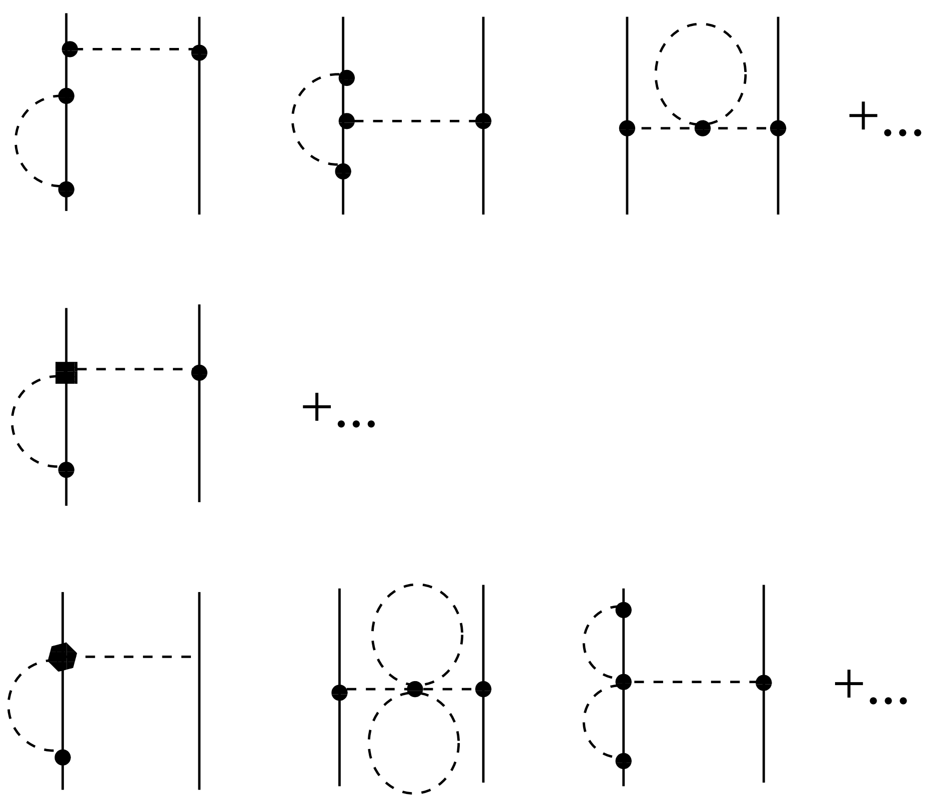
- NLO: renormalization of the one-pion-exchange (OPE)
  plus leading two-pion exchange (TPE)
  plus renormalization of the leading contact interactions
  plus contact interactions w/ 2 derivatives **7 LECs**

- N^{2}LO: further renormalization of the one-pion-exchange (OPE)
  plus subleading two-pion exchange (TPE) ($\sim$ LECs c_i of the πN sector)

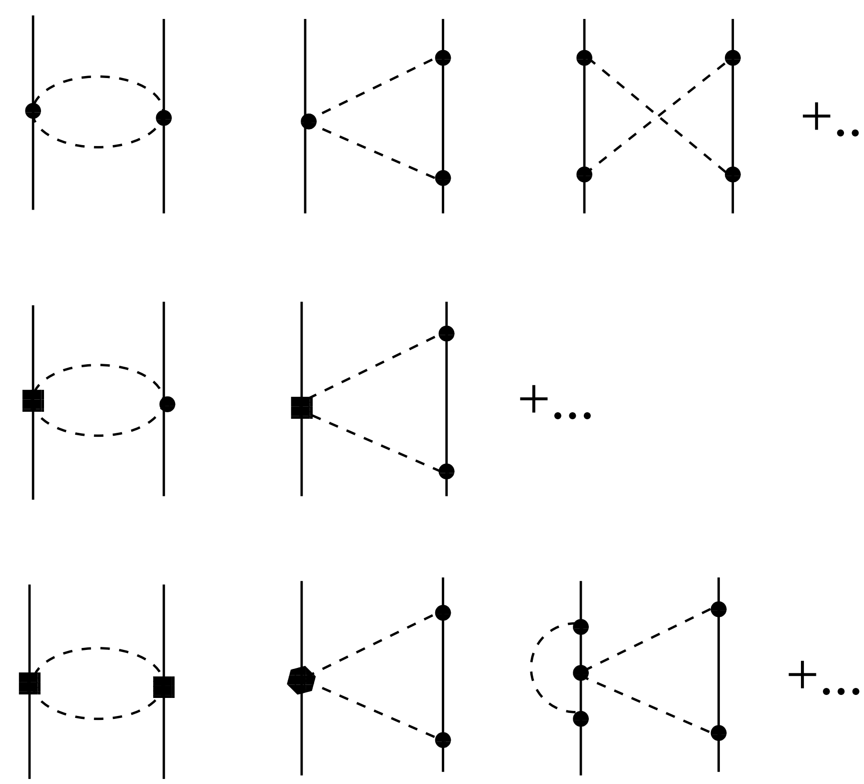
- N^{3}LO: further renormalization of the one-pion-exchange (OPE)
  plus sub-subleading two-pion exchange (TPE)
  plus leading three-pion exchange (TPE) **(very small)** Kaiser 2000
  plus renormalization of dim. two contact interactions
  plus contact interactions w/ 4 derivatives **12 LECs** Reinert et al. 2017

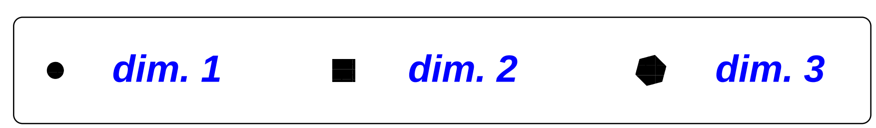
TYPICAL DIAGRAMS

- renormalization of OPEP



- TPEP

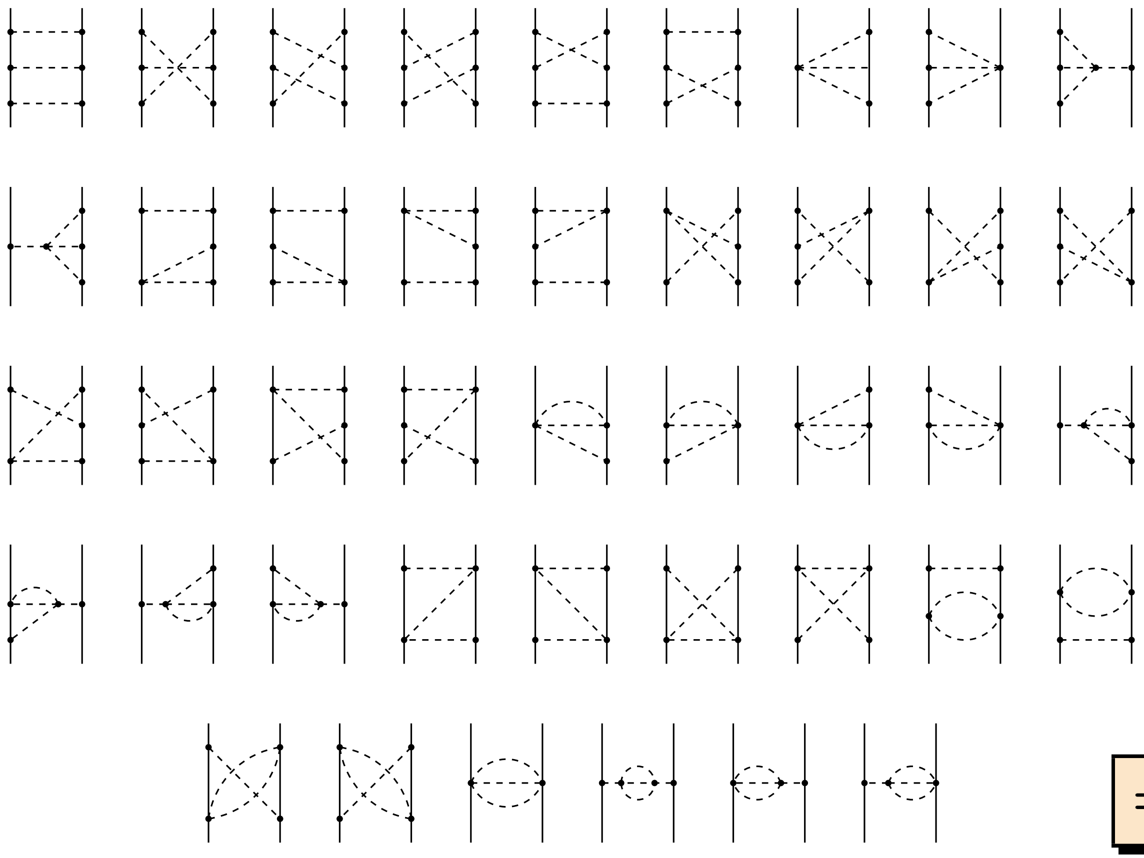




TYPICAL DIAGRAMS continued

Kaiser, Phys. Rev. C **61** (2000) 014003; C **62** (2000) 024001; C **63** (2001) 044010

- three-pion exchange (starts at N^{3}LO)



$\Rightarrow$ insignificant for $r \geq 1$ fm

SHORT-DISTANCE STRUCTURE of the POTENTIAL

- consider chiral 2π potential $\propto g_A^4$

$$V_{2\pi}^{(2)} = \frac{g_A^4}{32F_\pi^4} \int \frac{d^3l}{(2\pi)^3} \frac{\omega_+^2 + \omega_+\omega_- + \omega_-^2}{\omega_+^3 \omega_-^3 (\omega_+ + \omega_-)} \left\{ \tau_1^a \tau_2^a \left(\vec{l}^2 - \vec{q}^2 \right)^2 + 6\sigma_1^i (\vec{q} \times \vec{l})^i \, \sigma_2^j (\vec{q} \times \vec{l})^j \right\}$$

with $\omega_\pm = \sqrt{(\vec{q} \pm \vec{l})^2 + 4M_\pi^2}$

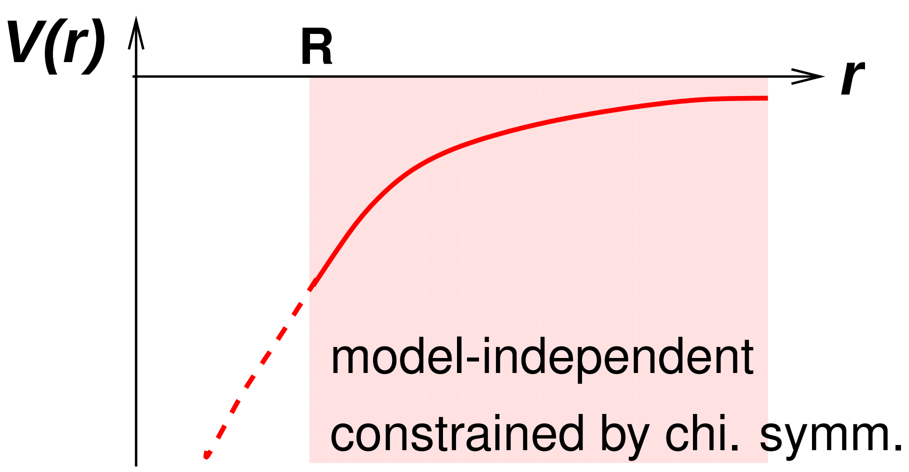
- log and quadratic divergences, absorb in short-range counterterms

$$V_{\rm cont} = (\alpha_1 + \alpha_2 \, q^2) \vec{\tau}_1 \cdot \vec{\tau}_2 + \alpha_3 \, \vec{\sigma}_1 \cdot \vec{q} \, \vec{\sigma}_1 \cdot \vec{q} + \alpha_4 \, q^2 \, \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

- co-ordinate space representation

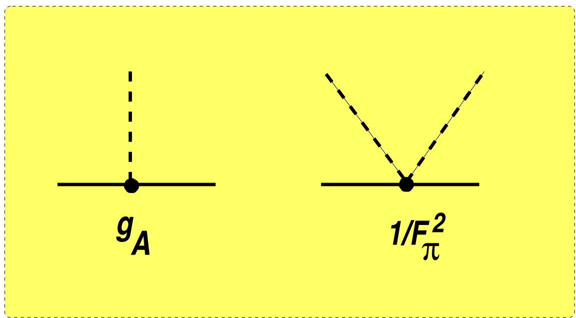
$$V_{2\pi}^{(2)}(q) \to V_{2\pi}^{(2)}(r)$$

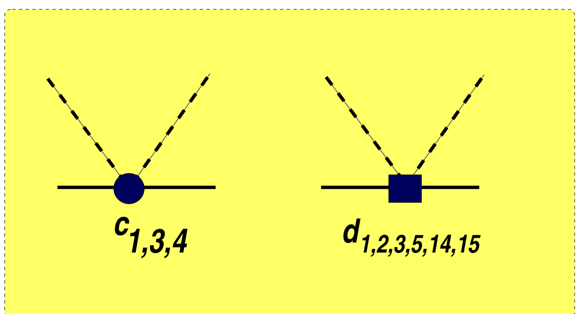
the large-r (long-range) behaviour is uniquely defined and does not depend on the regularization



LOW-ENERGY CONSTANTS

- Pion-nucleon system:





— g_A and F_π precisely known (chiral symmetry)

— dimension 2 & 3 couplings c_i & d_i known

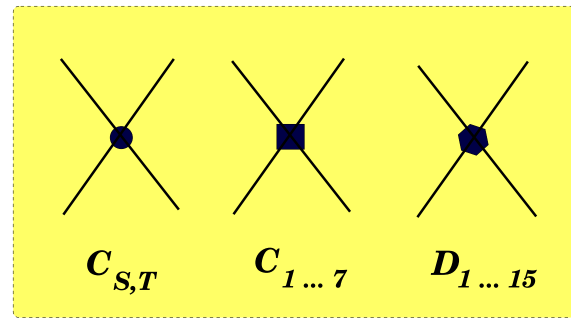
from CHPT/RS studies of $\pi N \to \pi N$

Büttiker, Fettes, M., Steininger, Mojžiš, Hoferichter, Kubis, Ruiz de Elvira, . . .

— physics understood: resonance saturation

Bernard, Kaiser, M., Nucl. Phys. A615 (1997) 483

- Nucleon-nucleon system:



— C_S and C_T: LO 4N couplings — Weinberg

— $C_{1,..,7}$: NLO 4N couplings

Ordonez et al., Epelbaum et al.

— $D_{1,..,12}$: N^{3}LO 4N couplings

Epelbaum, Glöckle, M., Reinert, Krebs, Entem, Machleidt

⇒ these must be fixed from NN data

⇒ fit to the low phases (S,P, ...)

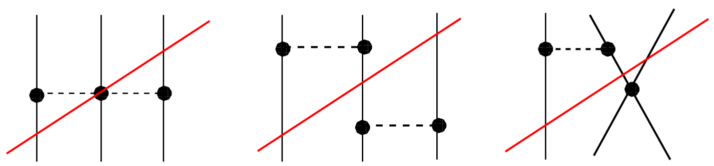
. . . and try to understand the physics behind their values

3N and 4N Potential

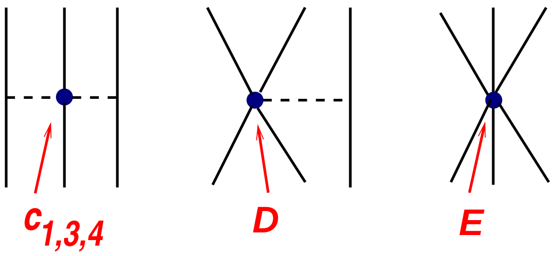
STRUCTURE of the 3N POTENTIAL

- LO: no 3NF

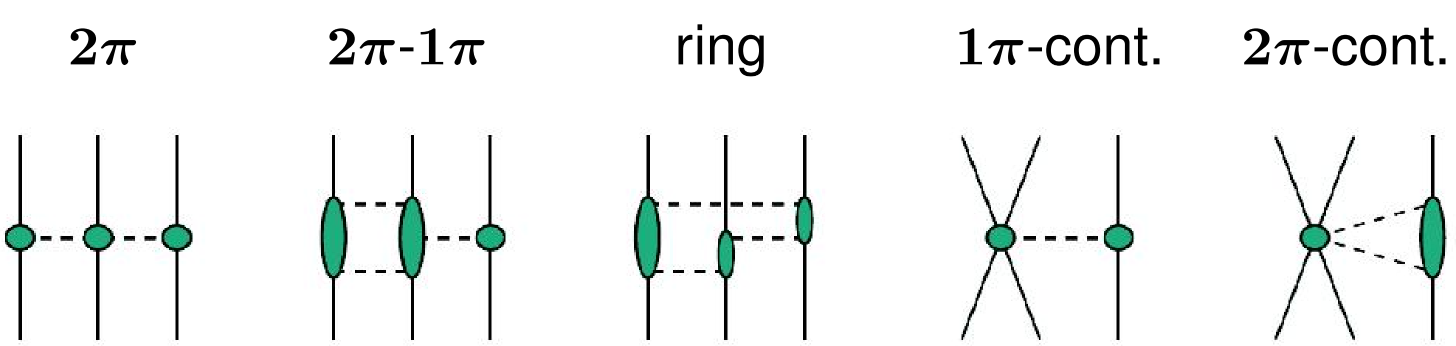
- NLO: 3NF vanishes for energy-independent formulation



- N^2LO: first nonvanishing 3NF
  $\rightarrow$ need two data points to fix the new two LECs D and E



- N^3LO: numerous one-loop corrections, 5 topologies, NO new parameters



LEADING 3N POTENTIAL

- Three different topologies

TPE
$$V^{\rm 3NF}_{\rm TPE} = \sum_{i\neq j\neq k} \frac{1}{2}\left(\frac{g_A}{2F_\pi}\right)^2 \frac{(\vec{\sigma}_i\cdot\vec{q}_i)(\vec{\sigma}_j\cdot\vec{q}_j)}{(\vec{q}_i^{\,2}+M_\pi^2)(\vec{q}_j^{\,2}+M_\pi^2)} F^{\alpha\beta}_{ijk}\tau^\alpha_i\tau^\beta_j$$

$$F^{\alpha\beta}_{ijk} = \delta^{\alpha\beta}\left[-\frac{4c_1M_\pi^2}{F_\pi^2}+\frac{2c_3}{F_\pi^2}\vec{q}_i\cdot\vec{q}_j\right]+\sum_\gamma \frac{c_4}{F_\pi^2}\epsilon^{\alpha\beta\gamma}\tau^\gamma_k\vec{\sigma}_k\cdot[\vec{q}_i\times\vec{q}_j]$$

OPE
$$V^{\rm 3NF}_{\rm OPE} = -\sum_{i\neq j\neq k}\frac{g_A}{8f_\pi^2}\,D\,\frac{\vec{\sigma}_j\cdot\vec{q}_j}{\vec{q}_j^{\,2}+M_\pi^2}\,(\vec{\tau}_i\cdot\vec{\tau}_j)\,(\vec{\sigma}_i\cdot\vec{q}_j)$$

cont
$$V^{\rm 3NF}_{\rm cont} = \frac{1}{2}\sum_{j\neq k} E\,(\vec{\tau}_j\cdot\vec{\tau}_k)$$

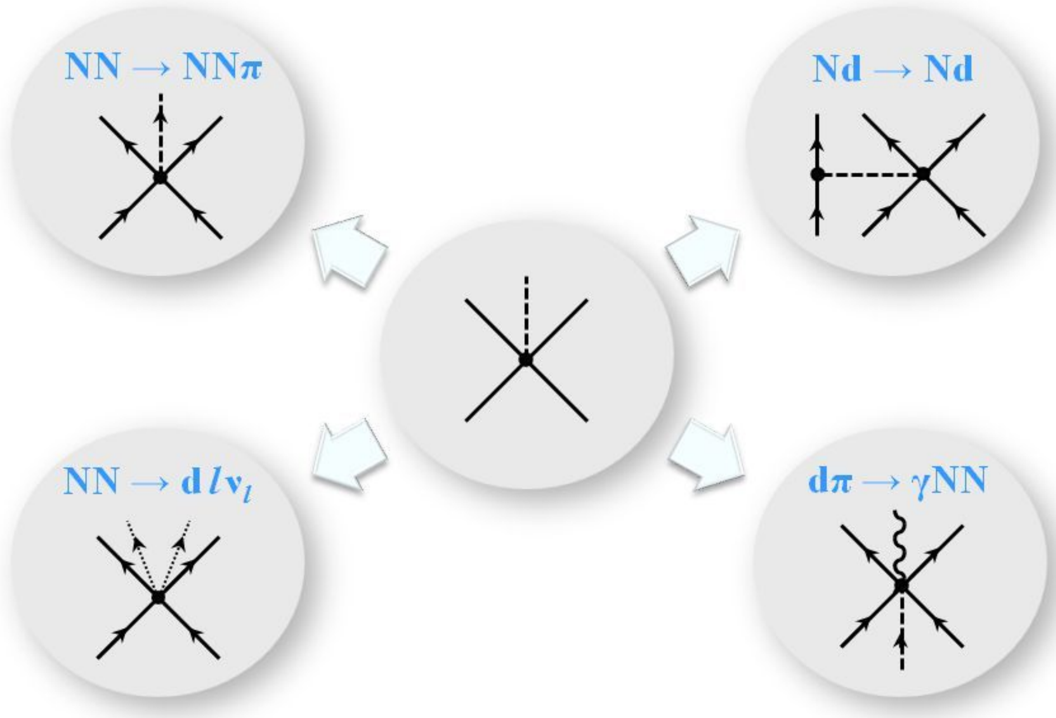
- LECs: c_1, c_3, c_4 from $\pi N\to\pi N$,
  D from $NN\to NN\pi$ or from 3N data or . . . (see next slide)
  E from 3N data

THE D-TERM

- The D-term figures prominently in various reactions:

Hanhart et al., 2000
Baru et al., 2009

Epelbaum et al., 2002
Nogga et al., 2005
Navratil et al., 2007



Ando et al., 2002,03
Park et al., 2003
Nakamura et al., 2007
Gazit et al., 2009

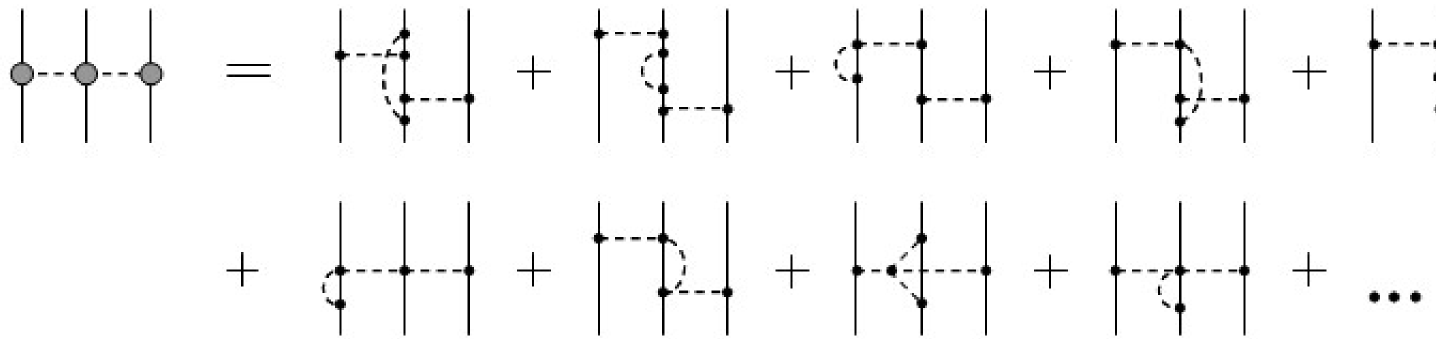
Lensky et al., 2005,07
Gardestig et al., 2006

→ power of EFT !

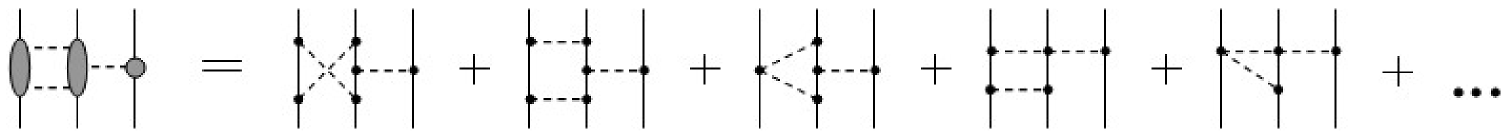
3N FORCES to N^{3}LO: DETAILED STRUCTURE

Bernard, Epelbaum, Krebs, UGM, Phys. Rev. C **77** (2008) 064004; Phys.Rev. C **84** (2011) 054001

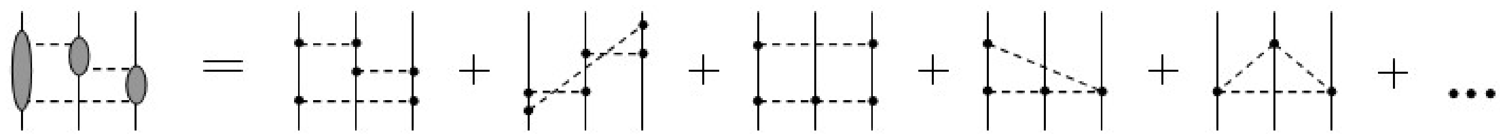
- 2π



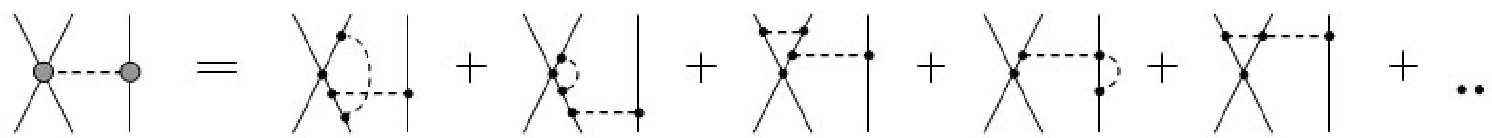
- 2π-1π



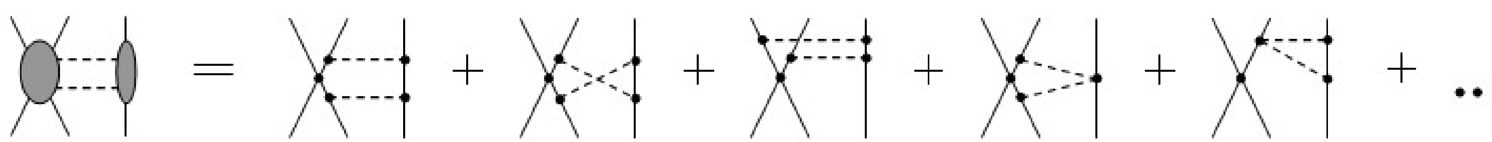
- 2π-$2N$



- 1π-contact



- 2π-contact



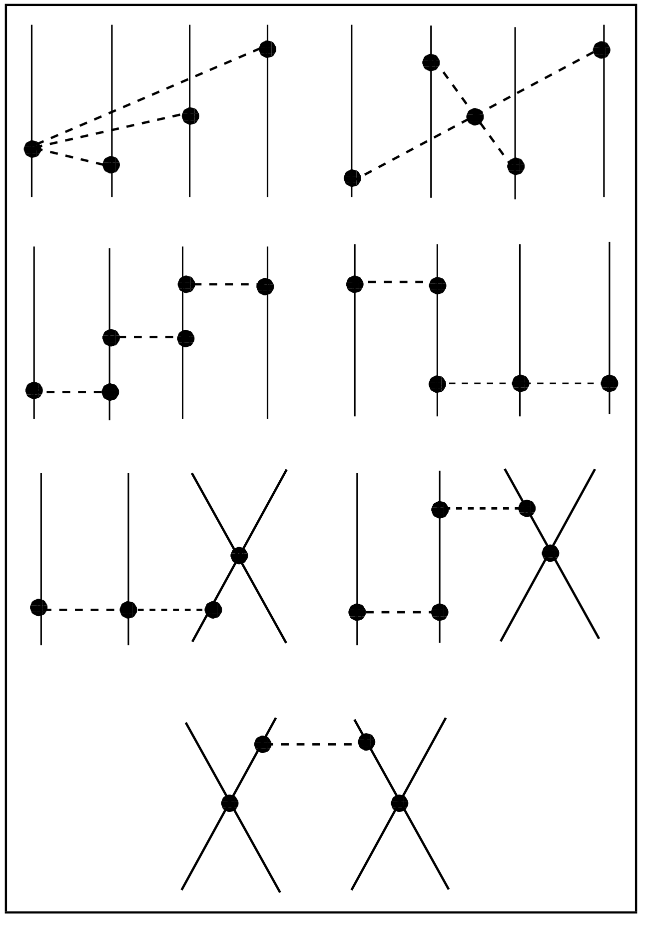
STRUCTURE of the 4N INTERACTION

Epelbaum, Phys. Lett. **B639** (2006) 456, Eur. Phys. J. A **34** (2007) 197

- first shows up at N^3LO
- chiral symmetry plays a crucial role
- at this order, no parameters
- contribution from reducible-like diagrams even in the static limit
- attractive contribution to the ^{4}He B.E.

$$\langle\psi(^4\mathrm{He})|V_{4N}|\psi(^4\mathrm{He})\rangle \sim 200\,\mathrm{keV}$$

A. Nogga *et al.*, unpublsihed



INTERMEDIATE SUMMARY

- Toy model study to capture the essence of nuclear EFT
- Nuclear forces from chiral EFT

  power counting $\longrightarrow$ correct hierarchy of the forces

  two-, three- and four-nucleon forces worked out up to N^{4}LO / N^{3}LO

  regularization of the short-distance components required

  isospin-breaking effects systematically included

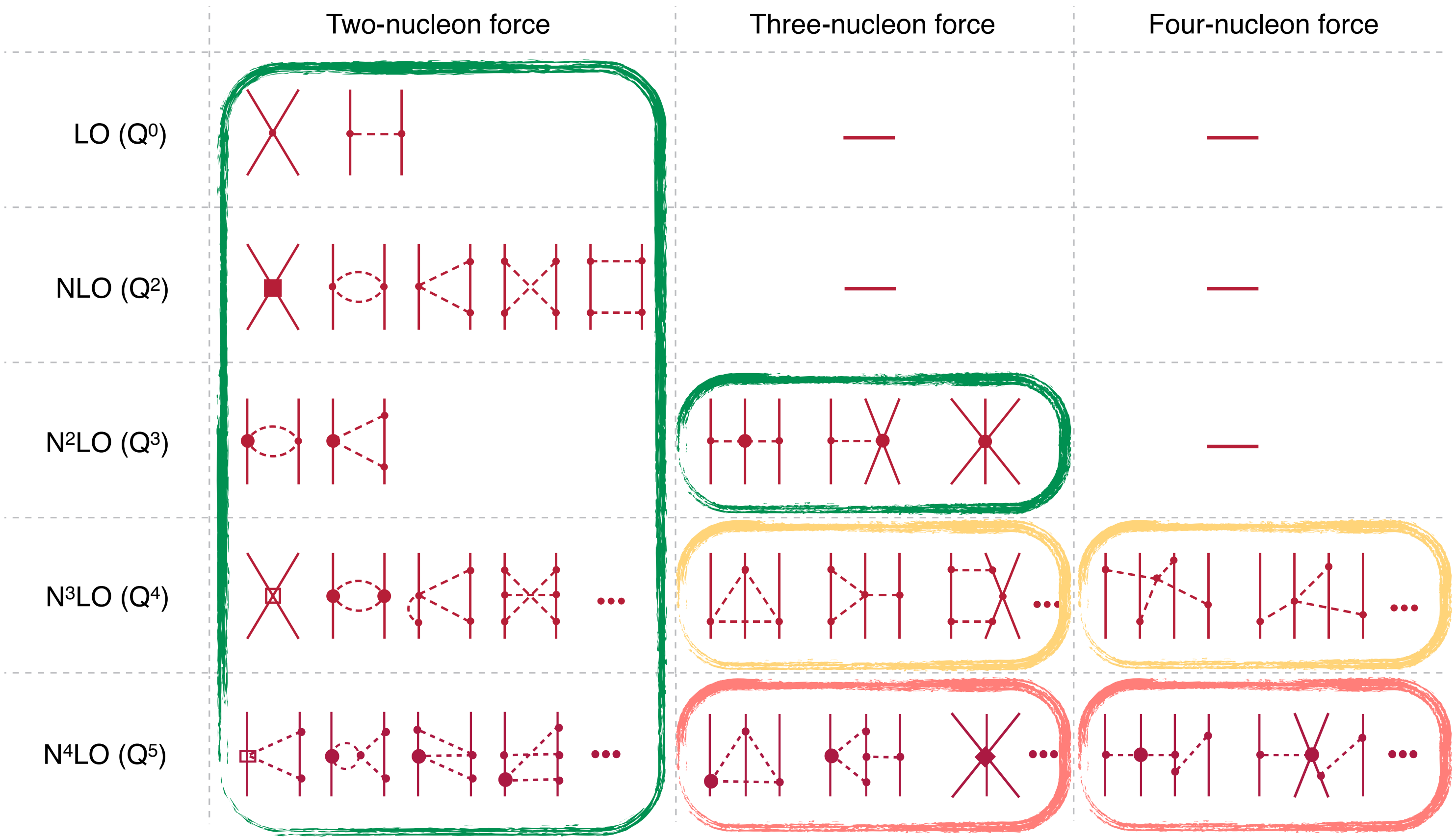
$\Rightarrow$ now let us see if/how these chiral forces work in nuclei

Continuum EFT: new developments

LENPIC www.lenpic.org

NUCLEAR FORCES in CHIRAL NUCLEAR EFT

- expansion of the potential in powers of Q [small parameter]
- explains observed hierarchy of the nuclear forces



worked out and applied   worked out and to be applied   calculations in progress

NN FORCES to FOURTH ORDER

Epelbaum, Krebs, UGM, Eur. Phys. J. **A 51**: 53 (2015)

- new regularization of long-range physics [coordinate space cut-off]:

$$V^{\rm reg}_{\rm long-range}(\vec{r}) = V_{\rm long-range}(\vec{r}) f_{\rm reg}\left(\frac{r}{R}\right) \;, \quad f_{\rm reg} = \left[1 - \exp\left(-\frac{r^2}{R^2}\right)\right]^6$$

$\Longrightarrow$ No distortion of the long-range potential $\to$ better at higher energies

$\Longrightarrow$ No additional spectral function regularization in the TPEP required

$\Longrightarrow$ Study of the chiral expansion of multi-pion exchanges: $R = 0.8 \cdots 1.2$ fm

Baru et al., EPJ A48 (12) 69

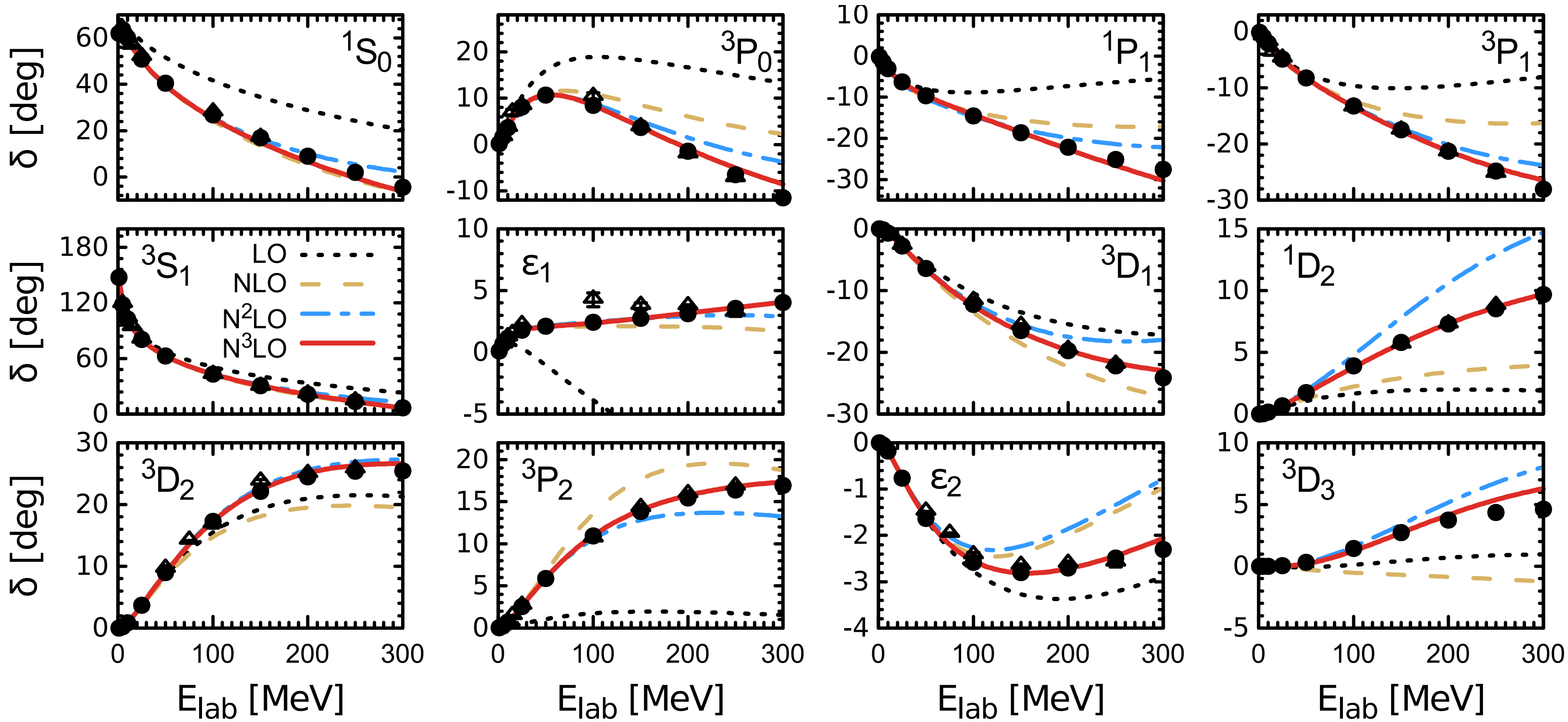
- new way of estimation the theoretical uncertainty [before: only cut-off variations]

$\Longrightarrow$ Expansion parameter depending on the region: $Q = \max\left(\frac{M_\pi}{\Lambda_b}, \frac{p}{\Lambda_b}\right)$

$\Longrightarrow$ Breakdown scale $\Lambda_b = 600$ MeV for $R = 0.8 \cdots 1.0$ fm

CONVERGENCE of the CHIRAL SERIES

- phase shifts show expected convergence [large N2LO corrections understood]

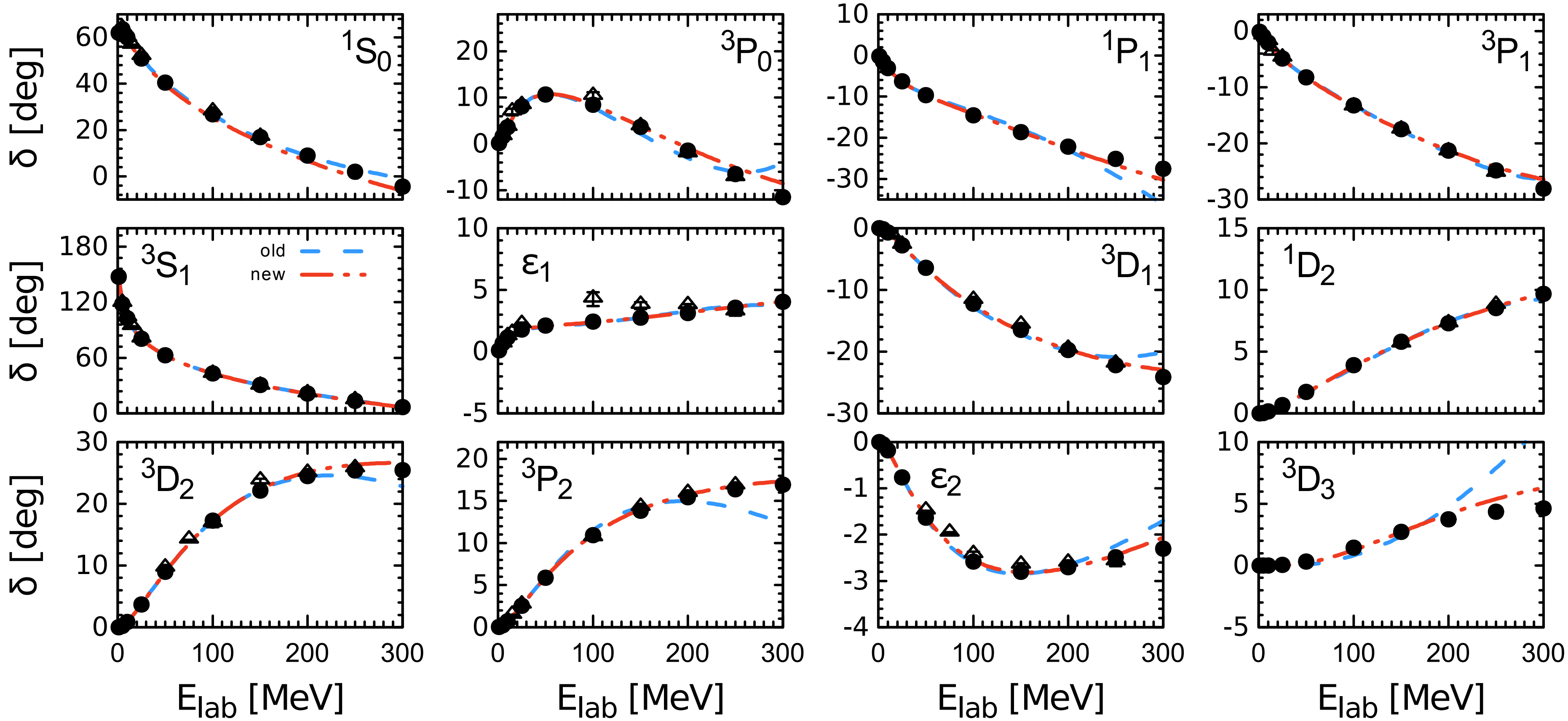


$\Longrightarrow$ clear improvement comp. to earlier N3LO potentials [momentum space reg.]

Entem, Machleidt; Epelbaum, Glöckle, UGM

COMPARISON to EARLIER WORK

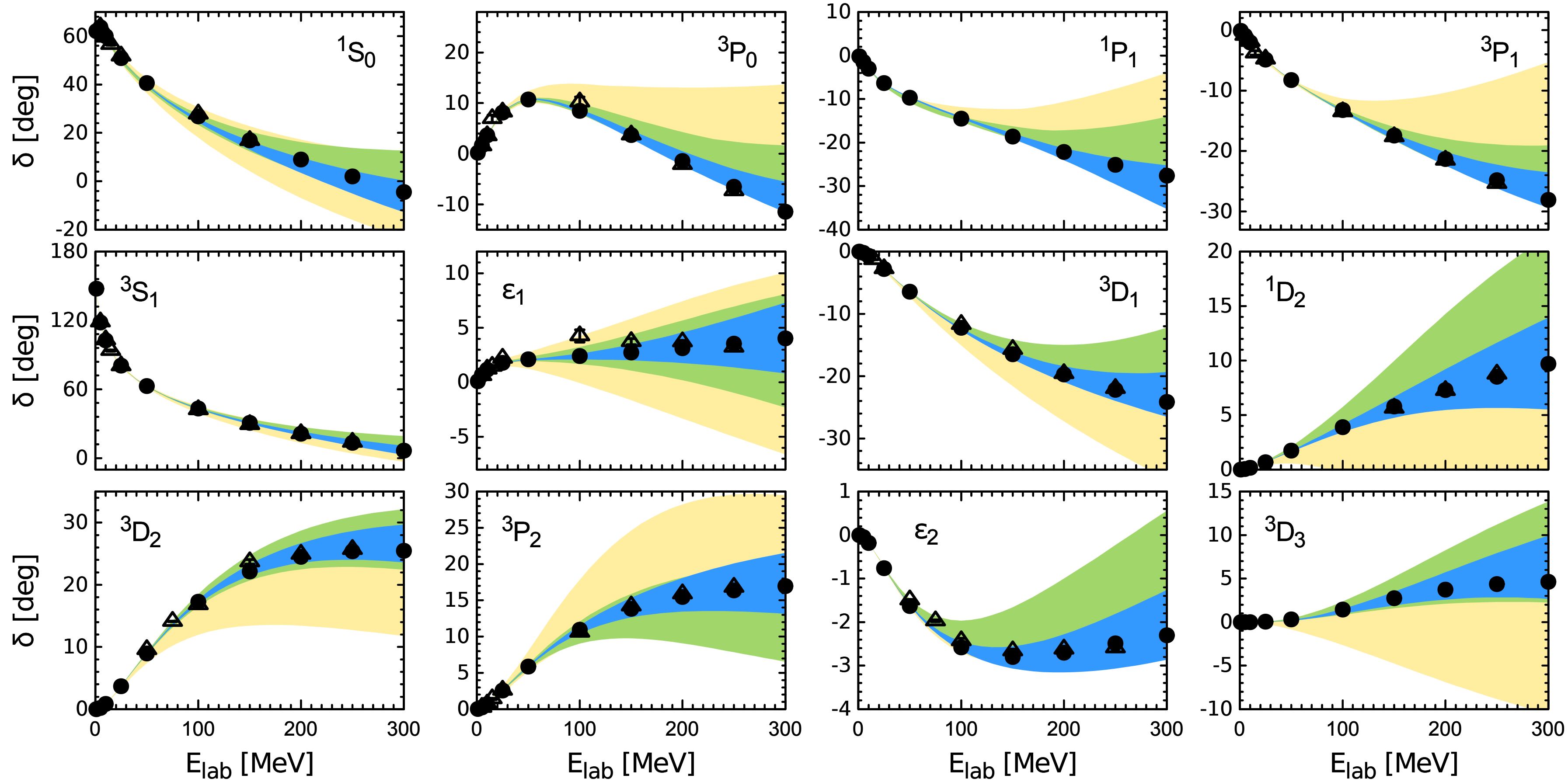
- phase shifts at N3LO based on a momentum-space regularization EGM (2005)



$\Longrightarrow$ clear improvement comp. to earlier N3LO potentials [momentum space reg.]

UNCERTAINTIES

- uncertainties show expected pattern



NLO N2LO N3LO

CALCULATION of the UNCERTAINTIES

Binder et al. [LENPIC coll.], PRC93 (2016) 044002

- define: $\Delta X^{(2)} \equiv X^{(2)} - X^{(0)}\,,\;\; \Delta X^{(i)} \equiv X^{(i)} - X^{(i-1)}\,,\; i \geq 3$

$\hookrightarrow$ chiral series: $X^{(i)} = X^{(0)} + \Delta X^{(2)} + \ldots + \Delta X^{(i)}\,,\;\; i \geq 3$

- note peculiarity of NN chiral expansion, method can easily accomodate term of $\mathcal{O}(Q)$

$\hookrightarrow$ expectation: $\Delta X^{(i)} = \mathcal{O}(Q^i |X^{(0)}|)$

$\hookrightarrow$ including also actual sizes: $\delta X^{(0)} = Q^2\,|X^{(0)}|$

$$\delta X^{(i)} = \max_{2 \leq j \leq i} \left(Q^{i+1} |X^{(0)}|,\; Q^{i+1-j} |\Delta X^{(j)}| \right)$$

- small parameter: $Q = \max(p/\Lambda_b, M_\pi/\Lambda)$

- size of the higher order corrections provide extra information:

$$\delta X^{(i)} \geq \max_{j,k} \left(|X^{(j \geq 1)} - X^{(k \geq 1)}| \right)$$

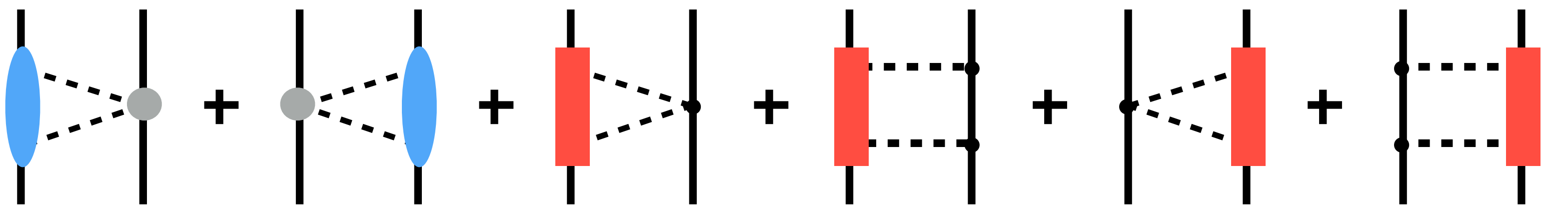
NN FORCES to FIFTH ORDER

Epelbaum, Krebs, UGM, Phys. Rev. Lett. **115** (2015) 122301

- No contact interactions at this order - odd in Q

- New contributions fixed from πN scattering, LECs c_i, d_i, e_i:

Büttiker, Fettes, UGM, Steininger (1998-2000); Krebs, Gasparian, Epelbaum (2012), Hoferichter, Ruiz de Elvira, Kubis, UGM (2015)



$$\mathcal{L}_{\pi N} = \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N}(c_i) + \mathcal{L}^{(3)}_{\pi N}(d_i) + \mathcal{L}^{(4)}_{\pi N}(e_i)$$

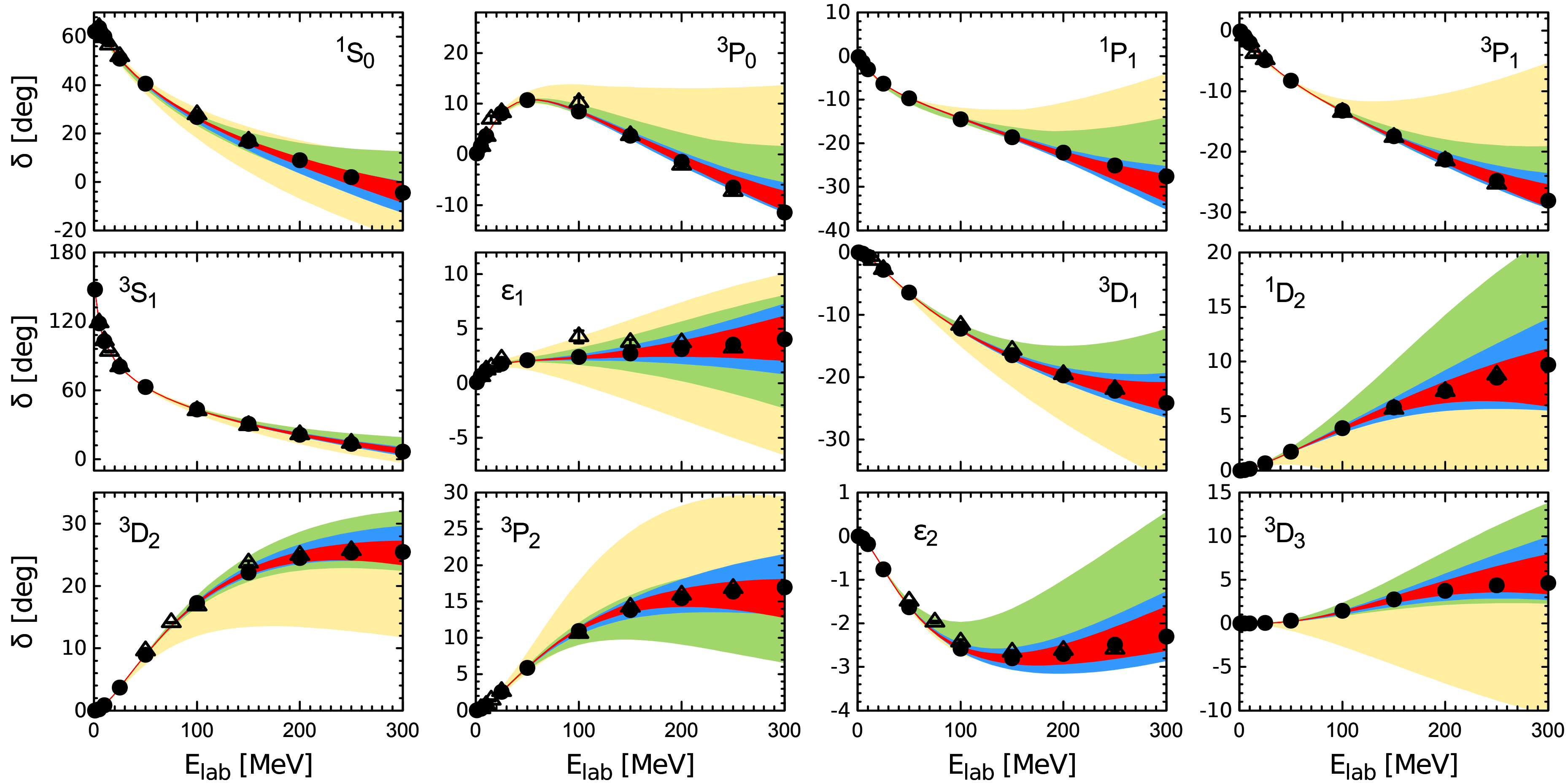
- Three-pion exchange can be neglected

  $\to$ explicit calculation of the dominant NLO contribution — Kaiser (2001)

  $\to$ no influence on phase shifts or deuteron properties

PHASE SHIFTS at N4LO

⇒ Precision phase shifts with small uncertainties up to $E_{\rm lab} = 300$ MeV



NLO N2LO N3LO N4LO

SOME N4LO RESULTS in the 2N SYSTEM

- description of the np and pp phase shifts

$E_{\rm lab}$ bin	LO	NLO	$\rm N^2LO$	$\rm N^3LO$	$\rm N^4LO$
neutron-proton phase shifts					
0–100	360	31	4.5	0.7	0.3
0–200	480	63	21	0.7	0.3
proton-proton phase shifts					
0–100	5750	102	15	0.8	0.3
0–200	9150	560	130	0.7	0.6

- deuteron properties

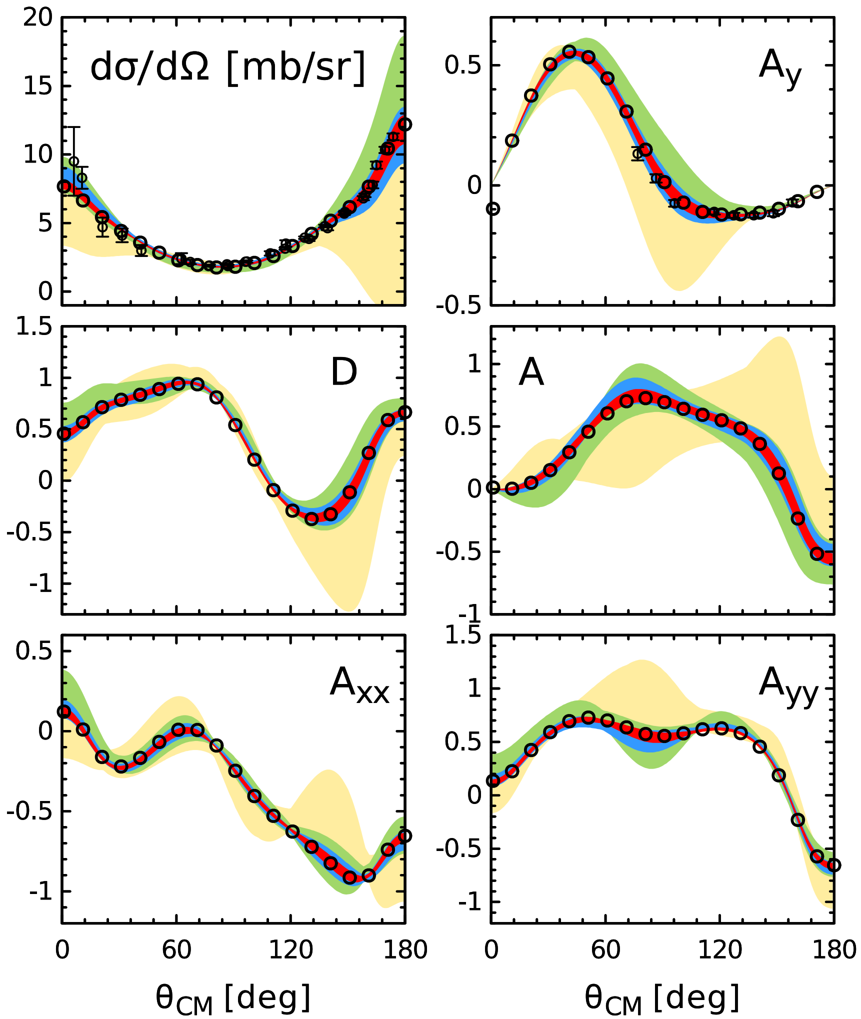
	LO	NLO	$\rm N^2LO$	$\rm N^3LO$	$\rm N^4LO$	Empirical
B_d [MeV]	2.0235	2.1987	2.2311	2.2246*	2.2246*	2.224575(9)
A_S [fm$^{-1/2}$]	0.8333	0.8772	0.8865	0.8845	0.8844	0.8846(9)
η	0.0212	0.0256	0.0256	0.0255	0.0255	0.0256(4)
r_d [fm]	1.990	1.968	1.966	1.972	1.972	1.97535(85)
Q [fm^2]	0.230	0.273	0.270	0.271	0.271	0.2859(3)
P_D [%]	2.54	4.73	4.50	4.19	4.29	

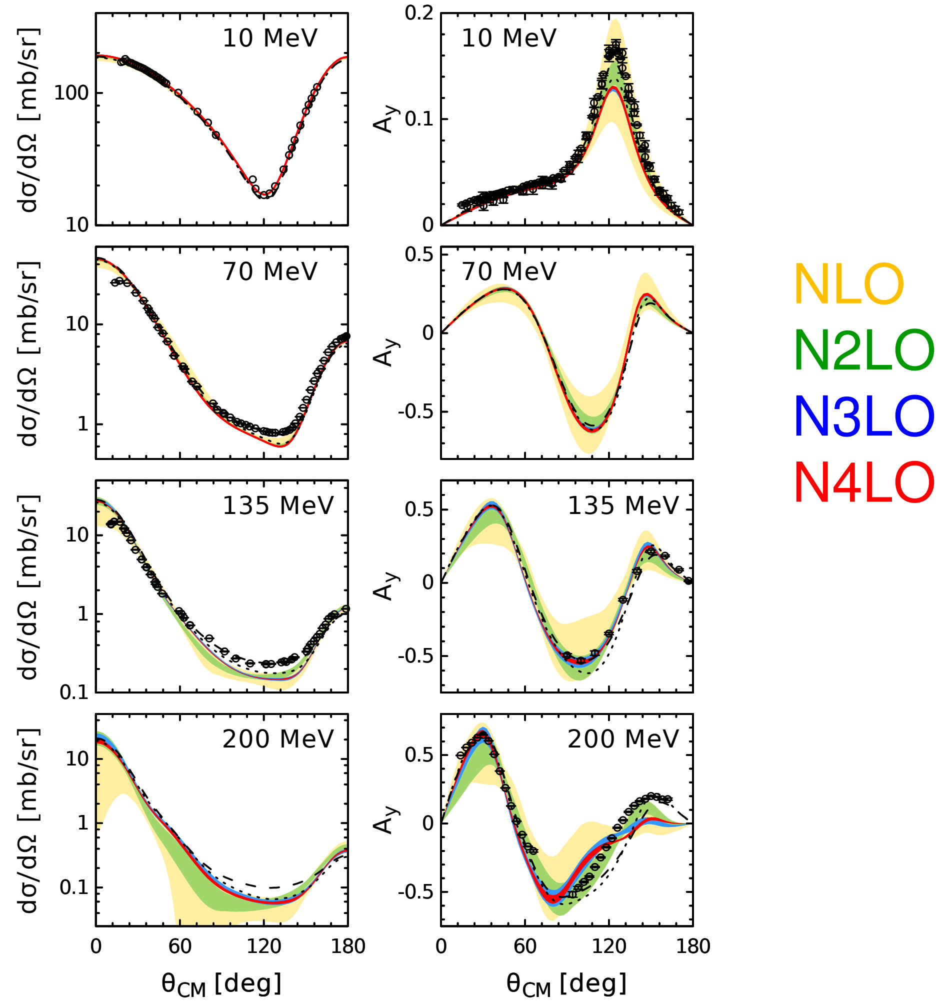
EVIDENCE for THREE-NUCLEON FORCES

- Two-nucleon system under control, three-nucleon system requires 3NFs!

  $\rightarrow$ being implemented [LENPIC collaboration]

- np scattering at 200 MeV
- nd scattering [2NFs only]

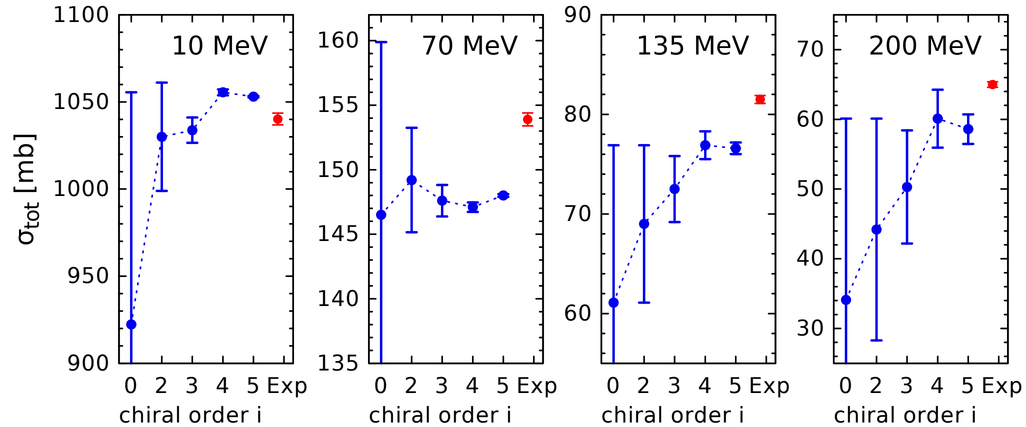




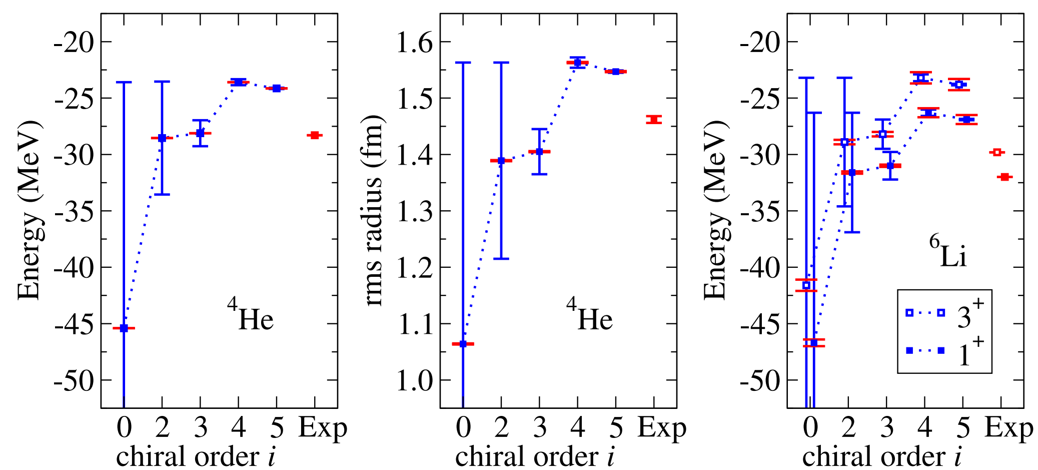
MORE EVIDENCE for THREE-NUCLEON FORCES

Binder et al. [LENPIC collaboration], Phys.Rev. C**93** (2016) 044002

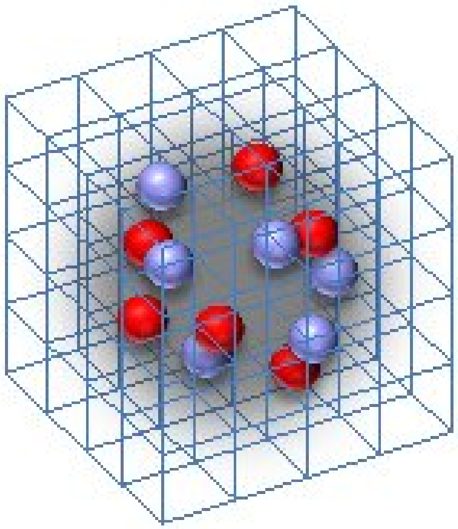
- Total cross section for Nd scattering [2NFs only]



- Binding energy and rms radius of ^{4}He, lowest levels in ^{6}Li [2NFs only]



Lattice chiral EFT physics



NLEFT

MANY–BODY APPROACHES

- nuclear physics = notoriously difficult problem: strongly interacting fermions

- two different approaches followed in the literature:

  ⋆ combine chiral NN(N) forces with standard many-body techniques

  Dean, Hagen, Navratil, Nogga, Papenbrock, Schwenk, . . .

  → show one example

  ⋆ combine chiral forces and lattice simulations methods

  → this new method is called *nuclear lattice simulations*

  Borasoy, Epelbaum, Krebs, Lee, Lände, UGM, Rupak, . . .

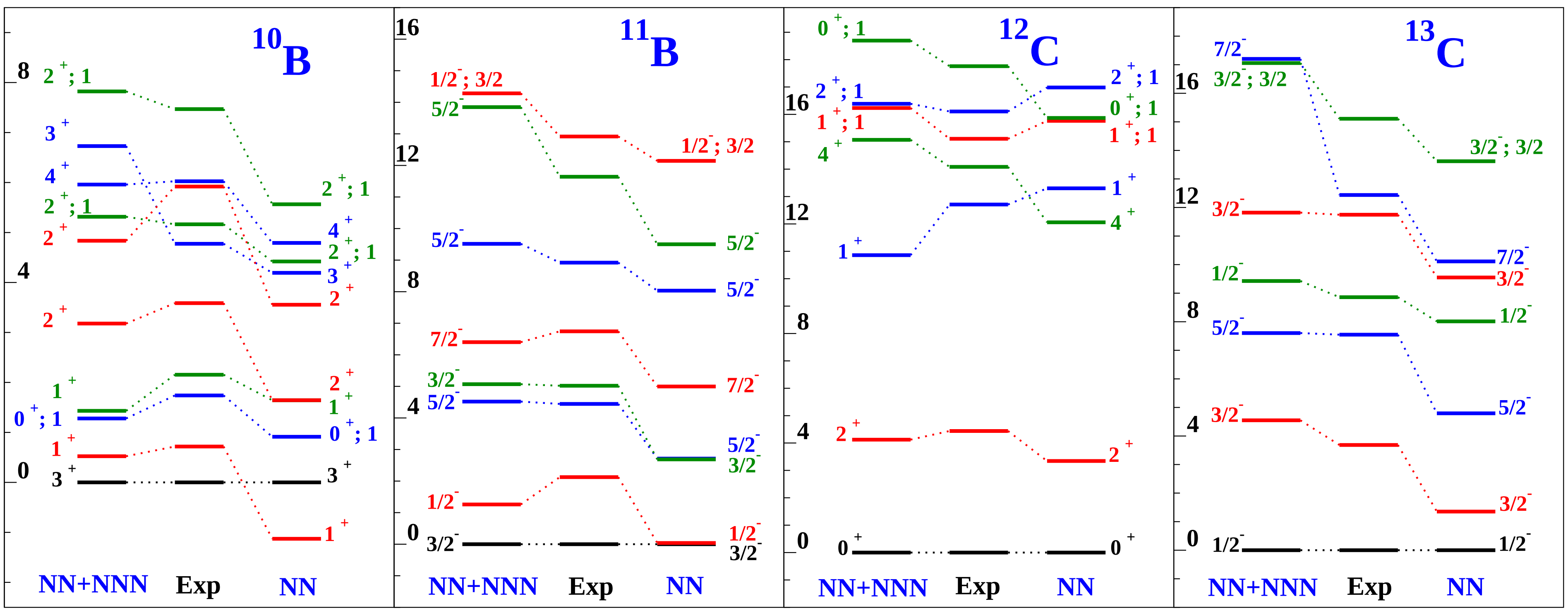
  → rest of the lectures

NO-CORE-SHELL MODEL: p-SHELL NUCLEI

- No-core-shell-model calculation — Navratil *et al.*, Phys. Rev. Lett. **99**, 042501 (2007)

- NN interaction at N^3LO and NNN interaction at N^2LO

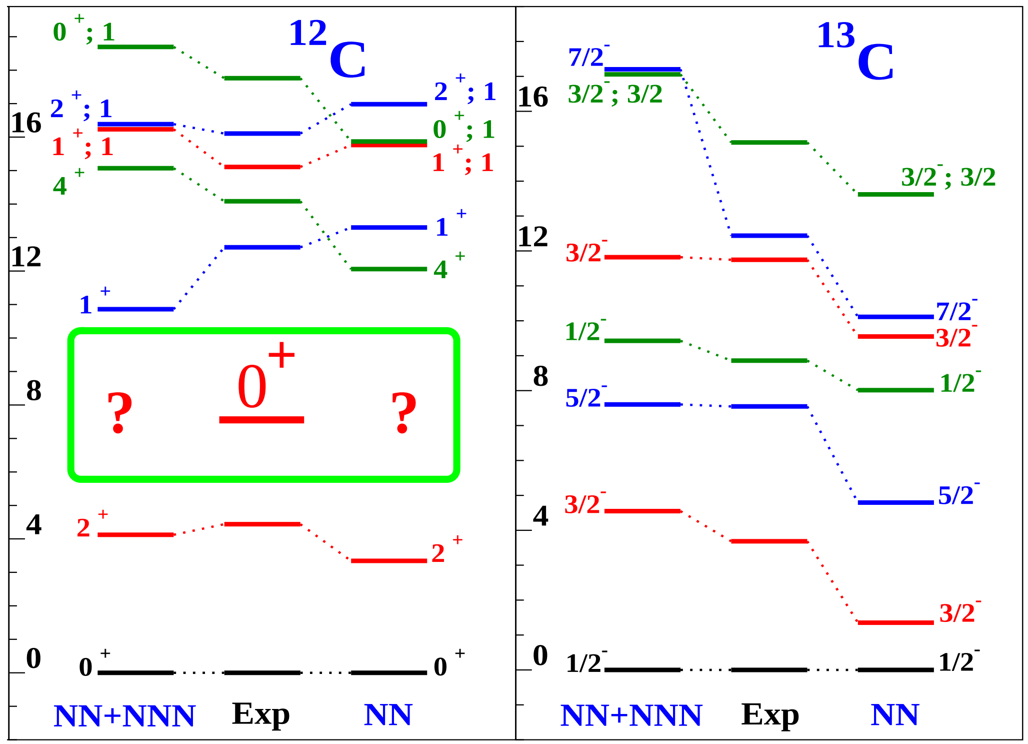
- Fix $D\&E$ from BE of ^{3}H and level structure of ^{4}He, ^{6}Li, ^{10,11}B and ^{12,13}C



MODERN MANY-BODY THEORY and the HOYLE STATE

- one of the most sophisticated many-body theory (No-Core-Shell-Model)

P. Navratil et al., Phys. Rev. Lett. 99 (2007) 042501



$\Rightarrow$ NO signal of the Hoyle state (i.g. α-cluster states)

$\Rightarrow$ must develop a better method

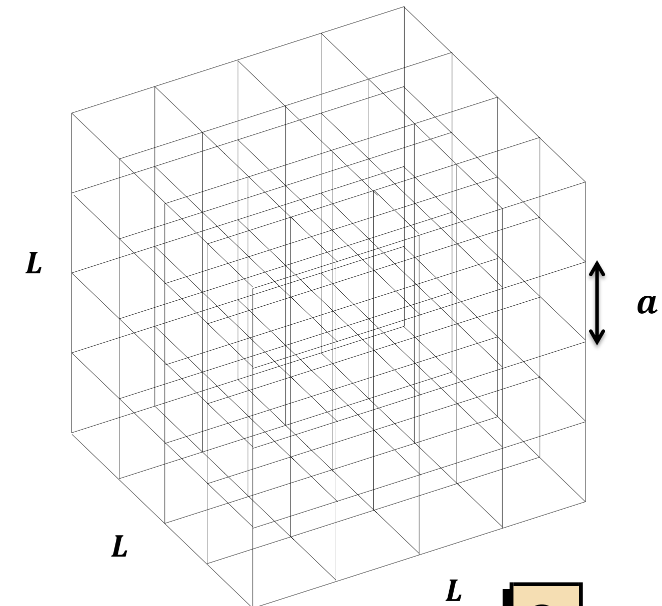
INTRO: WHAT IS A SPACE-TIME LATTICE ?

- Euclidean time: $\tau = it \Rightarrow \exp(-iHt) \to \exp(-H\tau)$
- Space-time volume: $V = L \times L \times L \times L_t$
- Lattice spacings a, a_t: $L = N\,a,\ L_t = N_t\,a_t,\ N, N_t \in \mathbb{N}$
- discrete space-time points:

$$\vec{x} = a\,(n_x, n_y, n_z),\ \tau = a_t\,n_t$$

$$n_x, n_y, n_z \in (0, 1, \ldots, N)$$

$$n_t \in (0, 1, \ldots, N_t)$$



- discrete momenta:

$$\vec{k} = \frac{2\pi}{L}\,\vec{n}$$ $\to$ UV cut-off (largest momentum $= \pi/a$, edge of the Brillouin zone)

- integrations become momentum sums: $\int dk_0 \int d^3k \to \sum_{k_0} \sum_{\vec{k}}$
- nucleon annihilation (creation) operators:

$$a^{(\dagger)}_{0,0} \equiv a^{(\dagger)}_{\uparrow,p},\ a^{(\dagger)}_{1,0} \equiv a^{(\dagger)}_{\downarrow,p},\ a^{(\dagger)}_{0,1} \equiv a^{(\dagger)}_{\uparrow,n},\ a^{(\dagger)}_{1,1} \equiv a^{(\dagger)}_{\downarrow,n}$$

exercise: derivatives on a lattice

LATTICE NOTATION: DERIVATIVES

- any derivative operator requires *improvement*, as the simplest representation in terms of two neighboring points is afflicted by the largest discretization errors

↪ definition of the first order spatial derivative:

$$\nabla_{l,(\nu)} f(\vec{n}) \equiv \frac{1}{2}\sum_{j=1}^{\nu+1}(-1)^{j+1}\theta_{\nu,j}\left[f(\vec{n}+j\hat{e}_l) - f(\vec{n}-j\hat{e}_l)\right]$$

↪ second order spatial derivative:

$$\tilde{\nabla}^2_{l,(\nu)} f(\vec{n}) \equiv -\sum_{j=0}^{\nu+1}(-1)^{j}\omega_{\nu,j}\left[f(\vec{n}+j\hat{e}_l) + f(\vec{n}-j\hat{e}_l)\right]$$

- no improvement ($\nu = 0$): $\theta_{0,1} = 1,\ \omega_{0,0} = 1,\ \omega_{0,1} = 1$
- Order a^2 improvement ($\nu = 1$): $\theta_{1,1} = \frac{4}{3},\ \theta_{1,2} = \frac{1}{6},\ \omega_{1,0} = \frac{5}{4},\ \omega_{1,1} = \frac{4}{3},\ \omega_{1,2} = \frac{1}{12}$
- Order a^4 improvement ($\nu = 2$): $\theta_{2,1} = \frac{3}{2},\ \theta_{2,2} = \frac{3}{10},\ \theta_{2,3} = \frac{1}{30}$

  $\omega_{2,0} = \frac{49}{36},\ \omega_{2,1} = \frac{3}{2},\ \omega_{2,2} = \frac{3}{20},\ \omega_{2,3} = \frac{1}{90}$

↪ improved lattice dispersion relation: $\omega^{(\nu)}(\vec{p}) \equiv \frac{1}{\tilde{m}_N}\sum_{j=0}^{\nu+1}\sum_{l=1}^{3}(-1)^j\omega_{\nu,j}\ \cos(jp_l)$

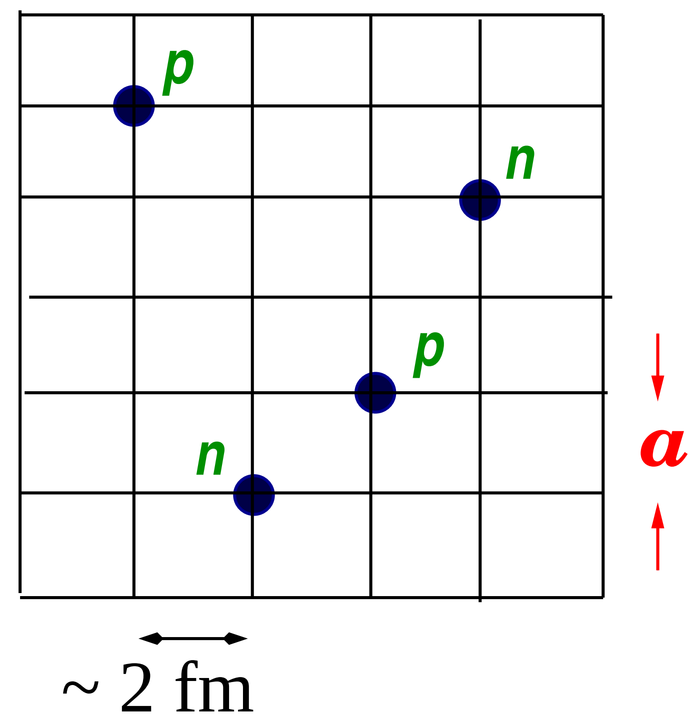
$$\boxed{\tilde{m}_N \equiv m_N a}$$

THE TOOL: NUCLEAR LATTICE SIMULATIONS

Frank, Brockmann (1992), Koonin, Müller, Seki, van Kolck (2000) , Lee, Schäfer (2004), . . .
Borasoy, Krebs, Lee, UGM, Nucl. Phys. **A768** (2006) 179; Borasoy, Epelbaum, Krebs, Lee, UGM, Eur. Phys. J. **A31** (2007) 105

- *new method* to tackle the nuclear many-body problem
- discretize space-time $V = L_s \times L_s \times L_s \times L_t$:
  nucleons are point-like fields on the sites
- discretized chiral potential w/ pion exchanges
  and contact interactions + Coulomb
- typical lattice parameters

$$\Lambda = \frac{\pi}{a} \simeq 300 \text{ MeV [UV cutoff]}$$



- strong suppression of sign oscillations due to approximate Wigner SU(4) symmetry

J. W. Chen, D. Lee and T. Schäfer, Phys. Rev. Lett. **93** (2004) 242302, T. Lähde et al., Eur. Phys. J. A**51** (2015) 92

- hybrid Monte Carlo & transfer matrix (similar to LQCD)

DIGRESSION: WIGNER SU(4) SYMMETRY

- Wigner's super-multiplet theory (1936 ff): Wigner, Phys. Rev. **51** (1937) 106; *ibid* 947

  Nuclear forces approximately spin- and isospin-independent

- Analysis in pionless EFT: $\mathcal{L}_2 = -\frac{1}{2}C_0(N^\dagger N)^2 - -\frac{1}{2}C_T(N^\dagger\sigma_i N)^2$

- Wigner trafo: $N \mapsto UN,\ \ U = \exp[i\alpha_{\mu\nu}\sigma_\mu\tau_\nu],\ \ \sigma_\mu = \{1,\sigma_i\},\ \ \tau_\nu = \{1,\tau_a\}$

  $\alpha_{\mu\nu} = 4 \times 4$ real matrix, $\alpha_{00} = 0$ [otherwise baryon number]

  $\hookrightarrow$ The C_0 term is invariant under a W.T., the C_T term is not

- in a partial-wave basis: $C(^1S_0) = C_0 - 3C_T\ ,\ \ C(^3S_1) = C_0 - C_T$

  $\hookrightarrow$ in the Wigner symmetry limit, we have: $C(^1S_0) = C(^3S_1)$

  $\hookrightarrow$ in the Wigner symmetry limit, we thus have: $1/a_{np}^{S=1} = 1/a_{np}^{S=0}$

  $\hookrightarrow$ Wigner symmetry breaking governed by: $\delta = \frac{1}{2}(1/a_{np}^{S=1} - 1/a_{np}^{S=0})$

  $$= \frac{1}{2}\left(\frac{1}{36.5\ \mathrm{MeV}} - \frac{1}{8.3\ \mathrm{MeV}}\right)$$

- **No** sign problem for spin-isopspin saturated nuclei in the W.S. limit!

  J. W. Chen et al., Phys. Rev. Lett. **93** (2004) 242302

- further breaking through OPE, Coulomb,..., but still an **approximate** symmetry

TRANSFER MATRIX METHOD

- Correlation–function for A nucleons: $Z_A(\tau) = \langle \Psi_A | \exp(-\tau H) | \Psi_A \rangle$

  with Ψ_A a Slater determinant for A free nucleons

  [or a more sophisticated (correlated) initial/final state]

Euclidean time

- Ground state energy from the time derivative of the correlator

$$E_A(\tau) = -\frac{d}{d\tau} \ln Z_A(\tau)$$

$\rightarrow$ ground state filtered out at large times: $E_A^0 = \lim_{\tau\to\infty} E_A(\tau)$

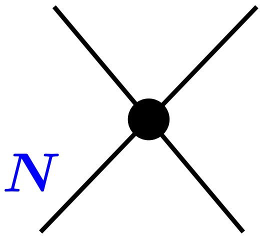
- Expectation value of any normal–ordered operator $\mathcal{O}$

$$Z_A^{\mathcal{O}} = \langle \Psi_A | \exp(-\tau H/2)\, \mathcal{O}\, \exp(-\tau H/2)\, | \Psi_A \rangle$$

$$\lim_{\tau\to\infty} \frac{Z_A^{\mathcal{O}}(\tau)}{Z_A(\tau)} = \langle \Psi_A | \mathcal{O} \, | \Psi_A \rangle$$

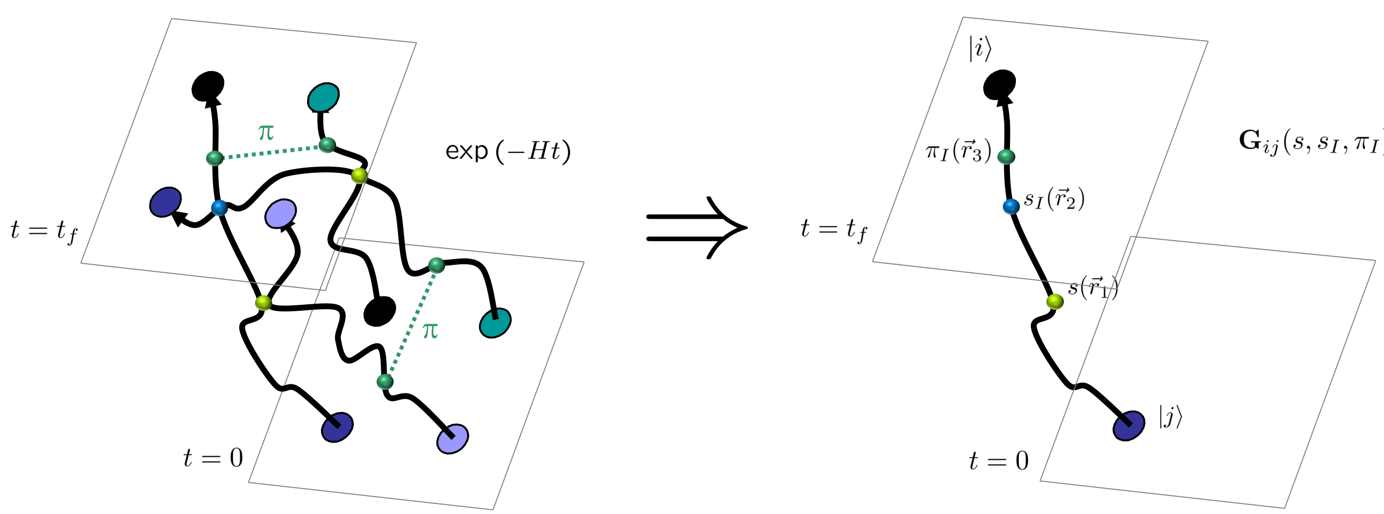
MONTE CARLO with AUXILIARY FILEDS

- Contact interactions represented by auxiliary fields s, s_I

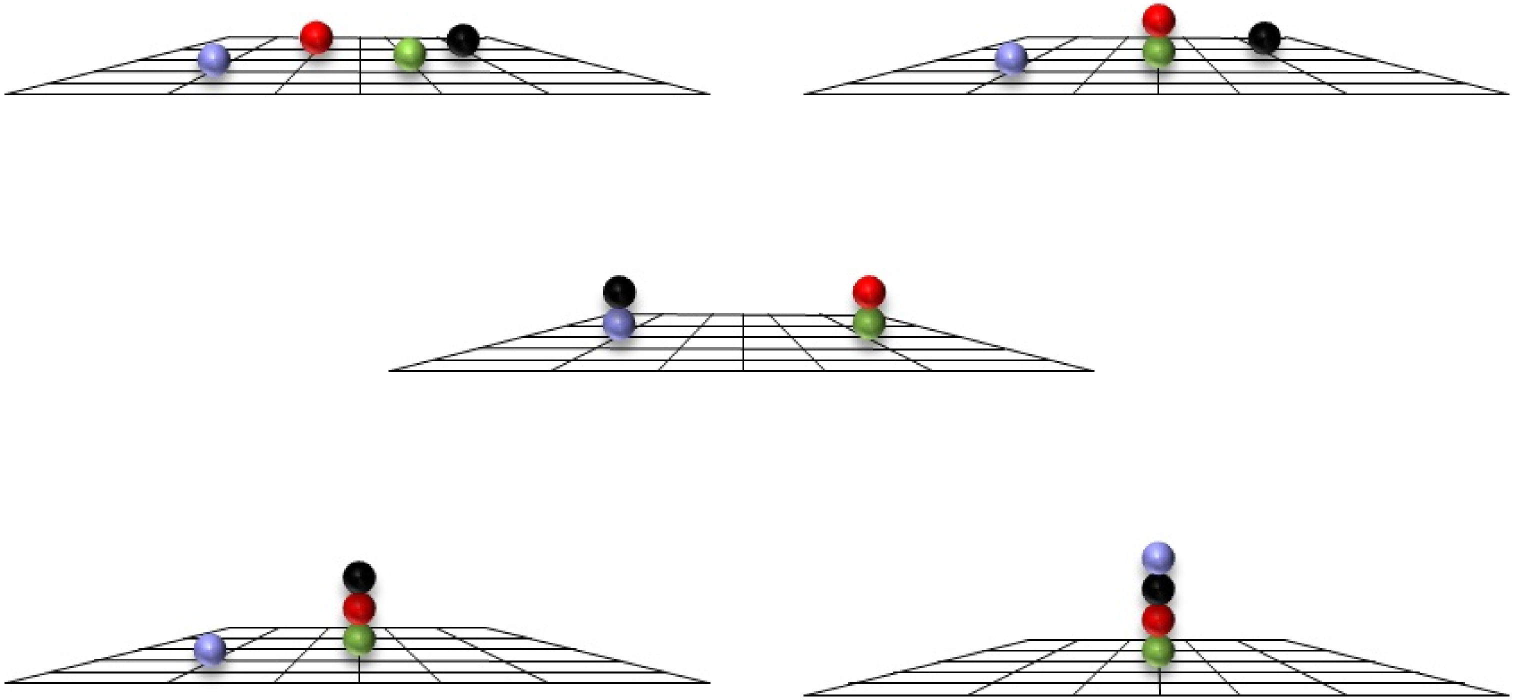


$$\exp(-C\rho^2/2) = \sqrt{\frac{1}{2\pi}} \int_{-\infty}^{+\infty} ds\, \exp(-s^2/2 + \sqrt{C}s\rho)\ , \quad \rho = N^\dagger N$$

- Correlation function = path-integral over pions & auxiliary fields



CONFIGURATIONS



⇒ all *possible* configurations are sampled

⇒ *clustering* emerges *naturally*

INITIAL STATES

- Zero momentum standing waves for ^{4}He to define $|\psi_A\rangle = |\psi^{\rm free}_{Z,N}\rangle$

$$\langle 0|a_{i,j}(\vec{n}\,)|\psi_1\rangle = L^{-3/2}\,\delta_{i,0}\delta_{j,1}$$

$$\langle 0|a_{i,j}(\vec{n}\,)|\psi_2\rangle = L^{-3/2}\,\delta_{i,0}\delta_{j,0}$$

$$\langle 0|a_{i,j}(\vec{n}\,)|\psi_3\rangle = L^{-3/2}\,\delta_{i,1}\delta_{j,1}$$

$$\langle 0|a_{i,j}(\vec{n}\,)|\psi_4\rangle = L^{-3/2}\,\delta_{i,1}\delta_{j,0}$$

- Wave packets with small momentum spread for ^{4}He to define $|\psi^{\rm free}_{Z,N}\rangle$

$$\langle 0|a_{i,j}(\vec{n}\,)|\psi_1\rangle = L^{-3/2}\sqrt{2}\cos\left(\frac{2\pi n_z}{L}\right)\,\delta_{i,0}\delta_{j,1}$$

$$\langle 0|a_{i,j}(\vec{n}\,)|\psi_2\rangle = L^{-3/2}\sqrt{2}\cos\left(\frac{2\pi n_z}{L}\right)\,\delta_{i,0}\delta_{j,0}$$

$$\langle 0|a_{i,j}(\vec{n}\,)|\psi_3\rangle = L^{-3/2}\sqrt{2}\cos\left(\frac{2\pi n_z}{L}\right)\,\delta_{i,1}\delta_{j,1}$$

$$\langle 0|a_{i,j}(\vec{n}\,)|\psi_4\rangle = L^{-3/2}\sqrt{2}\cos\left(\frac{2\pi n_z}{L}\right)\,\delta_{i,1}\delta_{j,0}$$

- or more complex initial states ...

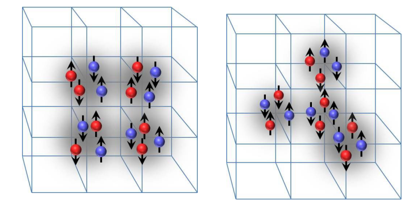
COMPUTATIONAL EQUIPMENT

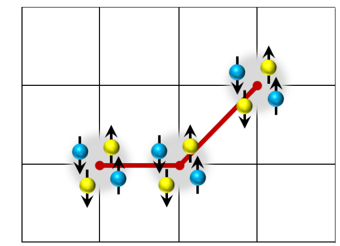
- Appropriate tool = JUQUEEN (BlueGene/Q)

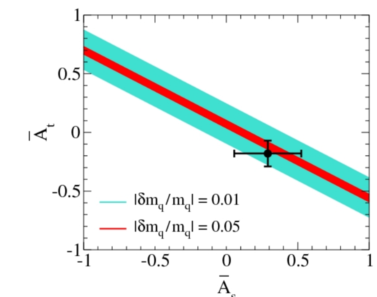
6 Pflops

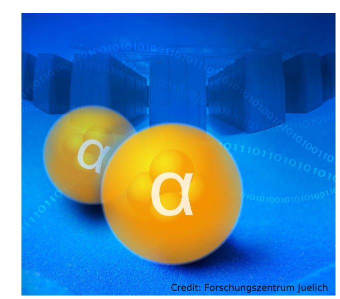
RESULTS from LATTICE NUCLEAR EFT

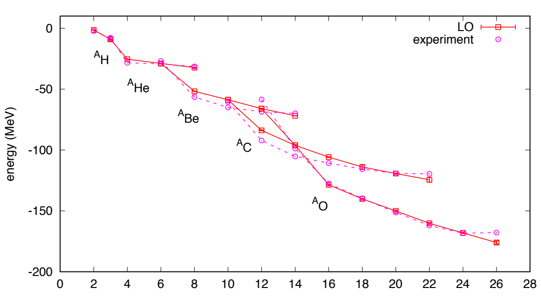
- ☐ Lattice EFT calculations for A=3,4,6,12 nuclei, PRL 104 (2010) 142501
- ☐ *Ab initio* calculation of the Hoyle state, PRL 106 (2011) 192501
- ☐ Structure and rotations of the Hoyle state, PRL 109 (2012) 142501
- ☐ Validity of Carbon-Based Life as a Function of the Light Quark Mass
  PRL 110 (2013) 142501
- ☐ *Ab initio* calculation of the Spectrum and Structure of ^{16}O,
  PRL 112 (2014) 142501
- ☐ Lattice Effective Field Theory for Medium-Mass Nuclei
  PLB 732 (2014) 110
- ☐ *Ab initio* alpha-alpha scattering, Nature 528 (2015) 111
- ☐ Nuclear Binding Near a Quantum Phase Transition, PRL 117 (2016) 132501
- ☐ *Ab initio* calculations of the isotopic dependence of nuclear clustering,
  PRL 119 (2017) 222505

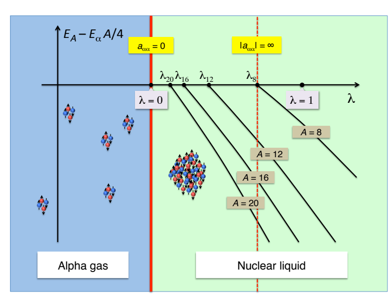


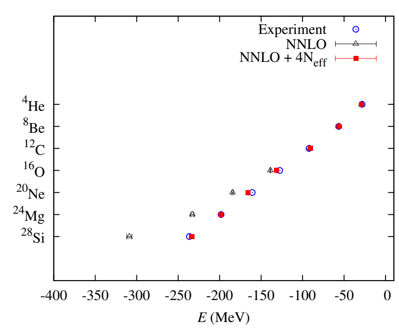










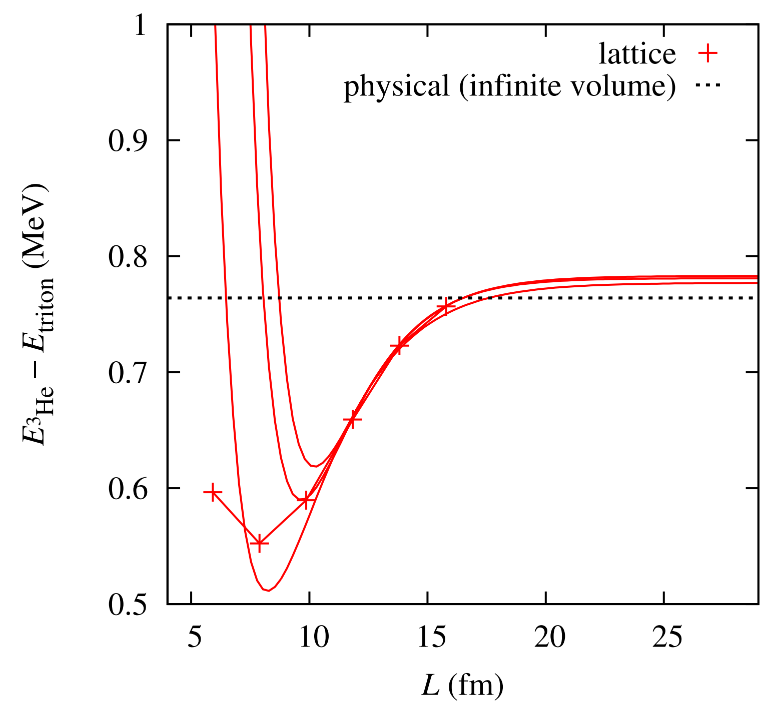


RESULTS

Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. 104 (2010) 142501; Eur. Phys. J. A45 (2010) 335

- some groundstate energies and differences [NNLO, 11+2 LECs]

	E [MeV]	NLEFT	Exp.
old algorithm	^{3}He - ^{3}H	0.78(5)	0.76
	^{4}He	−28.3(6)	−28.3
	^{8}Be	−55(2)	−56.5
	^{12}C	−92(3)	−92.2
new algorithm	^{16}O	−131(1)	−127.6
	^{20}Ne	−166(1)	−160.6
	^{24}Mg	−198(2)	−198.3
	^{28}Si	−234(3)	−236.5

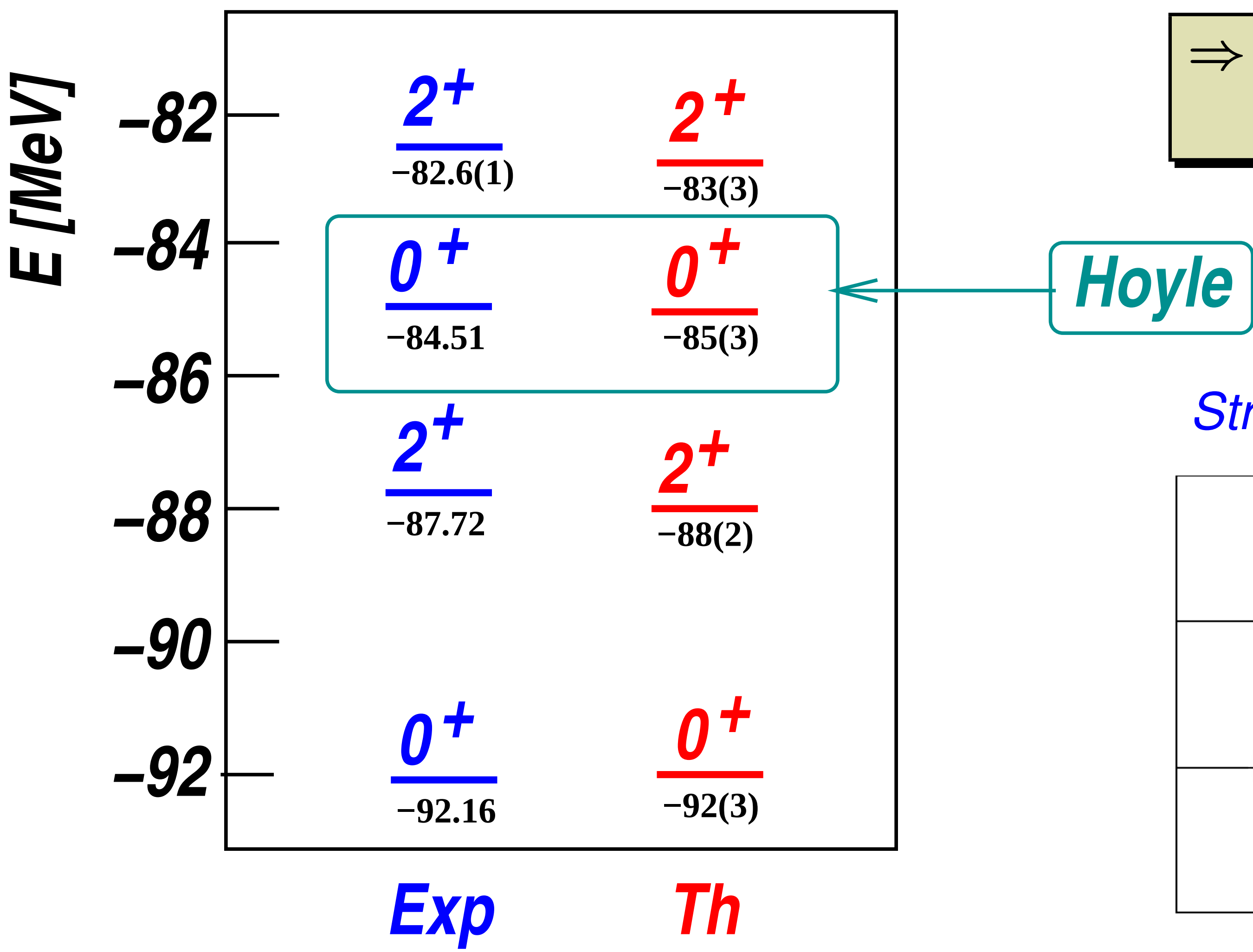


- promising results ⇒ uncertainties down to the 1% level
- excited states more difficult ⇒ projection MC method + triangulation

The SPECTRUM of CARBON-12

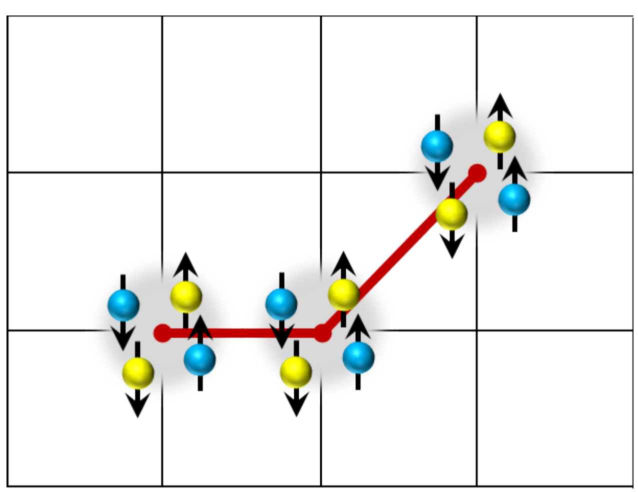
Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. 106 (2011) 192501
Epelbaum, Krebs, Lähde, Lee, UGM, Phys. Rev. Lett. 109 (2012) 252501

- After $8 \cdot 10^6$ hrs JUGENE/JUQUEEN (and “some” human work)



$\Rightarrow$ First ab initio calculation of the Hoyle state $\surd$

Structure of the Hoyle state:



A SHORT HISTORY of the HOYLE STATE

- Heavy element generation in massive stars: triple-α process

Bethe 1938, Öpik 1952, Salpeter 1952, Hoyle 1954, ...

$$^4\text{He} + {}^4\text{He} \rightleftharpoons {}^8\text{Be}$$
$$^8\text{Be} + {}^4\text{He} \rightleftharpoons {}^{12}\text{C}^* \rightarrow {}^{12}\text{C} + \gamma$$
$$^{12}\text{C} + {}^4\text{He} \rightleftharpoons {}^{16}\text{O} + \gamma$$

- Hoyle's contribution: calculation of the relative abundances of ^{4}He, ^{12}C and ^{16}O
  $\Rightarrow$ need a resonance close to the ^{8}Be + ^{4}He threshold at $E_R \simeq 0.37$ MeV
  $\Rightarrow$ this corresponds to a $J^P = 0^+$ excited state 7.7 MeV above the g.s.

- a corresponding state was experimentally confirmed at Caltech at
  $E - E(\text{g.s.}) = 7.653 \pm 0.008$ MeV   Dunbar et al. 1953, Cook et al. 1957

- still on-going experimental activity, e.g. EM transitions at SDALINAC

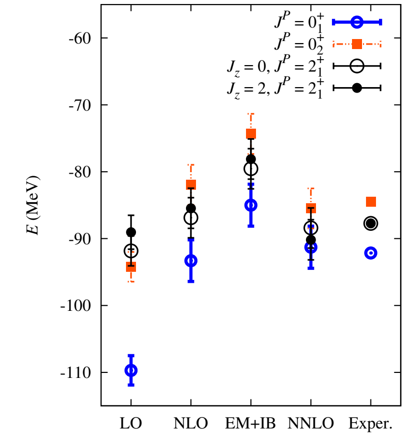
M. Chernykh et al., Phys. Rev. Lett. 98 (2007) 032501

- side remark: relevance to the anthropic principle?

H. Kragh, An anthropic myth: Fred Hoyle's carbon-12 resonance level,
Arch. Hist. Exact Sci. 64 (2010) 721

The RELEVANT QUESTION

Date: Sat, 25 Dec 2010 20:03:42 -0600
From: Steven Weinberg ⟨weinberg@zippy.ph.utexas.edu⟩
To: Ulf-G. Meissner ⟨meissner@hiskp.uni-bonn.de⟩
Subject: Re: Hoyle state in 12C



Dear Professor Meissner,

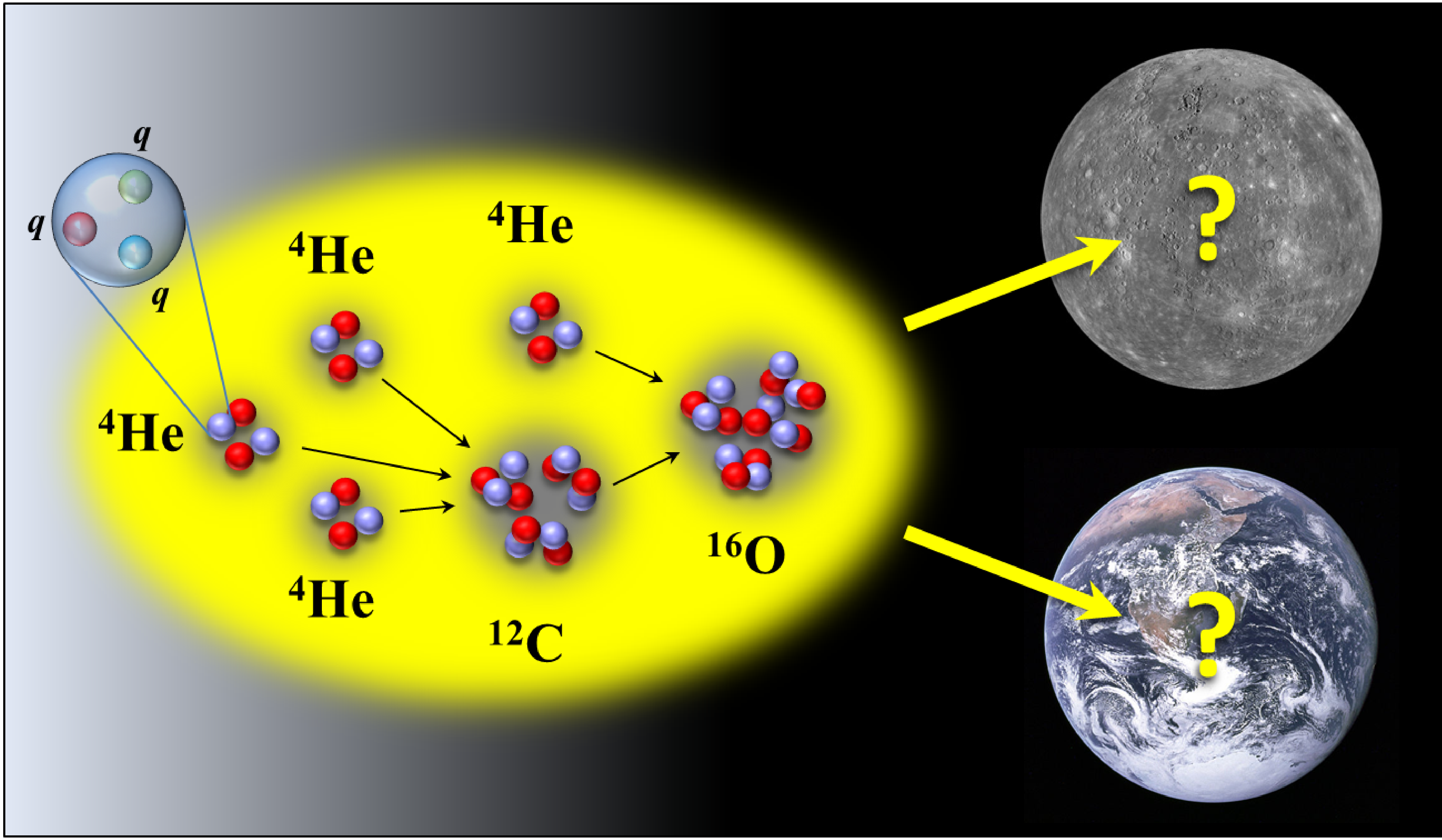
Thanks for the colorful graph. It makes a nice Christmas card. But I have a detailed question. Suppose you calculate not only the energy of the Hoyle state in C12, but also of the ground states of He4 and Be8. How sensitive is the result that the energy of the Hoyle state is near the sum of the rest energies of He4 and Be8 to the parameters of the theory? I ask because I suspect that for a pretty broad range of parameters, the Hoyle state can be well represented as a nearly bound state of Be8 and He4.

All best,
Steve Weinberg

- How does the Hoyle state move relative to the ^{4}He+^{8}Be threshold, if we change the fundamental parameters of QCD+QED?
- not possible in nature, *but on a high-performance computer!*

FINE-TUNING of FUNDAMENTAL PARAMETERS

Fig. courtesy Dean Lee



EARLIER STUDIES of the ANTHROPIC PRINCIPLE

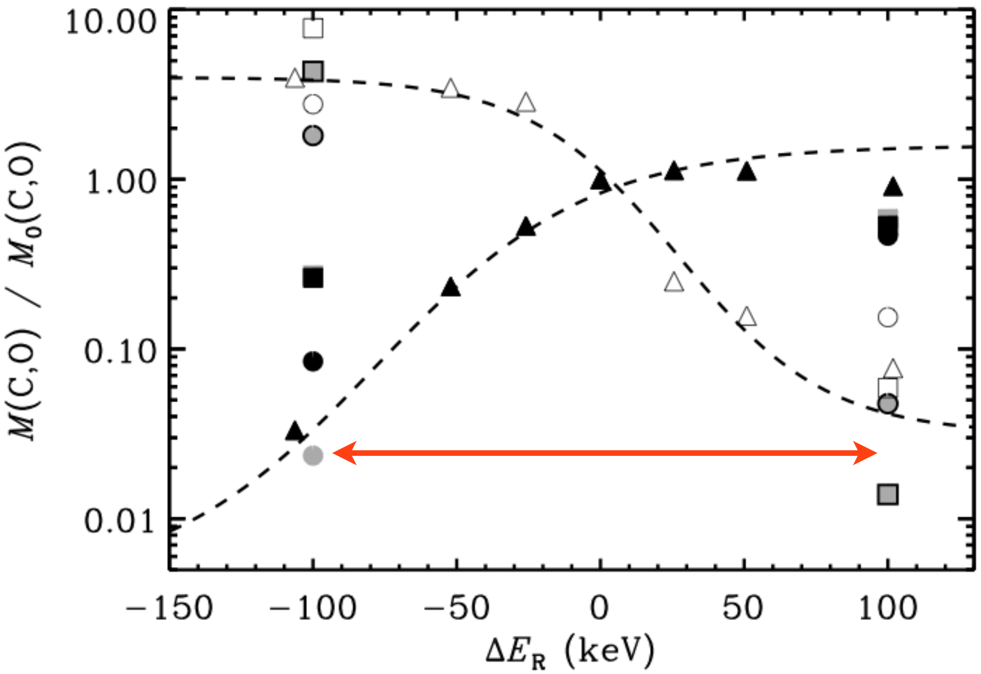
- rate of the 3α-process: $r_{3\alpha} \sim \Gamma_\gamma \exp\left(-\frac{\Delta E_{h+b}}{kT}\right)$

$$\Delta E_{h+b} = E^\star_{12} - 3E_\alpha = 379.47(18)\ \text{keV}$$

- how much can ΔE_{h+b} be changed so that there is still enough ^{12}C and ^{16}O?

$\Rightarrow$ $\boxed{\delta|\Delta E_{h+b}| \lesssim 100\ \text{keV}}$

too few ^{16}O too few ^{12}C



Oberhummer et al., Science **289** (2000) 88

Csoto et al., Nucl. Phys. A **688** (2001) 560

Schlattl et al., Astrophys. Space Sci. **291** (2004) 27

[Livio et al., Nature **340** (1989) 281]

FINE-TUNING: MONTE-CARLO ANALYSIS

Epelbaum, Krebs, Lähde, Lee, UGM, PRL **110** (2013) 112502

- consider first QCD only $\to$ calculate $\partial \Delta E / \partial M_\pi$

- relevant quantities (energy *differences*)

$$\boxed{\Delta E_h \equiv E_{12}^* - E_8 - E_4, \quad \Delta E_b \equiv E_8 - 2E_4}$$

- energy differences depend on parameters of QCD (LO analysis)

$$\boxed{E_i = E_i\Big(M_\pi^{\rm OPE}, m_N(M_\pi), g_{\pi N}(M_\pi), C_0(M_\pi), C_I(M_\pi)\Big)}$$

$$g_{\pi N} \equiv g_A/(2F_\pi)$$

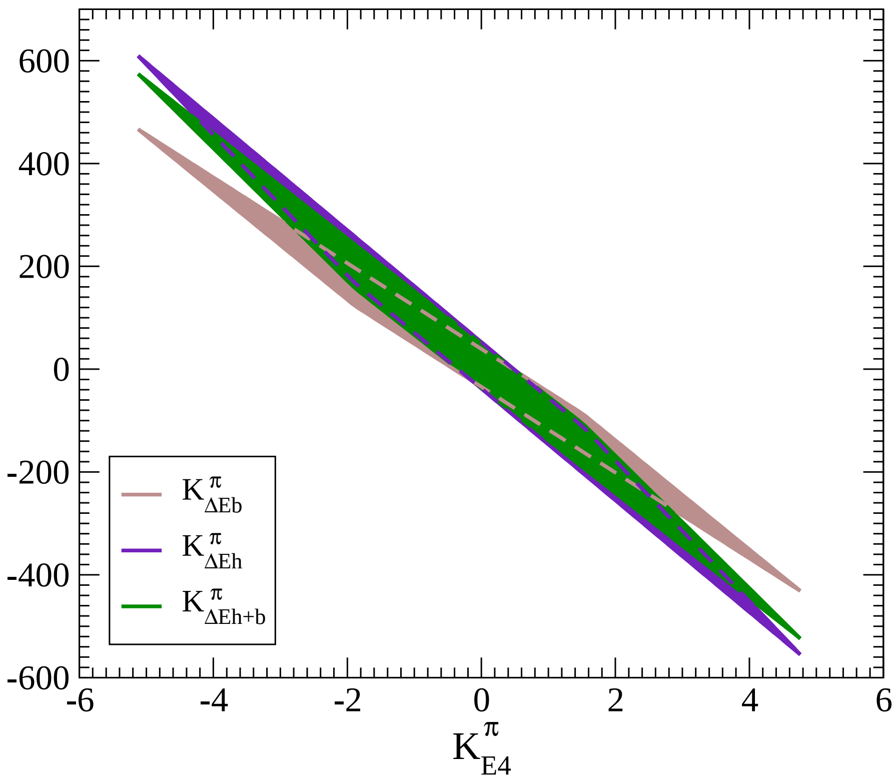
- remember: $M_{\pi^\pm}^2 \sim (m_u + m_d)$ Gell-Mann, Oakes, Renner (1968)

$\Rightarrow$ quark mass dependence $\equiv$ pion mass dependence

CORRELATIONS

- map $C_{0,I}(M_\pi)$ onto $\bar{A}_{s,t} \equiv \partial a_{s,t}^{-1}/\partial M_\pi\big|_{M_\pi^{\rm phys}}$ [singlet/triplet scatt. length]

- vary the derivatives $\bar{A}_{s,t} \equiv \partial a_{s,t}^{-1}/\partial M_\pi\big|_{M_\pi^{\rm phys}}$ within $-1,\ldots,+1$:



$$\Delta E_b = E(^8\text{Be}) - 2E(^4\text{He})$$

$$\Delta E_h = E(^{12}\text{C}^*) - E(^8\text{Be}) - E(^4\text{He})$$

$$\Delta E_{h+b} = E(^{12}\text{C}^*) - 3E(^4\text{He})$$

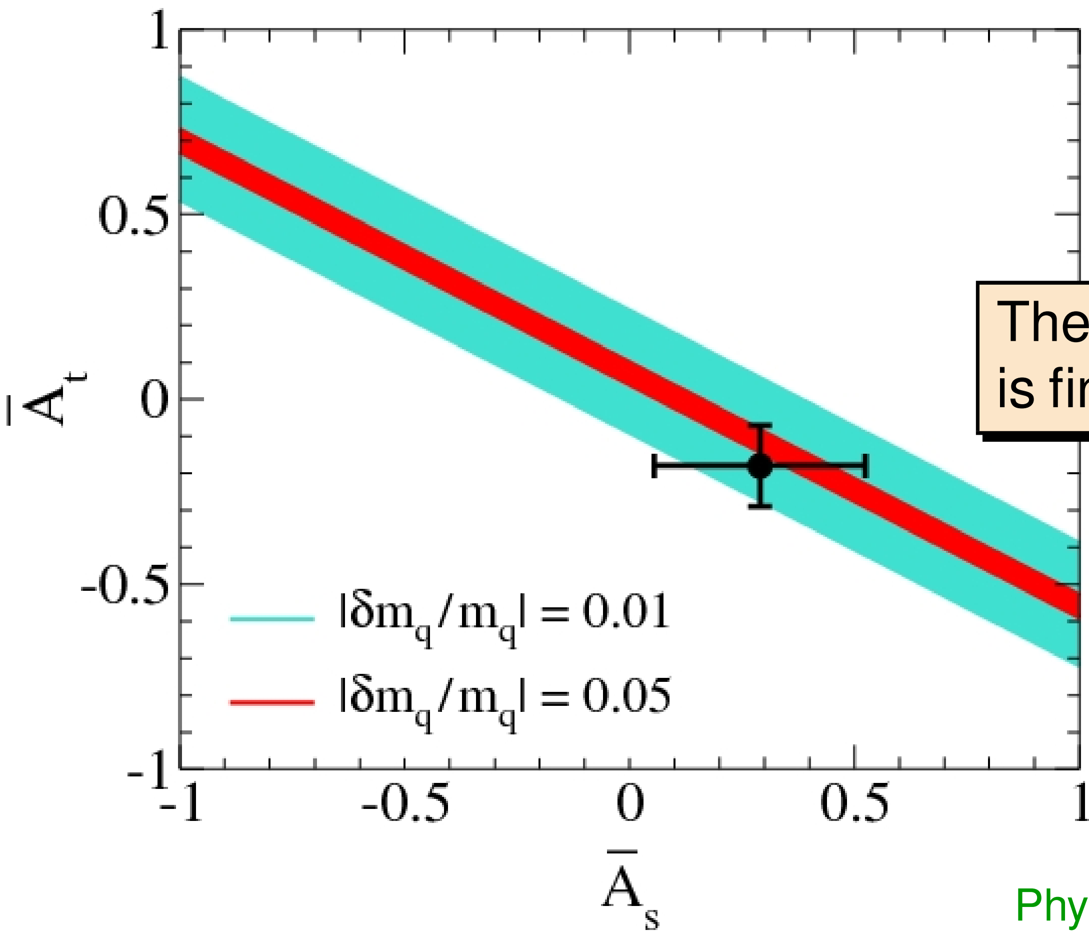
$$\boxed{\frac{\partial O_H}{\partial M_\pi} = K_H^\pi \, \frac{O_H}{M_\pi}}$$

- clear correlations: α-particle BE and the energies/energy differences

THE END-OF-THE-WORLD PLOT

- $|\delta(\Delta E_{h+b})| < 100$ keV [exp: 387 keV] Oberhummer et al., Science (2000)

$$\rightarrow \boxed{\left|\left(0.571(14)\bar{A}_s + 0.934(11)\bar{A}_t - 0.069(6)\right)\frac{\delta m_q}{m_q}\right| < 0.0015}$$



$$\bar{A}_{s,t} \equiv \left.\frac{\partial a_{s,t}^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}$$

The light quark mass is fine-tuned to $\simeq 2-3\,\%$

Similarly: $\alpha_{\rm EM}$ is fine-tuned to $\simeq 2.5\%$

Berengut et al., Phys. Rev. **D 87** (2013) 085018

Ab initio calculation of α-α scattering

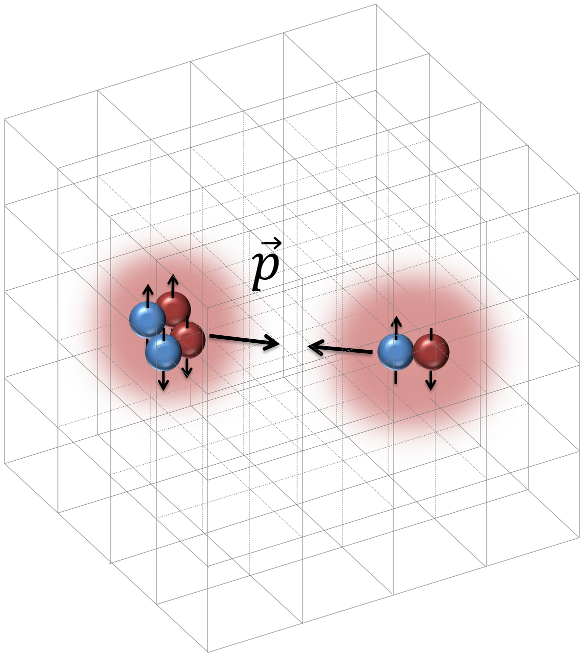
Elhatisari, Lee, Rupak, Epelbaum, Krebs, Lähde, Luu, UGM,

Nature **528** (2015) 111 [arXiv:1506.03513]

TWO-BODY SCATTERING on the LATTICE

- Processes involving α-particles and α-type nuclei comprise a major part of stellar nucleosynthesis, and control the production of certain elements in stars

- Ab initio calculations of scattering and reactions using standard many-body methods suffer from computational scaling with the of nucleons in the clusters



Lattice EFT computational scaling $\Rightarrow (A_1 + A_2)^2$

Rupak, Lee, Phys. Rev. Lett. 111 (2013) 032502
Pine, Lee, Rupak, Eur. Phys. J. A49 (2013) 151
Elhatisari, Lee, Phys. Rev. C90 (2014) 064001
Elhatisari, et al., Eur.Phys.J. A52 (2016) 174

ADIABATIC PROJECTION METHOD

- Basic idea to treat scattering and inelastic reactions:
  split the problem into two parts

First part:
use Euclidean time projection to construct an *ab initio* low-energy cluster Hamiltonian, called the **adiabatic Hamiltonian**

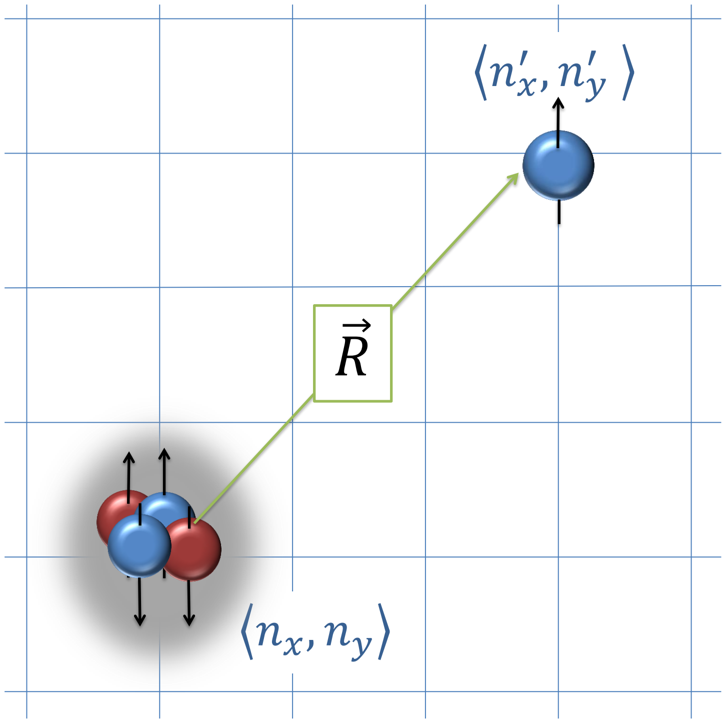
Second part:
compute the two-cluster scattering phase shifts or reaction amplitudes using the adiabatic Hamiltonian

ADIABATIC PROJECTION METHOD II

- Construct a low-energy effective theory for clusters

- Use initial states parameterized by the relative separation between clusters

$$|\vec{R}\rangle = \sum_{\vec{r}} |\vec{r} + \vec{R}\rangle \otimes \vec{r}$$



- project them in Euclidean time with the chiral EFT Hamiltonian H

$$|\vec{R}\rangle_\tau = \exp(-H\tau)|\vec{R}\rangle$$

$\to$ "dressed cluster states"

- Construct the adiabatic Hamiltonian from the dressed cluster states:

$$[H_\tau]_{\vec{R}\vec{R}'} = {}_\tau\langle\vec{R}|H|\vec{R}'\rangle_\tau$$

ADIABATIC HAMILTONIAN

- Construct the adiabatic Hamiltonian from the dressed cluster states:

$$[H_\tau]_{\vec{R}\vec{R}'} = {}_\tau\langle\vec{R}|H|\vec{R}'\rangle_\tau$$

- States are i.g. not normalized, require *norm matrix*:

$$[N_\tau]_{\vec{R}\vec{R}'} = {}_\tau\langle\vec{R}|\vec{R}'\rangle_\tau$$

- construct the full adiabatic Hamiltonian:

$$\left[H^a_\tau\right]_{\vec{R}\vec{R}'} = \sum_{\vec{R}_n\vec{R}_m} \left[N_\tau^{-1/2}\right]_{\vec{R}\vec{R}_n} \left[H_\tau\right]_{\vec{R}_n\vec{R}_m} \left[N_\tau^{-1/2}\right]_{\vec{R}_m\vec{R}'}$$

→ The structure of the adiabatic Hamiltonian is similar to the Hamiltonian matrix used in recent ab initio NCSM/RGM calculations

Navratil, Quaglioni, Phys. Rev. C **83** (2011) 044609
Navratil, Roth, Quaglioni, Phys. Lett. B **704** (2011) 379
Navratil, Quaglioni, Phys. Rev. Lett. **108** (2012) 042503

SCATTERING CLUSTER WAVE FUNCTIONS

- During Euclidean time interval τ_ε, each cluster undergoes spatial diffusion:

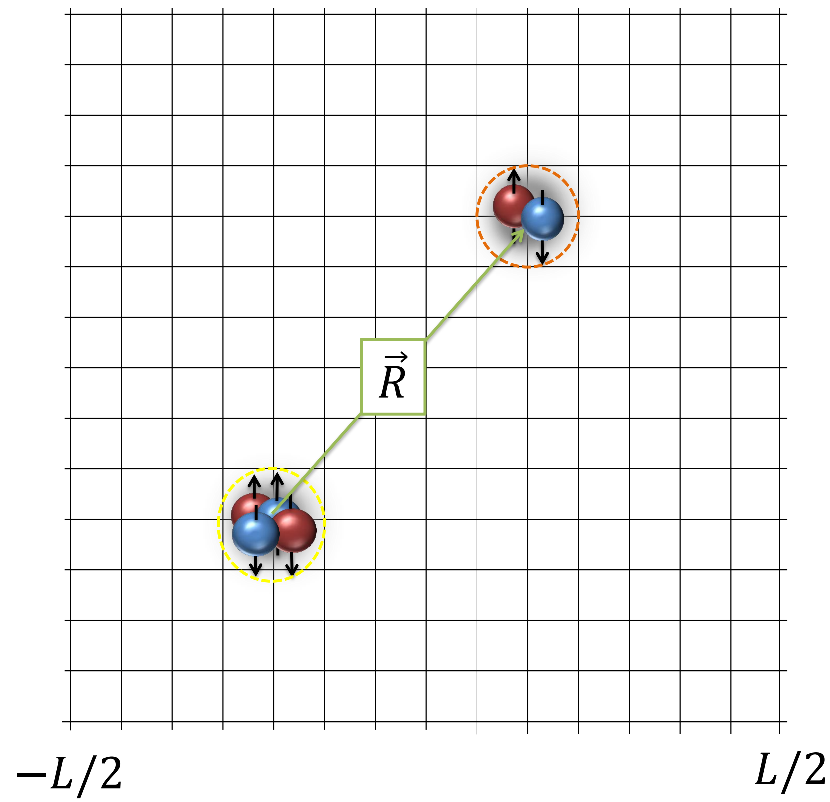
$$d_{\varepsilon,i} = \sqrt{\tau_\varepsilon / M_i}$$

- Only non-overlapping clusters if

$$|\vec{R}| \gg d_{\varepsilon,i} \quad \Rightarrow \quad |\vec{R}\rangle_{\tau_\varepsilon}$$

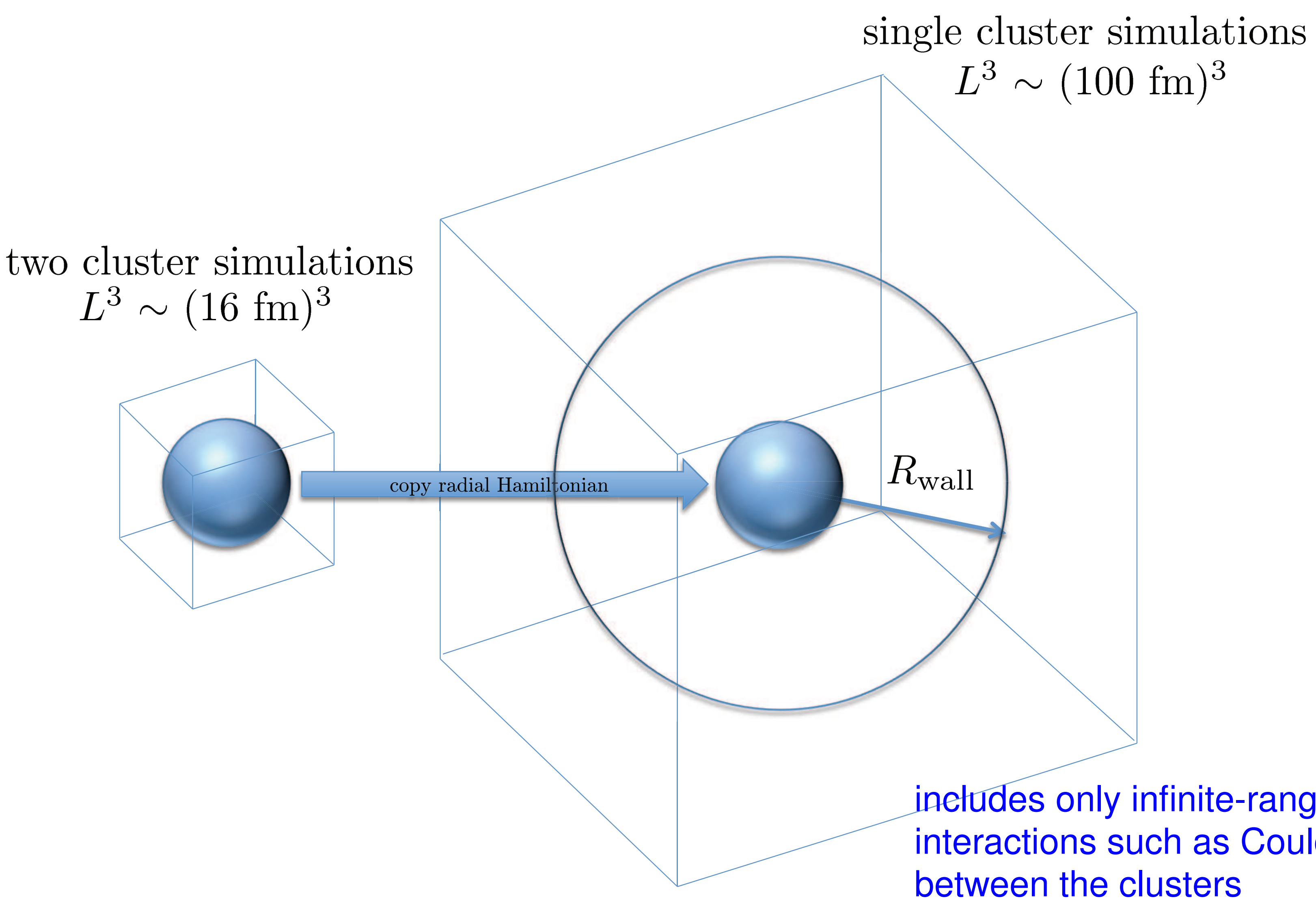
- Defines asymptotic region, where the amount of overlap between clusters is less than ε

$$|\vec{R}| > R_\varepsilon$$



$\Rightarrow$ In the asymptotic region we can describe the system in terms of an effective cluster Hamiltonian (the free lattice Hamiltonian for two clusters) plus infinite-range interactions (like the Coulomb int.)

ADIABATIC HAMILTONIAN plus COULOMB



includes only infinite-range interactions such as Coulomb between the clusters

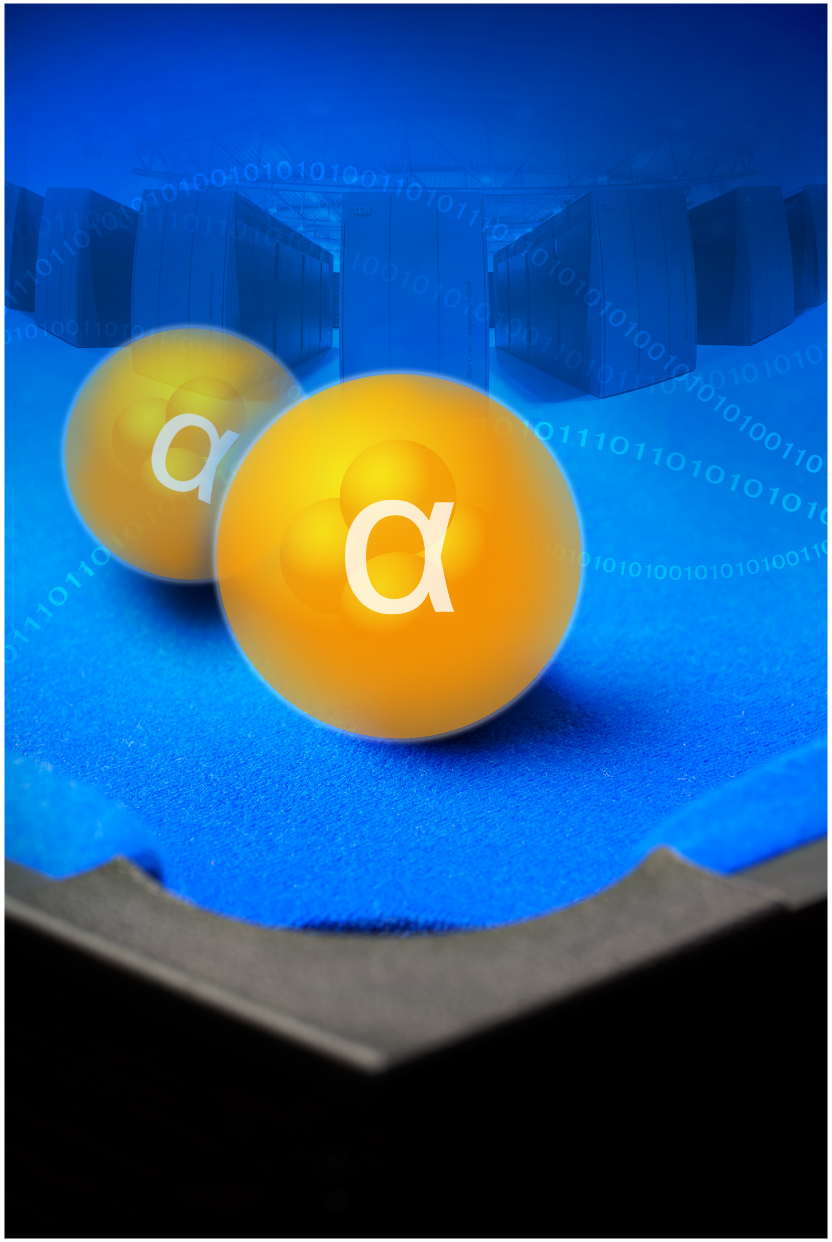
25

ALPHA–ALPHA SCATTERING

- same lattice action as for the Hoyle state in ^{12}C and the structure of ^{16}O
- 11 NN + 2 3N LECs, coarse lattice $a = 1.97$ fm, $N = 8$
- new algorithm for Monte Carlo updates and alpha clusters
- adiabatic projection method to construct a two-alpha Hamiltonian
- spherical wall method to extract the phase shifts using radial Hamiltonian

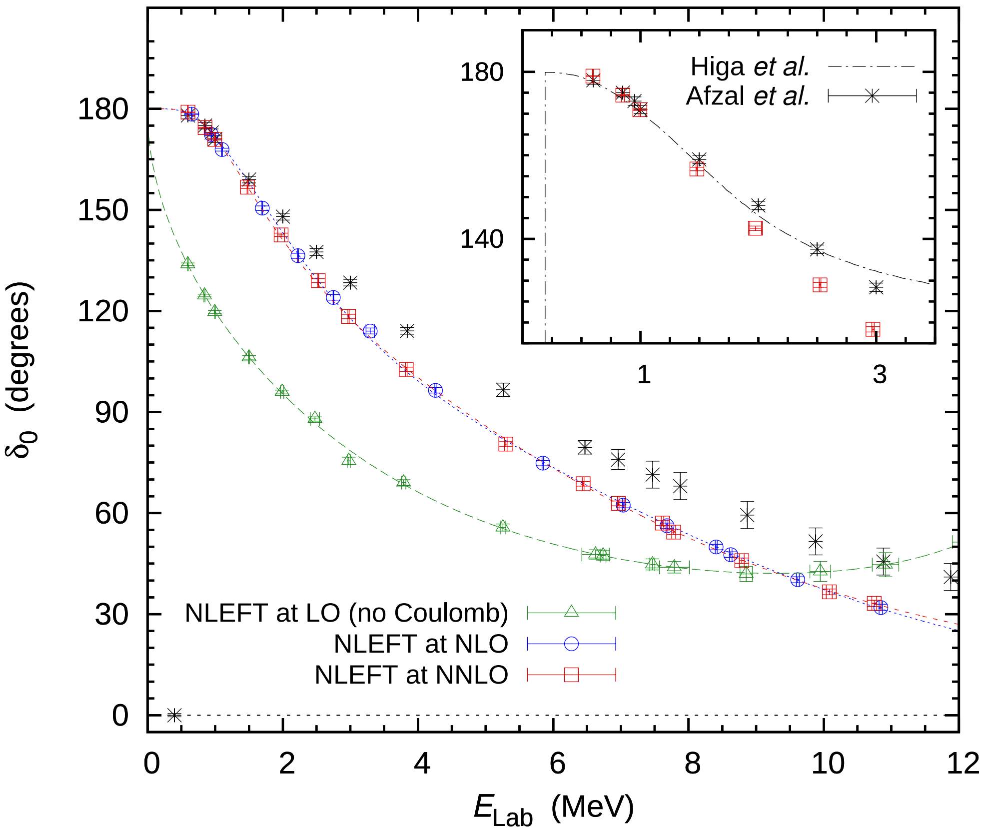
  Borasoy, Epelbaum, Krebs, Lee, UGM, Eur. Phys. J. A**34** (2007) 185;

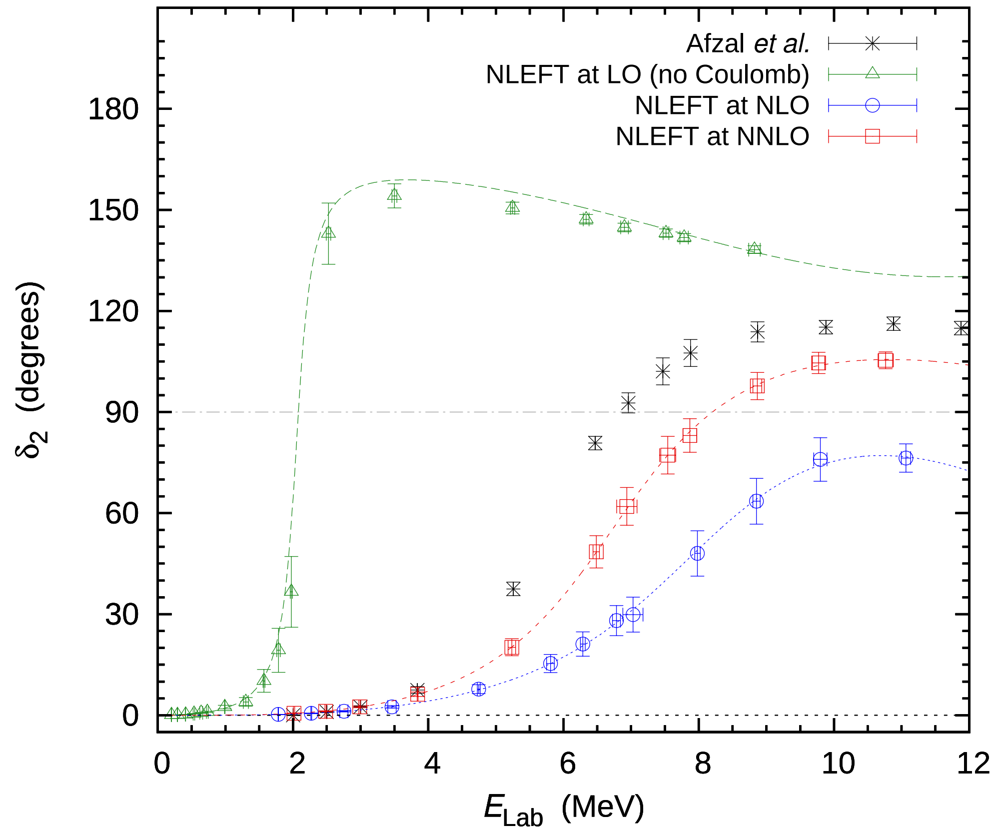
  Lu, Lähde, Lee, UGM, Phys.Lett. **B760** (2016) 309



PHASE SHIFTS

- S-wave and D-wave phase shifts (LO has no Coulomb)





$$E_R^{\rm NNLO} = -0.11(1)\ \text{MeV} \quad [+0.09\ \text{MeV}]$$

$$E_R^{\rm NNLO} = 3.27(12)\ \text{MeV} \quad [\,2.92(18)\ \text{MeV}]$$

$$\Gamma_R^{\rm NNLO} = 2.09(16)\ \text{MeV} \quad [1.35(50)\ \text{MeV}]$$

Afzal et al., Rev. Mod. Phys. **41** (1969) 247 [data]; Higa et al., Nucl.Phys. A**809** (2008) 171 [halo EFT]

INTERMEDIATE SUMMARY

- Chiral nuclear EFT: best approach to nuclear forces and few-body systems
  - $\rightarrow$ new, solid method to estimate the theoretical uncertainties
  - $\rightarrow$ high-precision NN potential to fifth order available
  - $\rightarrow$ pinning down the 3NFs under way
- Nuclear lattice simulations as a new quantum many-body approach
  - $\rightarrow$ many promising results at NNLO using coarse lattices
  - $\rightarrow$ clustering emerges naturally, α-cluster nuclei
  - $\rightarrow$ scattering and inelastic reactions can also be calculated *ab initio*
  - $\rightarrow$ holy grail of nuclear astrophysics (α+^{12}C $\rightarrow$ ^{16}O+γ) in reach

SPARES

Isospin symmetry and isospin violation

ISOSPIN SYMMETRY

- For $m_u = m_d$, QCD is invariant under *SU(2) isospin* transformations:

$$q \to q' = Uq\ , \quad q = \begin{pmatrix} u \\ d \end{pmatrix}, \quad U = \begin{pmatrix} a^* & b^* \\ -b & a \end{pmatrix}, \quad |a|^2 + |b|^2 = 1$$

— NB: Charge symmetry = 180° rotation in iso-space

- Rewriting of the QCD quark mass term:

$$\mathcal{H}_{\rm QCD}^{\rm SB} = m_u\,\bar{u}u + m_d\,\bar{d}d = \underbrace{\frac{1}{2}(m_u + m_d)(\bar{u}u + \bar{d}d)}_{I=0} + \underbrace{\frac{1}{2}(m_u - m_d)(\bar{u}u - \bar{d}d)}_{I=1}$$

Strong isospin violation (IV)

- Competing effect: QED $\to$ can be treated in CHPT
- Standard situation: small IV on top of large iso-symmetric background, requires precise machinery to perform accurate calculations

Gasser, Urech, Steininger, Fettes, M., Knecht, Kubis, . . .

ISOSPIN VIOLATION - PIONS & KAONS

Weinberg, Gasser, Leutwyler, Urech, Neufeld, Knecht, M., Müller, Steininger, . . .

- SU(2) effective Lagrangian w/ virtual photons to leading order:

Q = quark charge matrix

$$\mathcal{L}^{(2)} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\lambda}{2}(\partial_\mu A^\mu)^2 + \frac{F_\pi^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U\rangle + C\langle QUQU^\dagger\rangle$$

  ⋆ pion mass difference of em origin, $M_{\pi^+}^2 - M_{\pi^0}^2 = 2Ce^2/F_\pi^2$

  ⋆ no strong isospin breaking at LO, absence of D-symbol

  ⋆ strong and em corrections at NLO worked out

M., Müller, Steininger, Phys. Lett. B 406 (1997) 154

- Three-flavor chiral perturbation theory:

  ⋆ for $m_u = m_d \Rightarrow M_{K^+}^2 - M_{K^0}^2 = M_{\pi^+}^2 - M_{\pi^0}^2 = \frac{2Ce^2}{F_\pi^2}$ – Dashen's theorem

  ⋆ for $m_u \neq m_d \Rightarrow$ leading order strong kaon mass difference:

$$(M_{K^0}^2 - M_{K^+}^2)^{\text{strong}} = (m_u - m_d)B_0 + \mathcal{O}(m_q^2)$$

$B_0 = |\langle 0|\bar{q}q|0\rangle|/F_\pi^2$

  ⋆ strong and em corrections at NLO incl. leptons worked out

Urech, Nucl. Phys. B 433 (1995) 234
Knecht, Neufeld, Rupertsberger, Talavera, Eur. Phys. J. C 12 (2000) 469

ISOSPIN VIOLATION - NUCLEONS

Weinberg, . . ., Fettes, M., Müller, Steininger

- Effective Lagrangian for isospin violation (to leading order):

$$\mathcal{L}_{\pi N}^{(2,\mathrm{IV})} = \bar{N}\{c_5 \underbrace{(\chi_+ - \tfrac{1}{2}\langle\chi_+\rangle)}_{\sim m_u - m_d} + f_1 \underbrace{\langle\hat{Q}_+^2 - Q_-^2\rangle}_{\sim q_u - q_d} + f_2 \underbrace{\hat{Q}_+\langle Q_+\rangle}_{\sim q_u - q_d}\} N + \mathcal{O}(q^3)$$

– Three LECs parameterize the leading strong (c_5) & em (f_1, f_2) IV effects

– These LECs link various observables/processes:

$$m_n - m_p = 4\, c_5\, B_0(m_u - m_d) + 2\, e^2\, f_2\, F_\pi^2 + \ldots$$ fairly well known

Gasser, Leutwyler, . . .

$$a(\pi^0 p) - a(\pi^0 n) = \mathrm{const}\ (-4\, c_5\, B_0\, (m_u - m_d)) + \ldots$$

extremely hard to measure — Weinberg, M., Steininger

– IV in πN scattering analyzed in CHPT $\rightarrow$ intriguing results

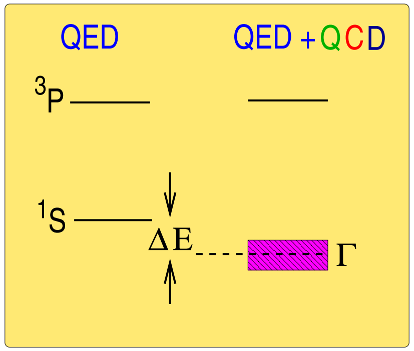
Fettes & M., Nucl. Phys. A 693 (2001) 693; Hoferichter, Kubis, M., Phys. Lett. B 678 (2009) 65

- also access to IV in $np \rightarrow d\pi^0$ and $dd \rightarrow \alpha\pi^0$ (spin-isospin filter)

$\rightarrow$ need to develop a high-precision EFT for few-nucleon systems

BOUND STATE EFT: HADRONIC ATOMS

- Hadronic atoms are bound by the static Coulomb force (QED)
- Many species: $\pi^+\pi^-$, $\pi^\pm K^\mp$, $\pi^- p$, $\boxed{\pi^- d,\ K^- p,\ K^- d,}$ $\ldots$
- Bohr radii $\gg$ typical scale of strong interactions
- Small average momenta $\Rightarrow$ non-relativistic approach
- Observable effects of QCD
  - $\star$ energy shift $\boldsymbol{\Delta E}$ from the Coulomb value
  - $\star$ decay width $\boldsymbol{\Gamma}$



$\Rightarrow$ access to scattering at zero energy! = S-wave scattering lengths

- These scattering lengths are very sensitive to the chiral & isospin symmetry breaking in QCD

  Weinberg, Gasser, Leutwyler, . . .

- can be analyzed systematically & consistently in the framework of low-energy Effective Field Theory (including virtual photons)

EFFECTIVE FIELD THEORY for HADRONIC ATOMS

- Three step procedure utilizing *nested* effective field theories

- Step 1:

  Construct non-relativistic effective Lagrangian (complex couplings)
  & solve Coulomb problem exactly, corrections in perturbation theory

- Step 2: *matching*

  relate complex couplings of $\mathcal{L}_{\rm eff}$ to QCD parameters, e.g. scattering lengths
  & express complex energy shift in terms of QCD parameters

- Step 3:

  extract scattering length(s) from the measured complex energy shift

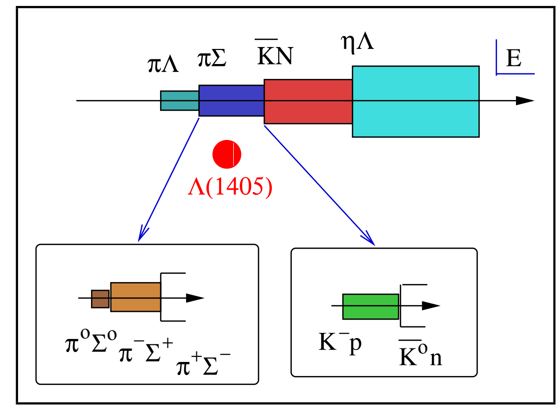
  $\Rightarrow$ most precise way of determining hadron-hadron scattering lengths

$\rightarrow$ study kaonic hydrogen as one example

FEATURES OF KAONIC HYDROGEN

- Strong ($K^- p \to \pi^0 \Lambda, \pi^\pm \Sigma^\mp, \ldots$) and weaker electromagnetic ($K^- p \to \gamma \Lambda, \gamma \Sigma^0, \ldots$) decays

  $\to$ complicated (interesting) analytical structure



- Average momentum $\langle p^2 \rangle = \alpha\, \mu \simeq 2$ MeV

  $\to$ highly non-relativistic

- Bohr radius $r_B = 1/(\alpha\, \mu) \simeq 1o0$ fm

- Binding energy $E_{1s} = \frac{1}{2}\, \alpha^2\, \mu + \ldots \simeq 8$ keV

- Width $\Gamma_{1s} \simeq 250$ eV $\ll E_{1s}$

- $\mathcal{M} = m_n + M_{K^0} - m_p + M_{K^-} > 0 \Rightarrow$ unitary cusp

- Isospin breaking, small parameter $\delta \sim \alpha \sim (m_d - m_u)$

$$\Delta E = \underbrace{\delta^3}_{\text{LO}} + \underbrace{\delta^4}_{\text{NLO}} + \ldots$$

NON-RELATIVISTIC EFFECTIVE LAGRANGIAN

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \psi^\dagger \Big\{ i\mathcal{D}_t - m_p + \frac{\mathcal{D}^2}{2m_p} + \frac{\mathcal{D}^4}{8m_p^3} + \cdots$$

$$- \mathbf{c_p^F} \frac{e\sigma\mathbf{B}}{2m_p} - \mathbf{c_p^D} \frac{e(\mathcal{D}\mathbf{E} - \mathbf{E}\mathcal{D})}{8m_p^2} - \mathbf{c_p^S} \frac{ie\sigma(\mathcal{D}\times\mathbf{E} - \mathbf{E}\times\mathcal{D})}{8m_p^2} + \cdots \Big\} \psi$$

proton

$$+ \chi^\dagger \Big\{ i\partial_t - m_n + \frac{\nabla^2}{2m_n} + \frac{\nabla^4}{8m_n^3} + \cdots \Big\} \chi$$

neutron

$$+ \sum_\pm (K^\pm)^\dagger \Big\{ iD_t - M_{K^+} + \frac{\mathbf{D}^2}{2M_{K^+}} + \frac{\mathbf{D}^4}{8M_{K^+}^3} + \cdots \mp \mathbf{c_K^R} \frac{e(\mathbf{DE} - \mathbf{ED})}{6M_{K^+}^2} + \cdots \Big\} K^\pm$$

$$+ (\bar{K}^0)^\dagger \Big\{ i\partial_t - M_{\bar{K}^0} + \frac{\nabla^2}{2M_{\bar{K}^0}} + \frac{\nabla^4}{8M_{\bar{K}^0}^3} + \cdots \Big\} \bar{K}^0$$

kaons

$$+ \tilde{\mathbf{d}}_\mathbf{1}\, \psi^\dagger\psi\,(K^-)^\dagger K^- + \tilde{\mathbf{d}}_\mathbf{2}(\psi^\dagger\chi\,(K^-)^\dagger\bar{K}^0 + h.c.) + \tilde{\mathbf{d}}_\mathbf{3}\chi^\dagger\chi\,(\bar{K}^0)^\dagger\bar{K}^0 + \cdots .$$

$\rightarrow$ calculate electromagnetic levels and the strong shift (note: $\tilde{d}_i$ complex!)

ENERGY SHIFT in KAONIC HYDROGEN

a) recoil corrections

b) transverse photon exchange

c) finite size corrections

d) vacuum polarisation

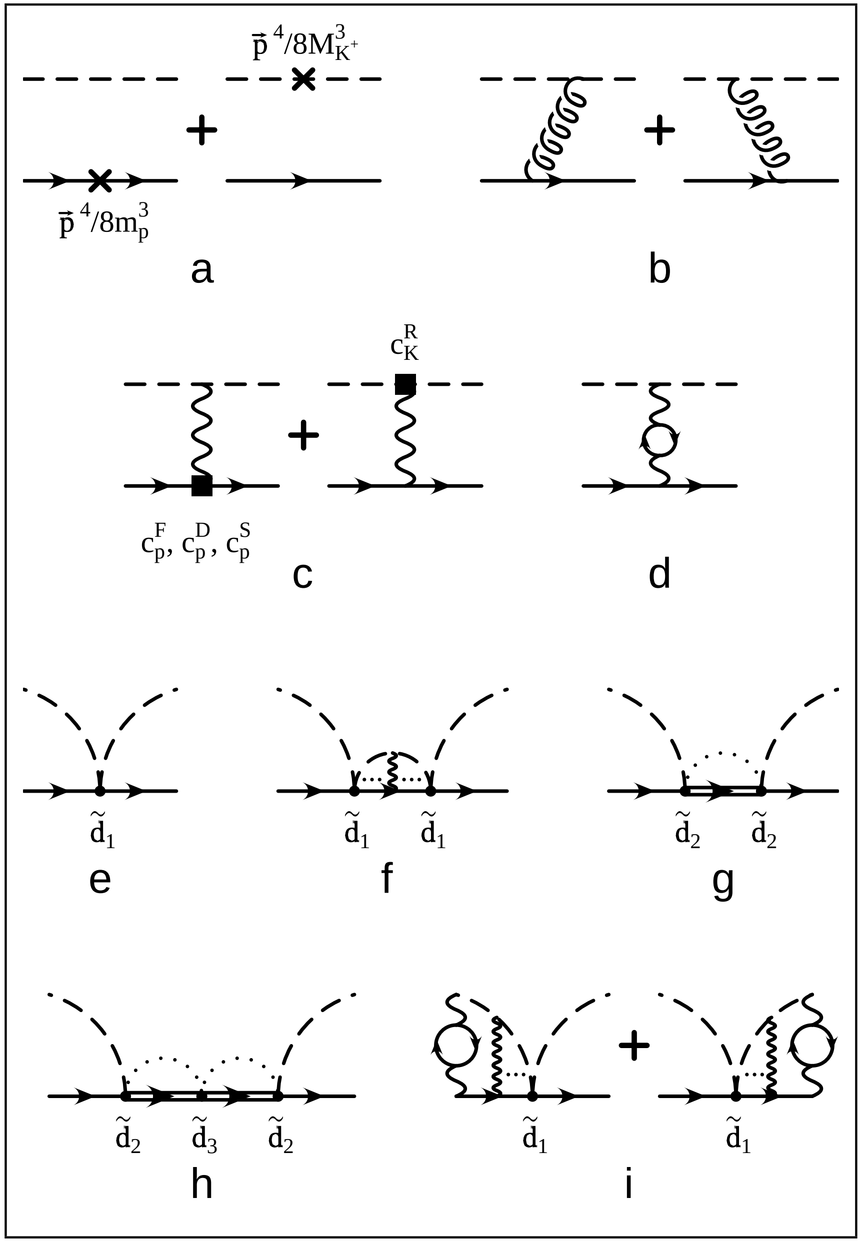
e) leading $K^- p$ interaction

f) $K^- p$ interaction w/ Coulomb ladders

g) leading $\bar{K}^0 n$ intermediate state

h) iterated $\bar{K}^0 n$ intermediate state

i) Coulomb ladders in the $K^- p$ interaction

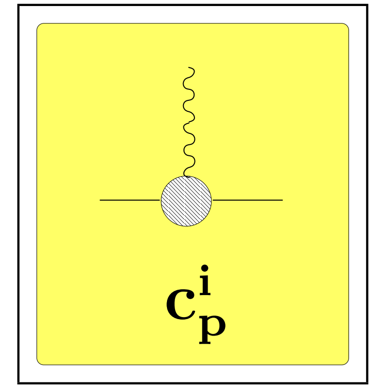


MATCHING CONDITIONS

- Electromagnetic form factors

$$c_p^F = 1 + \mu_p,\ c_p^D = 1 + 2\mu_p + \tfrac{4}{3}\, m_p^2 \langle r_p^2 \rangle,\ c_p^S = 1 + 2\mu_p$$

$$c_K^R = M_{K^+}^2 \langle r_K^2 \rangle$$



- Kaon–nucleon scattering amplitude

matching allows to express the complex strong energy shift in terms of the threshold amplitude (kaon-nucleon scattering lengths a_0 and a_1)

$$\Delta E_n^s - \frac{i}{2}\,\Gamma_n = -\frac{\alpha^3 \mu_c^3}{2\pi M_{K^+} n^3}\, \mathcal{T}_{KN} \left\{ 1 - \frac{\alpha \mu_c^2}{4\pi M_{K^+}}\, \mathcal{T}_{KN} (s_n(\alpha) + 2\pi i) + \delta_n^{\rm vac} \right\}$$

with $\mathcal{T}_{KN} = 4\pi \left(1 + \frac{M_{K^+}}{m_p}\right) \frac{1}{2}\,(a_0 + a_1) + O(\sqrt{\delta})$

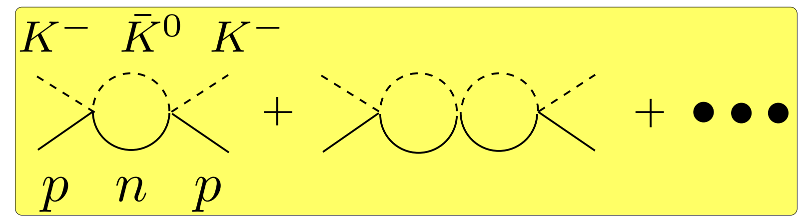
$$s_n(\alpha) = 2(\psi(n) - \psi(1) - \tfrac{1}{n} + \ln\alpha - \ln n)$$

$\Rightarrow$ correct, but not sufficiently accurate

UNITARY CUSP

- Corrections at $\mathcal{O}(\sqrt{\delta})$ can be expressed entirely in terms of a_0 and a_1

$\rightarrow$ resum the fundamental bubble
to account for the unitary cusp



$$\mathcal{T}_{KN}^{(0)} = 4\pi\left(1+\frac{M_{K^+}}{m_p}\right)\frac{\frac{1}{2}(a_0+a_1)+q_0 a_0 a_1}{1+\frac{q_0}{2}(a_0+a_1)}, \qquad q_0 = \sqrt{2\mu_0 \Delta\mathcal{M}}$$

$\star$ agrees with R.H. Dalitz and S.F.Tuan, Ann. Phys. **3** (1960) 307

$\star$ all corrections at $\mathcal{O}(\sqrt{\delta})$ included

$$\mathcal{T}_{KN} = \mathcal{T}_{KN}^{(0)} + \frac{i\alpha\mu_c^2}{2M_{K^+}}(\mathcal{T}_{KN}^{(0)})^2 + \underbrace{\delta\mathcal{T}_{KN}}_{\mathcal{O}(\delta)} + o(\delta)$$

$\Rightarrow$ These further $\mathcal{O}(\delta)$ corrections are expected to be small

FINAL FORMULA to ANALYZE the DATA

$$\Delta E_n^s - \frac{i}{2}\,\Gamma_n = -\frac{\alpha^3\mu_c^3}{2\pi M_{K^+} n^3}\,(\mathcal{T}_{KN}^{(0)} + \delta\mathcal{T}_{KN})\left\{1 - \frac{\alpha\mu_c^2 s_n(\alpha)}{4\pi M_{K^+}}\,\mathcal{T}_{KN}^{(0)} + \delta_n^{\rm vac}\right\}$$

- $\mathcal{O}(\sqrt{\delta})$ and $\mathcal{O}(\delta \ln \delta)$ terms:
  - ⋆ Parameter-free, expressed in terms of a_0 and a_1
  - ⋆ Numerically by far dominant
- Estimate of $\delta\mathcal{T}_{KN}$ in CHPT
  - ⋆ $\delta\mathcal{T}_{KN}/\mathcal{T}_{KN} = (-0.5 \pm 0.4)\cdot 10^{-2}$ at $O(p^2)$
  - ⋆ should be improved (loops, unitarization, influence of $\Lambda(1405)$, etc.)
- vacuum polarization calculation: $\delta_n^{\rm vac} \simeq 1\%$

D. Eiras and J. Soto, Phys. Lett. **B 491** (2000) 101 [hep-ph/0005066]

Spares ...

ISOSPIN BREAKING NN FORCES

- $V(pp) \simeq V(pn) \simeq V(nn) \to$ concept of isospin — Heisenberg 1932
- broken in the Standard Model by strong and electromagnetic effects
- hiearchy of isospin-breaking nuclear forces:

chiral order	2N force
$\nu = 2$	$V_{1\gamma} + V_{1\pi}$
$\nu = 3$	$V_{1\pi} + V_{\rm cont}$
$\nu = 4$	$V_{\pi\gamma} + V_{1\pi} + V_{2\pi} + V_{\rm cont}$
$\nu = 5$	$V_{1\pi} + V_{2\pi} + V_{\rm cont}$

- convenient counting:

van Kolck 1993, Friar et al., 1996, Epelbaum et al. 2004

$$\varepsilon = \frac{m_d - m_u}{m_d + m_u} \sim \frac{1}{3},\ e,\ \frac{e}{4\pi} \to \boxed{\varepsilon \sim e \sim \frac{q}{\Lambda},\ \frac{e}{4\pi} \sim \frac{q^2}{\Lambda^2}}$$

— captures the essence/size of em corrections

— different from what is done in the meson/single-nucleon sector

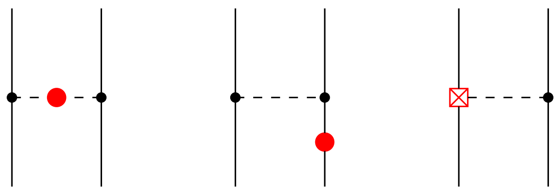
ISOSPIN BREAKING NN POTENTIALS I

- **Long-range em forces**: dominated by the Coulomb interaction ($\nu = 2$), vacuum polarization and magnetic moment interaction
  Uehling 1935, Durand 1957, Sokes, de Swart 1990
- **$\pi\gamma$-exchange**: LO contribution numerically small, NLO contribution of comparable size since $\kappa_V = 4.7$
  van Kolck et al. 1998, Kaiser 2006
- **IV contact terms**: contribute to $^1S_0$ and P-waves up to $\nu = 5$
  Friar et al. 2004, Epelbaum, M.. 2005
- **IV OPEP**: pion mass difference dominant (CIB), charge-dependent πN couplings (largely unknown, small effect)
  van Kolck 1993, van Kolck et al. 1996, Friar et al. 2004, Epelbaum, M.. 2005
- **IV TPEP**: pion and nucleon mass differences, LO ($\nu = 4$) and NLO ($\nu = 5$) contributions comparable (large c_i)
  Friar, van Kolck 1999, Niskanen 2002 1996, Friar et al. 2003, Epelbaum, M.. 2005

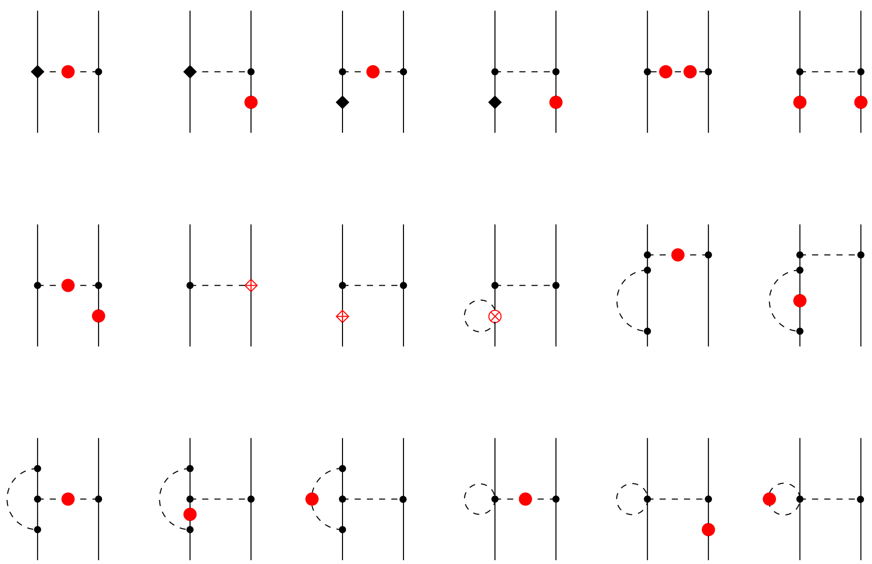
TYPICAL DIAGRAMS

- OPE

$\nu = 2, 3$

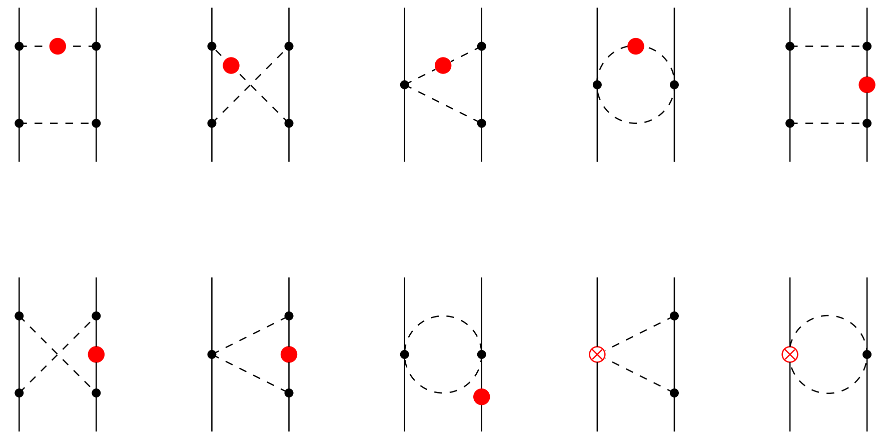


$\nu = 4$



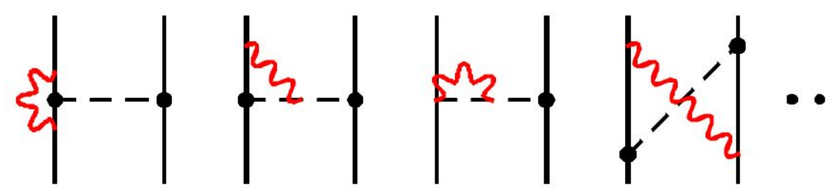
- TPE

$\nu = 4$



- $\pi\gamma$

$\nu = 4$



ISOSPIN-BREAKING 3N FORCES

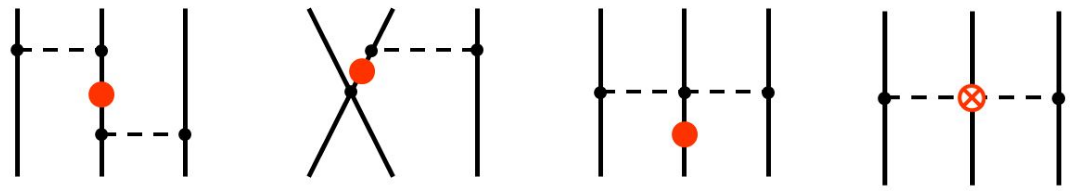
- LO ($\nu = 4$): Epelbaum, M., Palomar 2005, Friar et al. 2005

  all 3NFs at this order are CSB

  mostly driven by $m_p - m_n$

  etimated contribution to the
  $^3He -^3H$ B.E. difference:

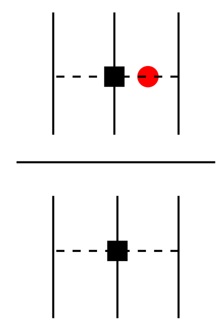
  $E(^3He) - E(^3H) \sim 5$ keV



- NLO ($\nu = 5$): Epelbaum, M.,Palomar 2005

  charge symmetry conserving and charge symmetry breaking corrections

  large effect form CSC class expected:



$$\propto \frac{2\Delta M_\pi^2}{M_\pi^2} \sim 15\%$$