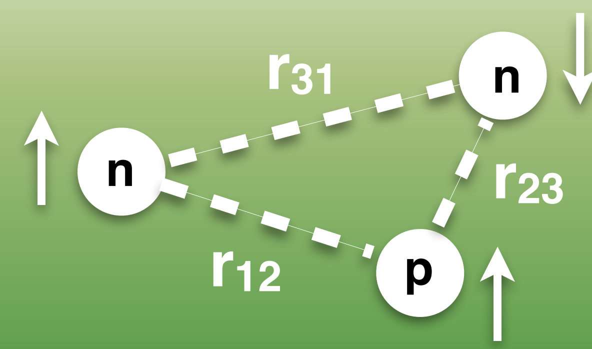
Evgeny Epelbaum, RUB

IHEP, Peking, September 2, 2014

Chiral three-nucleon force

- The three-nucleon force problem
- Chiral EFT for nuclear forces
- Chiral expansion of the nuclear force up to N^4LO
- Inclusion of the Δ(1232) isobar
- Summary & outlook



The three-nucleon force problem

JUNE 15, 1939 PHYSICAL REVIEW VOLUME 55

Many-Body Interactions in Atomic and Nuclear Systems

H. PRIMAKOFF, *Polytechnic Institute of Brooklyn, Brooklyn, New York*

AND

T. HOLSTEIN,* *New York University, University Heights, New York, New York*

(Received March 28, 1938)

„ ...replacement of field interactions by two-body action-at-a-distance potentials is a poor approximation in nuclear physics.“

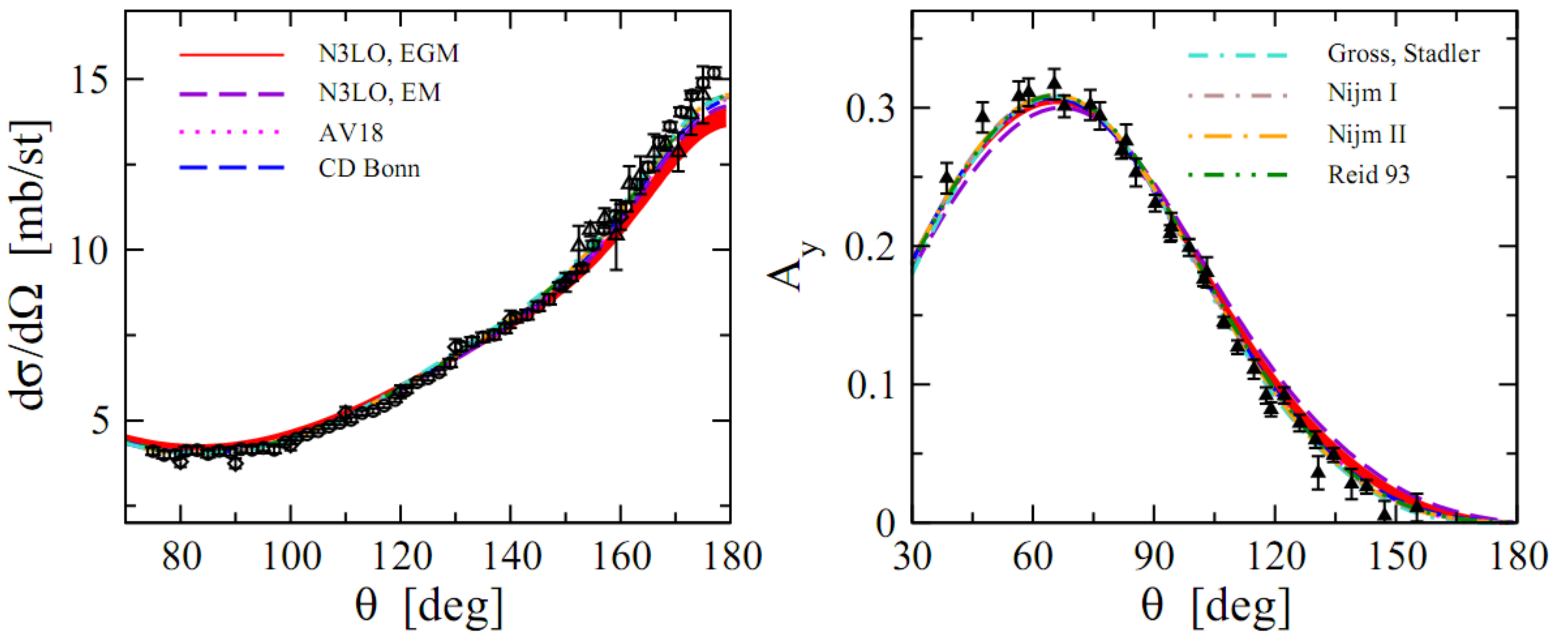
The three-nucleon force problem

Modern phenomenological NN potentials (AV18, CDBonn, Nijm I,II, Reid93, …)

- Long-range part due to EM interaction and the one-pion exchange potential
- Short-range pieces modeled phenomenological; benefit from the general form of the NN potential being rather simple:

$$V(\vec{p}\,', \vec{p}) \equiv \langle \vec{p}\,' | V | \vec{p} \rangle = \left\{ \mathbf{1}_{\rm spin},\ \vec{\sigma}_1 \cdot \vec{\sigma}_2,\ S_{12}(\vec{q}),\ S_{12}(\vec{k}),\ i\vec{S} \cdot \vec{q} \times \vec{k},\ \vec{\sigma}_1 \cdot \vec{q} \times \vec{k}\, \vec{\sigma}_2 \cdot \vec{q} \times \vec{k} \right\} \times \{ \mathbf{1}_{\rm isospin},\ \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \}$$

- Perfect fit to NN data ($\chi^2 \sim 1$)



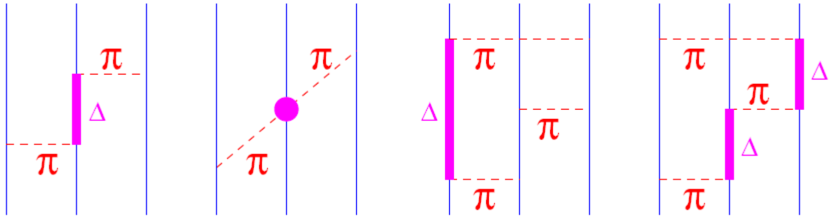
Chiral expansion of nuclear forces

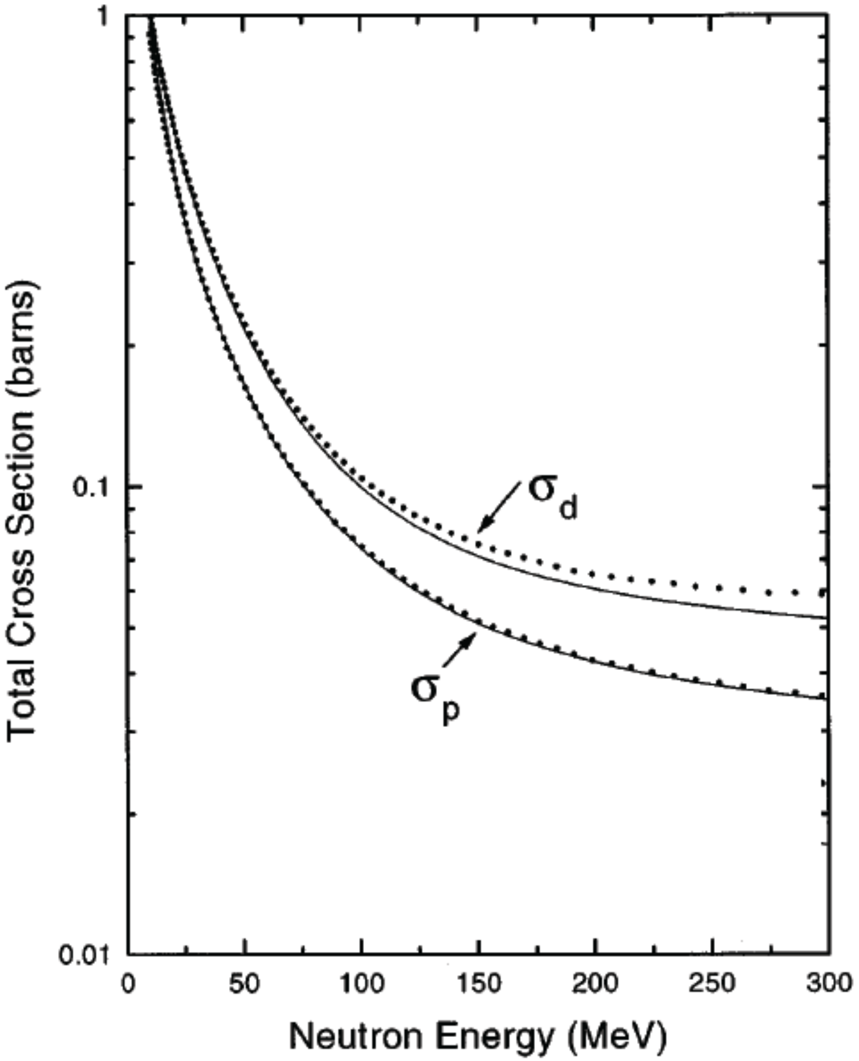
- Triton binding energy calculated based on V_{NN} is typically under-bound by the amount of ~ 1 MeV
- Large discrepancies for the total nucle-on-deuteron cross section

→ „evidence“ for missing 3NFs

Phenomenological 3NF models

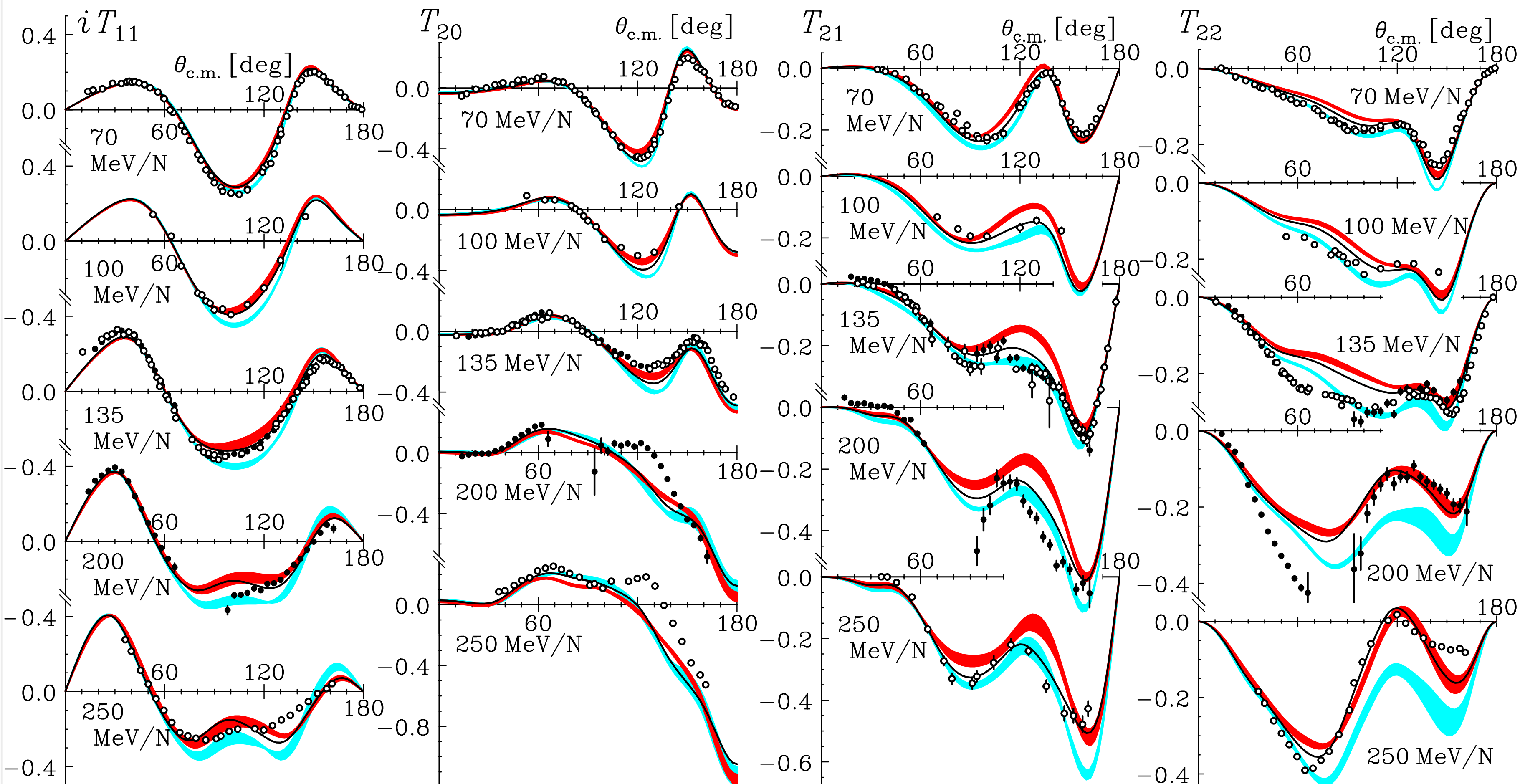
FM, Brasil, TM, Urbana, Illinois,…





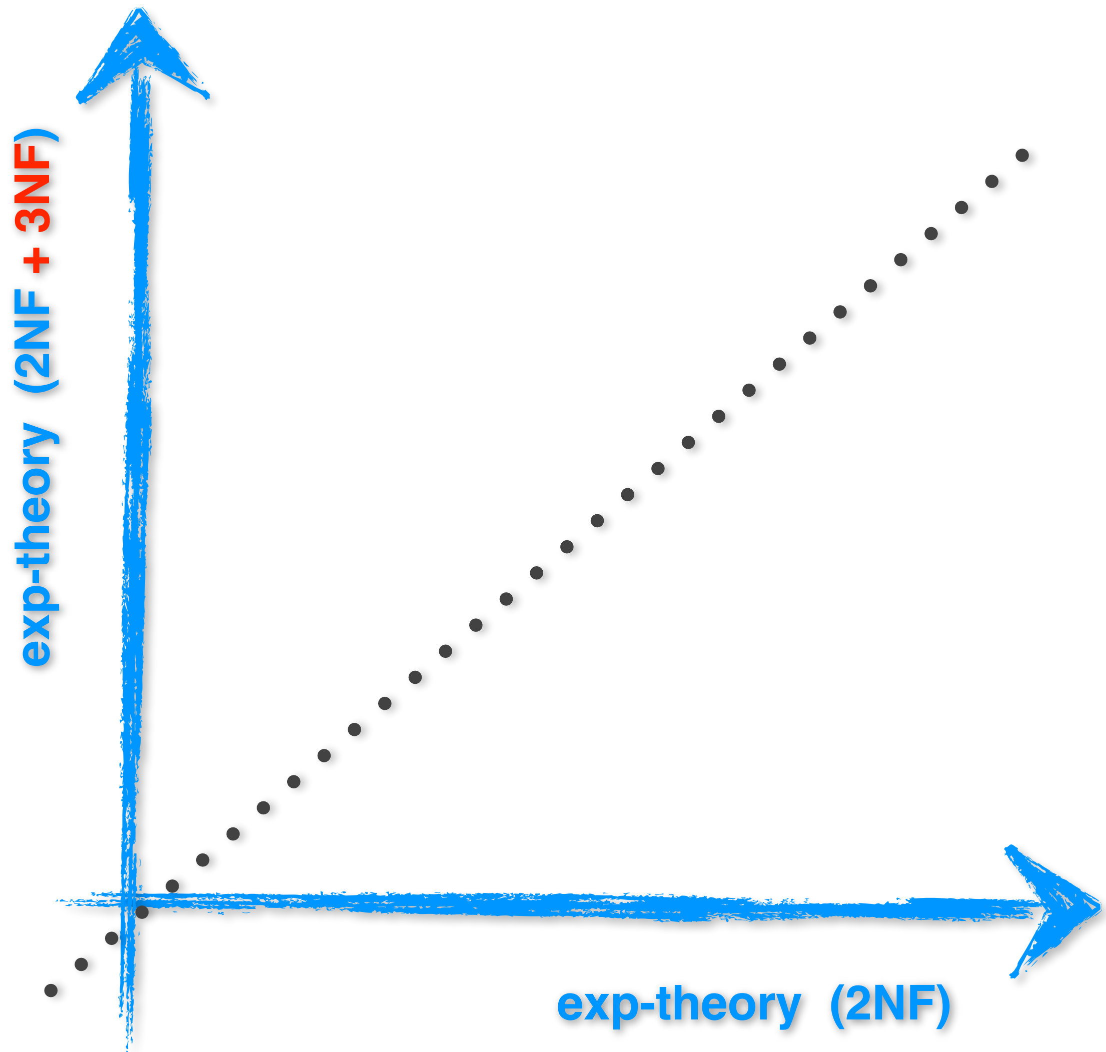
Chiral expansion of nuclear forces

Spin observables in elastic nucleon-deuteron scattering



Phenomenological 3NF models

Fugita-Miyazawa,
Tucson-Melbourne,
Brasil, Urbana IX, Illinois, ...



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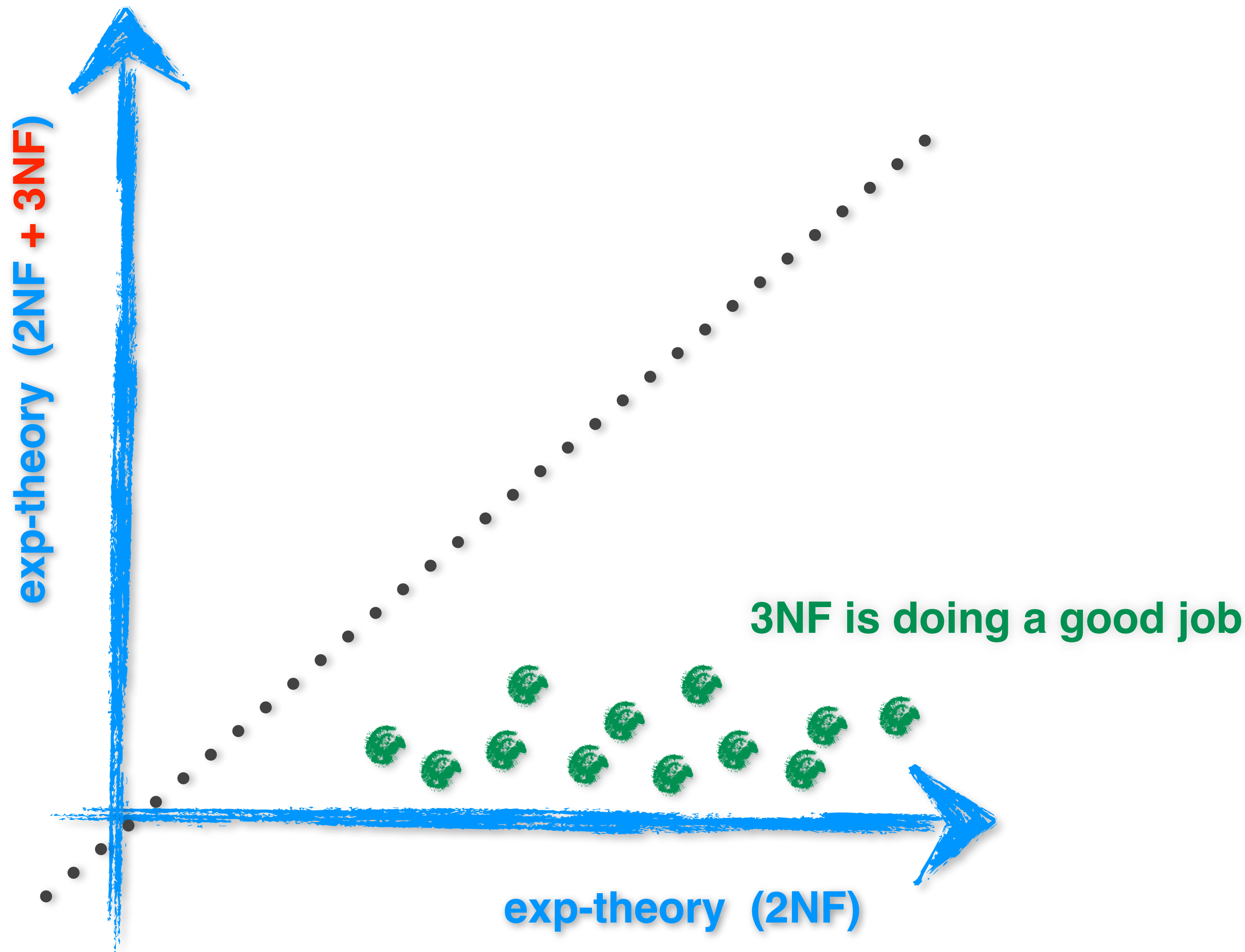

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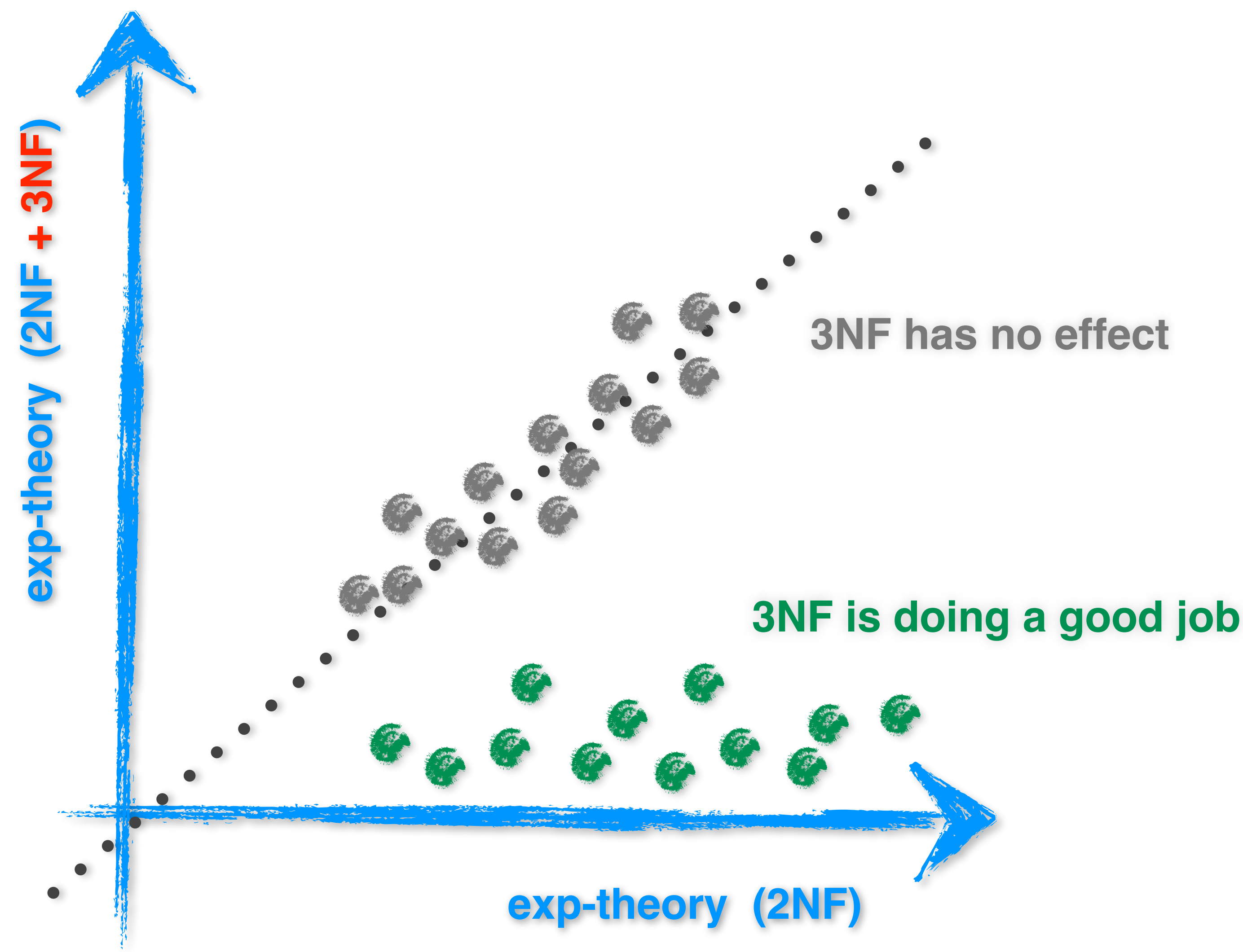

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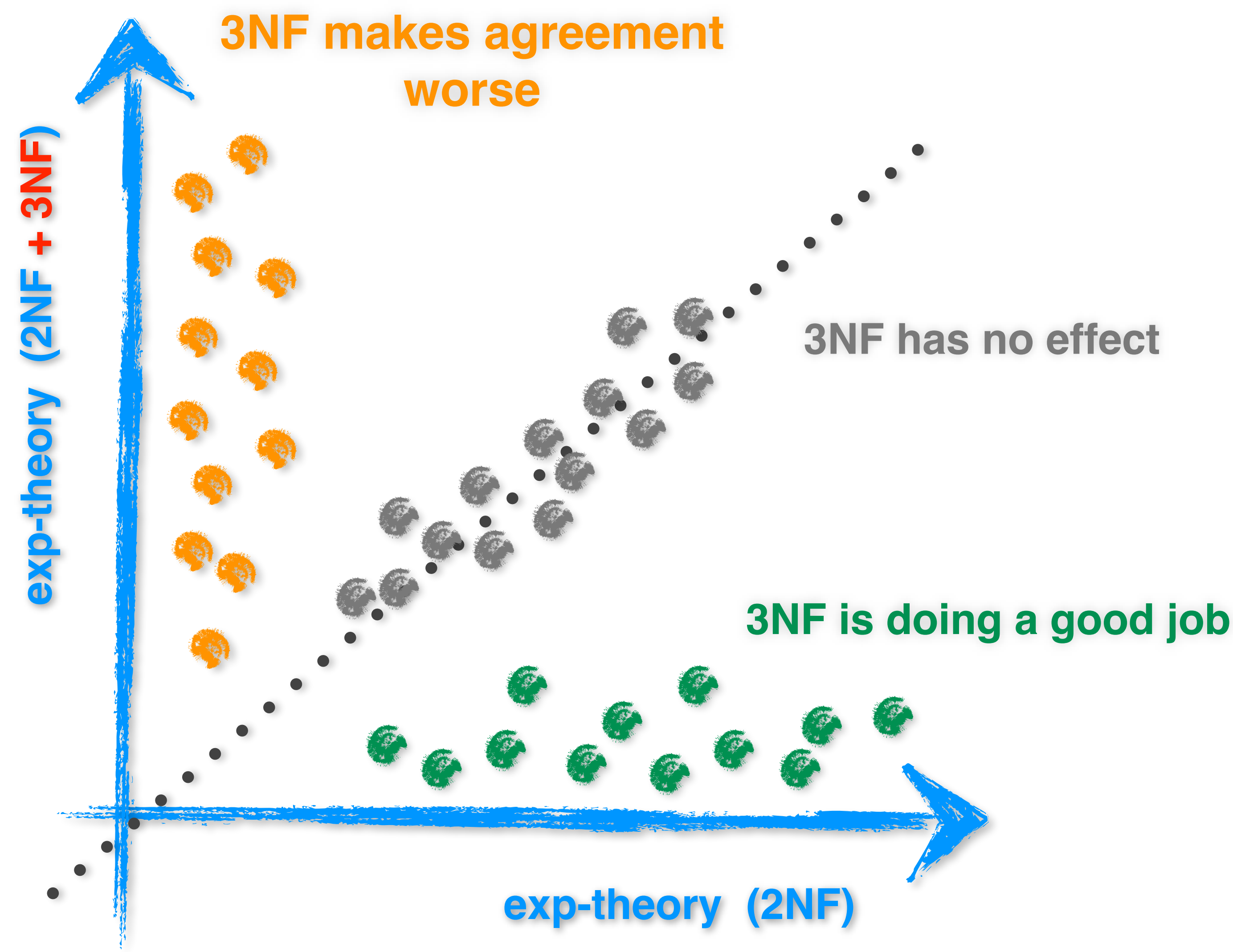

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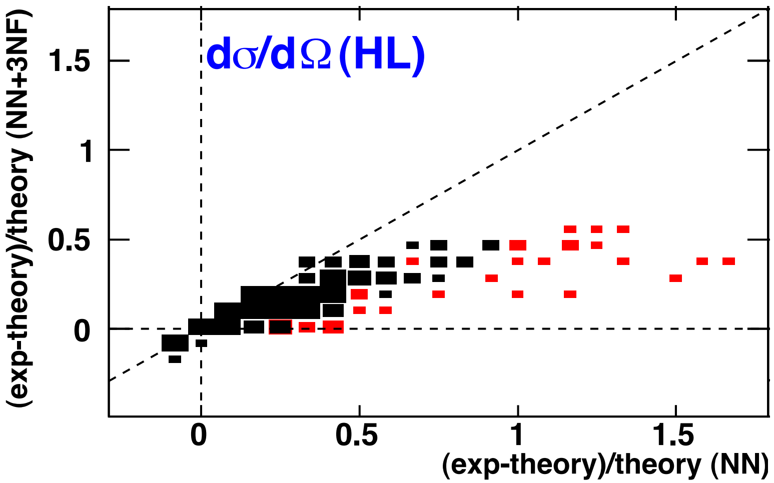
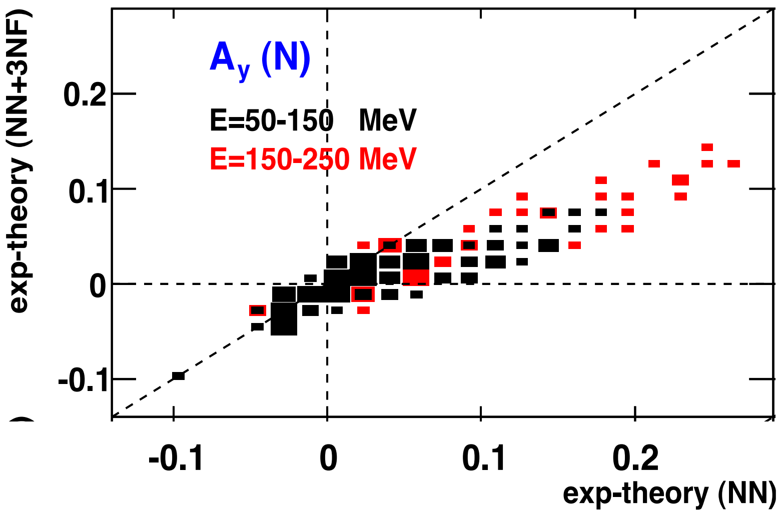
Elastic nucleon-deuteron scattering





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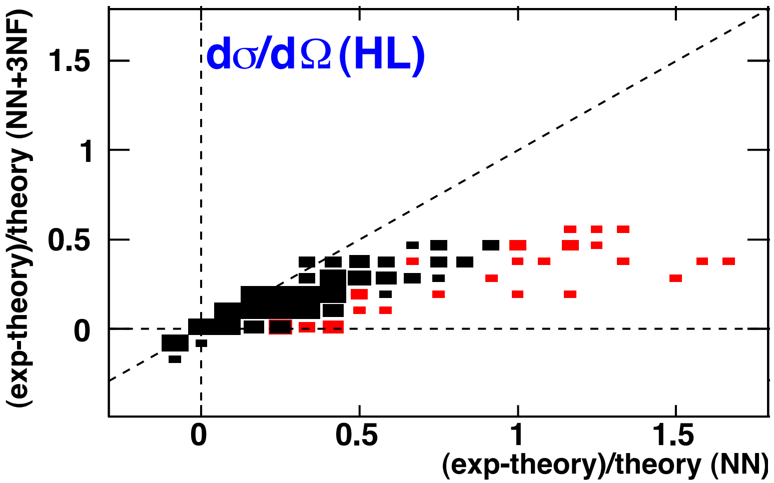
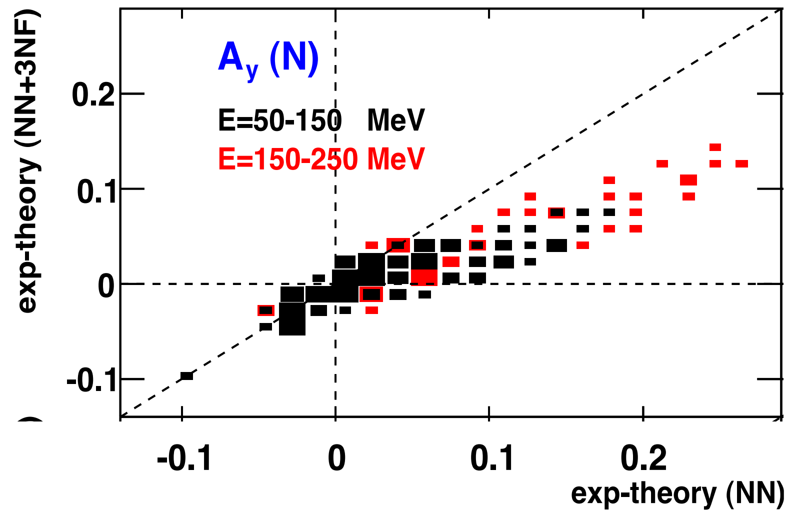
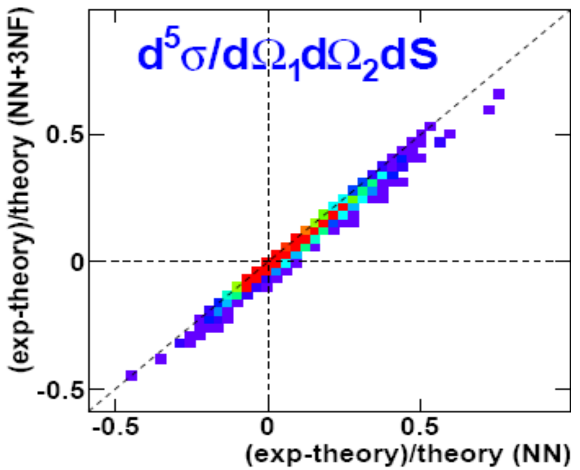
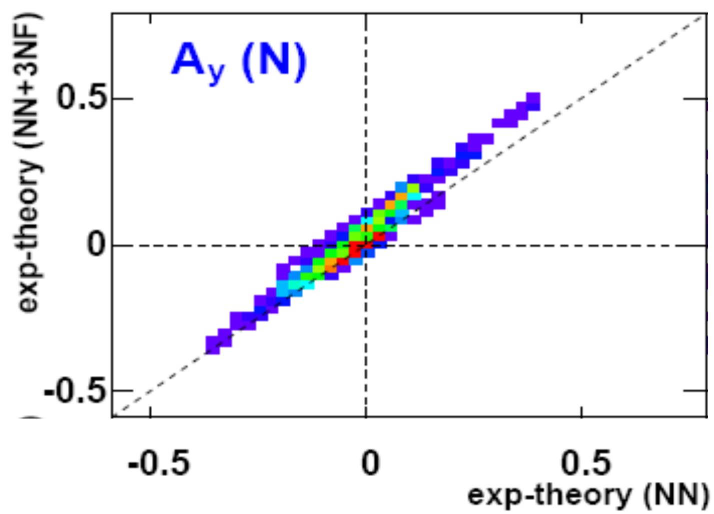




Deuteron breakup reaction





from: Kalantar-Nayestanaki, EE, Messchendorp, Nogga, Rept. Prog. Phys. 75 (2012) 016301

General structure of a 3NF

Why is it so difficult to model the 3NF as compared to NN potentials?

- More scarce Nd data base compared to np and pp data bases
- Solving the Faddeev equation for 3N more involved than solving the LS equation for NN
- **General structure of the 3NF is much more involved**

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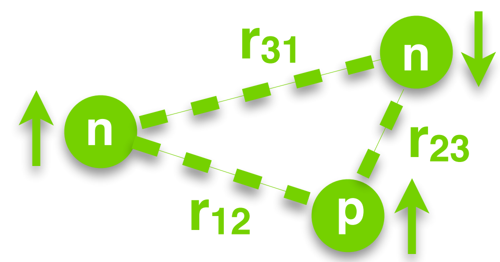
Most general structure of a local, isospin-symmetric 3NF

Krebs, Gasparyan, EE '13; Phillips, Schat '13; EE, Gasparyan, Krebs, Schat, in preparation

Generators $\mathcal{G}$ in momentum space	Generators $\tilde{\mathcal{G}}$ in coordinate space
$\mathcal{G}_1 = 1$	$\tilde{\mathcal{G}}_1 = 1$
$\mathcal{G}_2 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3$	$\tilde{\mathcal{G}}_2 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3$
$\mathcal{G}_3 = \vec{\sigma}_1 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_3 = \vec{\sigma}_1 \cdot \vec{\sigma}_3$
$\mathcal{G}_4 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_4 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \vec{\sigma}_1 \cdot \vec{\sigma}_3$
$\mathcal{G}_5 = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_5 = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \vec{\sigma}_1 \cdot \vec{\sigma}_2$
$\mathcal{G}_6 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3)$	$\tilde{\mathcal{G}}_6 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3)$
$\mathcal{G}_7 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	$\tilde{\mathcal{G}}_7 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})$
$\mathcal{G}_8 = \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_8 = \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{23} \cdot \vec{\sigma}_3$
$\mathcal{G}_9 = \vec{q}_1 \cdot \vec{\sigma}_3 \vec{q}_3 \cdot \vec{\sigma}_1$	$\tilde{\mathcal{G}}_9 = \hat{r}_{23} \cdot \vec{\sigma}_3\, \hat{r}_{12} \cdot \vec{\sigma}_1$
$\mathcal{G}_{10} = \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{10} = \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_3$
$\mathcal{G}_{11} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{11} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{23} \cdot \vec{\sigma}_2$
$\mathcal{G}_{12} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{12} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_2$
$\mathcal{G}_{13} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{13} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{12} \cdot \vec{\sigma}_1\, \hat{r}_{23} \cdot \vec{\sigma}_2$
$\mathcal{G}_{14} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{14} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{12} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_2$
$\mathcal{G}_{15} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{q}_2 \cdot \vec{\sigma}_1 \vec{q}_2 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{15} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \hat{r}_{13} \cdot \vec{\sigma}_1\, \hat{r}_{13} \cdot \vec{\sigma}_3$
$\mathcal{G}_{16} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_2 \vec{q}_3 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{16} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{12} \cdot \vec{\sigma}_2\, \hat{r}_{12} \cdot \vec{\sigma}_3$
$\mathcal{G}_{17} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{17} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_3$
$\mathcal{G}_{18} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{\sigma}_3 \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	$\tilde{\mathcal{G}}_{18} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \vec{\sigma}_3\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})$
$\mathcal{G}_{19} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_3 \cdot \vec{q}_1 \vec{q}_1 \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)$	$\tilde{\mathcal{G}}_{19} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_3 \cdot \hat{r}_{23}\, \hat{r}_{23} \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)$
$\mathcal{G}_{20} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot \vec{q}_3 \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	$\tilde{\mathcal{G}}_{20} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \hat{r}_{23}\, \vec{\sigma}_3 \cdot \hat{r}_{12}\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})$



Assuming hermiticity, time reversal & parity invariance, **20 structure functions** are needed:

$$V(r_{12}, r_{23}, r_{31}) = \sum_{i=1}^{20} \tilde{\mathcal{G}}_i \, F_i(r_{12}, r_{23}, r_{31}) + \text{permutations}$$

$$V(q_1, q_2, q_3) = \sum_{i=1}^{20} \mathcal{G}_i \, F_i(q_1, q_2, q_3) + \text{permutations}$$

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Phenomenological modeling seems not feasible!

Need a theoretical approach which would:

- **be based on QCD,**
- **yield consistent many-body forces,**
- **be systematically improvable,**
- **allow for error estimation**

→ Chiral Effective Field Theory

Chiral dynamics
and the pion-nucleon system

Chiral Perturbation Theory

Chiral Perturbation Theory: expansion of the scattering amplitude in powers of

Weinberg, Gasser, Leutwyler, Meißner, ...

$$Q = \frac{\text{momenta of pions and nucleons or } M_\pi \sim 140 \text{ MeV}}{\text{hard scales [at best } \Lambda_\chi = 4\pi F_\pi \sim 1 \text{ GeV]}}$$

Manohar, Georgi '84

Tool: Feynman calculus using the effective chiral Lagrangian

low-energy constants

$$\mathcal{L}_\pi = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \ldots$$

$$\mathcal{L}_{\pi N} = \underbrace{\bar{N}\left(i\gamma^\mu D_\mu[\pi] - m + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu[\pi]\right)N}_{\mathcal{L}_{\pi N}^{(1)}} + \underbrace{\sum_i \boldsymbol{c}_i \bar{N}\hat{O}_i^{(2)}[\pi]N}_{\mathcal{L}_{\pi N}^{(2)}} + \underbrace{\sum_i \boldsymbol{d}_i \bar{N}\hat{O}_i^{(3)}[\pi]N}_{\mathcal{L}_{\pi N}^{(3)}} + \ldots$$

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Pion-nucleon scattering up to Q^4 in heavy-baryon ChPT

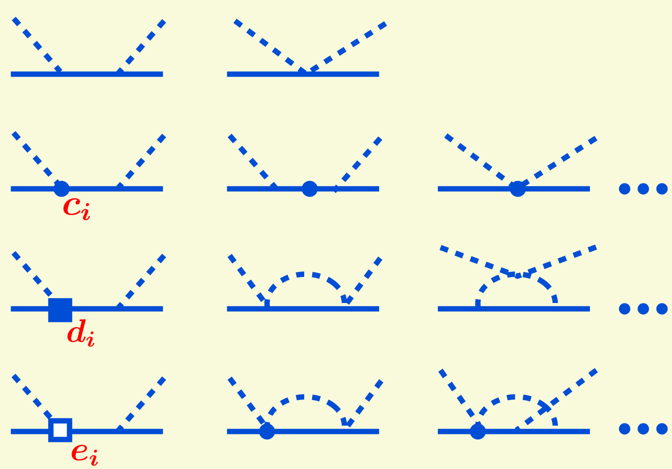
Fettes, Meißner '00; Krebs, Gasparyan, EE '12

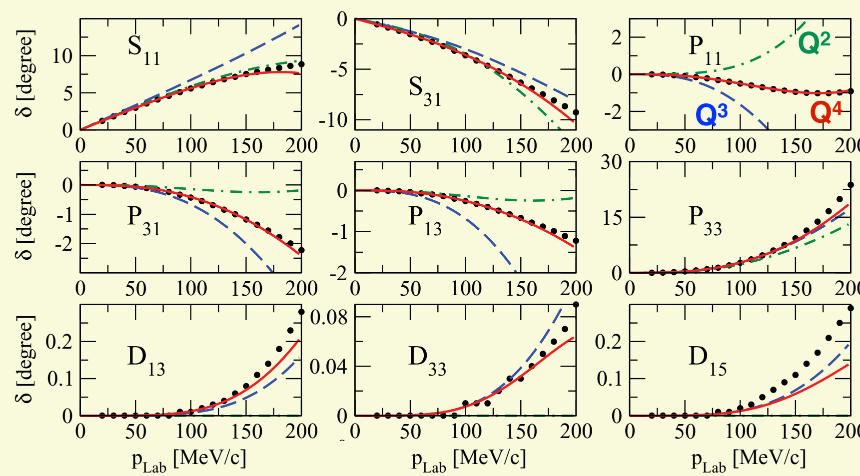
Order Q:

Order Q^2:

Order Q^3:

Order Q^4:





Chiral Perturbation Theory

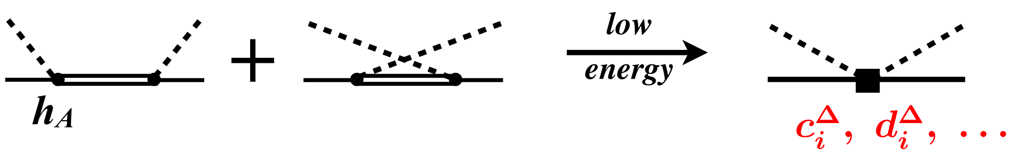
Improving the convergence of ChPT:

- **Covariant formulations (no 1/m_N-expansion)** Becher, Leutwyler, Fuchs, Gegelia, Japaridze, Scherer ...

- **Explicit treatment of the Δ(1232) isobar** Jenkens, Manohar, Hemmert, Holstein, Kambor, ...

For ChPT to be useful, (renormalized) LECs must be natural, i.e. $\sim \alpha_i/\Lambda_\chi^n, \quad \alpha_i = \mathcal{O}(1)$

LECs contain information about short-range physics such as the Δ:



$$c_2^\Delta = -c_3^\Delta = 2c_4^\Delta = \frac{4h_A^2}{9(m_\Delta - m_N)} \simeq 2.8\,\mathrm{GeV}^{-1}$$ Bernard, Kaiser, Meißner '97

$$\bar{d}_{14}^\Delta - \bar{d}_{15}^\Delta = -2(\bar{d}_1^\Delta + \bar{d}_2^\Delta) = 2\bar{d}_3^\Delta = \frac{-2h_A^2}{9(m_\Delta - m_N)^2} \simeq -4.8\,\mathrm{GeV}^{-2}$$ Krebs, Gasparyan, EE, to appear

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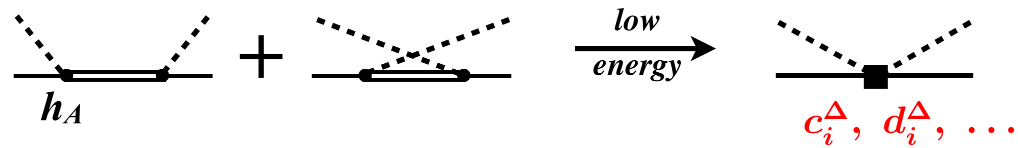

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LECs from pion-nucleon scattering (HB ChPT) in units of GeV^{-n}

Krebs, Gasparyan, EE, to appear; similar results found by Fettes, Meißner; Büttiker, Meißner, ...

	c_1	c_2	c_3	c_4	$\bar d_1 + \bar d_2$	$\bar d_3$	$\bar d_5$	$\bar d_{14} - \bar d_{15}$	$\bar e_{14}$	$\bar e_{15}$	$\bar e_{16}$	$\bar e_{17}$	$\bar e_{18}$
Δ-less approach	-0.75	3.49	-4.77	3.34	6.21	-6.83	0.78	-12.02	1.52	-10.41	6.08	-0.37	3.26
Δ-full approach	-0.95	1.90	-1.78	1.50	2.40	-3.87	1.21	-5.25	-0.24	-6.35	2.34	-0.39	2.81
Δ-contribution	0	2.81	-2.81	1.40	2.39	-2.39	0	-4.77	1.87	-4.15	4.15	-0.17	1.32

The hope: LECs of a more natural size $\longrightarrow$ better convergence of the EFT expansion...

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ChPT, DOF: π, N

Expansion in $q \in \left(\frac{M_\pi}{\Lambda_\chi}, \frac{p_i}{\Lambda_\chi} \right)$

☺ simple, smaller number of LECs

☹ certain LECs are unnaturally large

$$c_2 = -2.8, \; c_3 = -3.9, \; c_4 = 2.9 \; [\mathrm{GeV}^{-1}]$$

→ (sometimes) slow convergence…

ChPT mit Δ, DOF: π, N, Δ

Expansion in: $\epsilon \in \left(\frac{M_\pi}{\Lambda_\chi}, \frac{p_i}{\Lambda_\chi}, \frac{m_\Delta - m_N}{\Lambda_\chi} \right)$

☹ more LECs, considerably more extensive

$$S^{\mu\nu}(p) = \frac{p + m_\Delta}{p^2 - m_\Delta^2} \left(-g^{\mu\nu} + \frac{1}{3}\gamma^\mu\gamma^\nu + \frac{1}{3m_\Delta}(\gamma^\mu p^\nu - \gamma^\nu p^\mu) + \frac{2}{3m_\Delta^2} p^\mu p^\nu \right)$$

☺ more natural LECs, e.g.

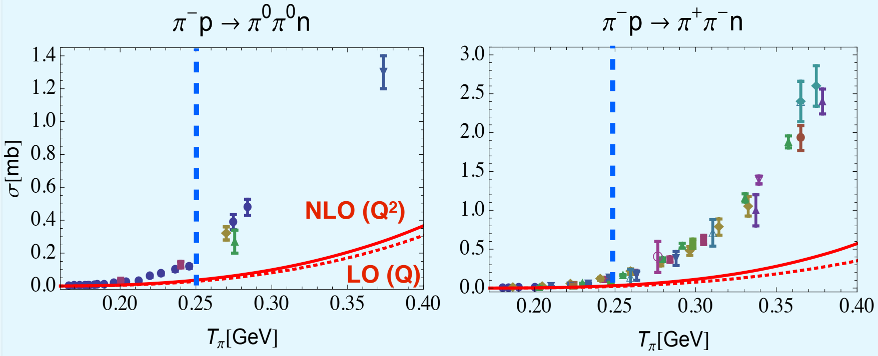
$$c_2 = -0.3, \; c_3 = -0.8, \; c_4 = 1.3 \; [\mathrm{GeV}^{-1}]$$

→ **expect faster convergence**

The reaction $\pi N \rightarrow \pi\pi N$

Siemens, Bernard, EE, Krebs, Meißner '14

Heavy-baryon ChPT



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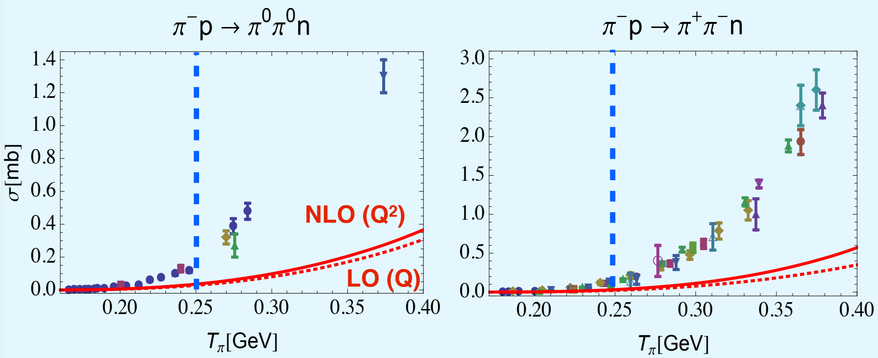
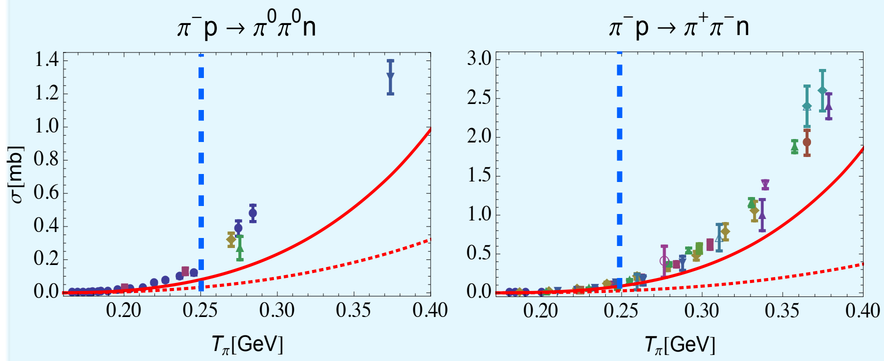


Covariant ChPT



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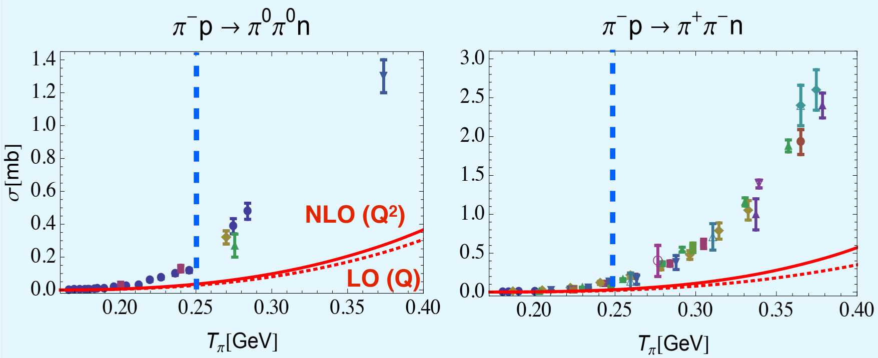
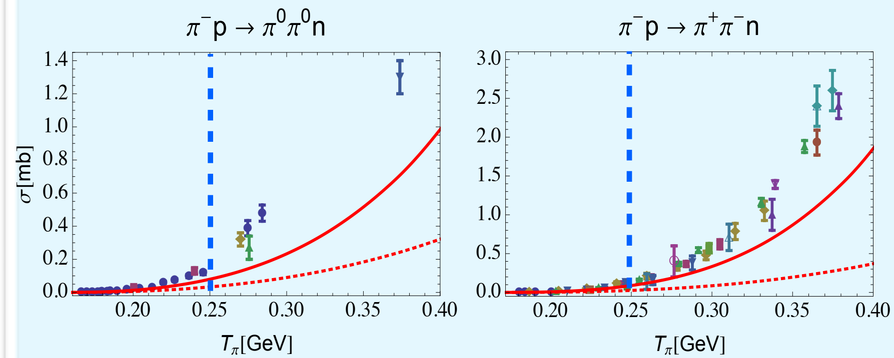
Siemens, Bernard, EE, Krebs, Meißner ’14

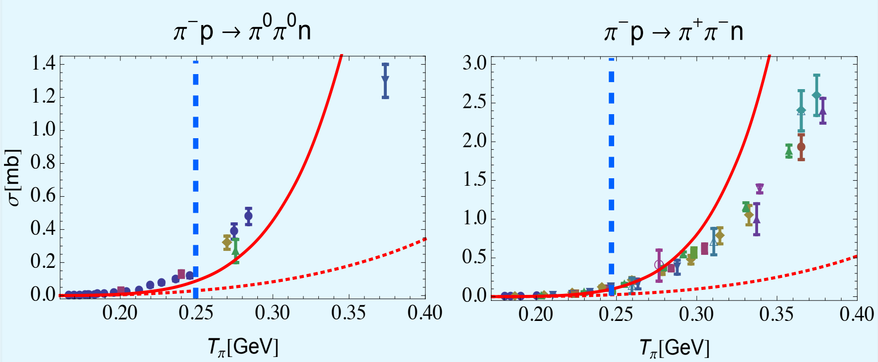
Heavy-baryon ChPT



Covariant ChPT



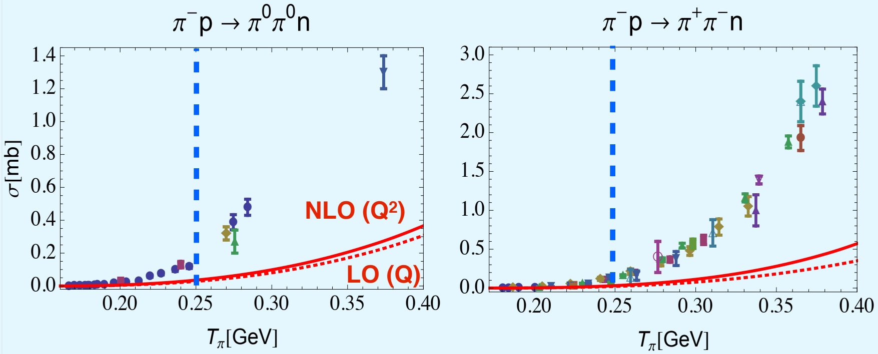
Heavy-baryon ChPT, explicit Δ



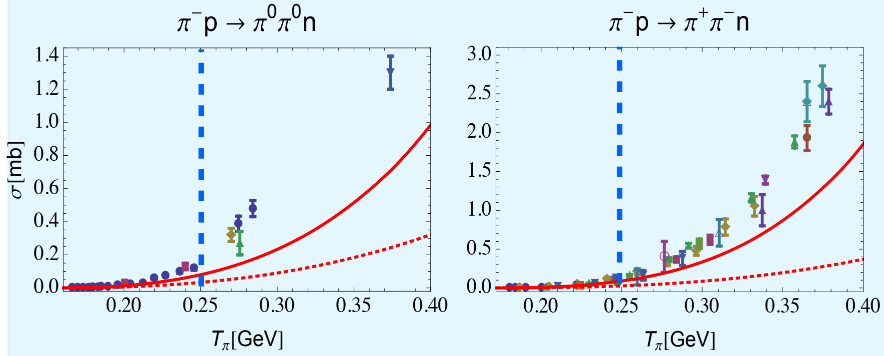
The reaction $\pi N \to \pi\pi N$

Siemens, Bernard, EE, Krebs, Meißner ’14

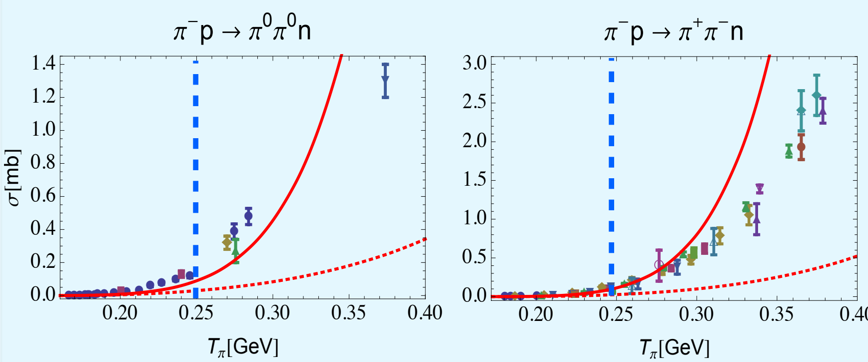
Heavy-baryon ChPT



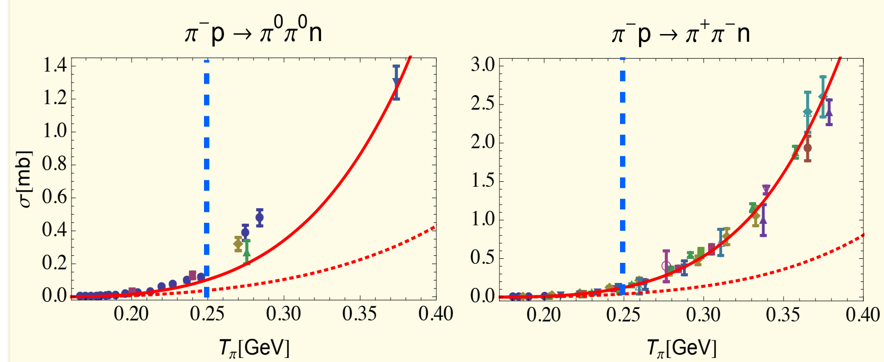
Covariant ChPT



Heavy-baryon ChPT, explicit Δ

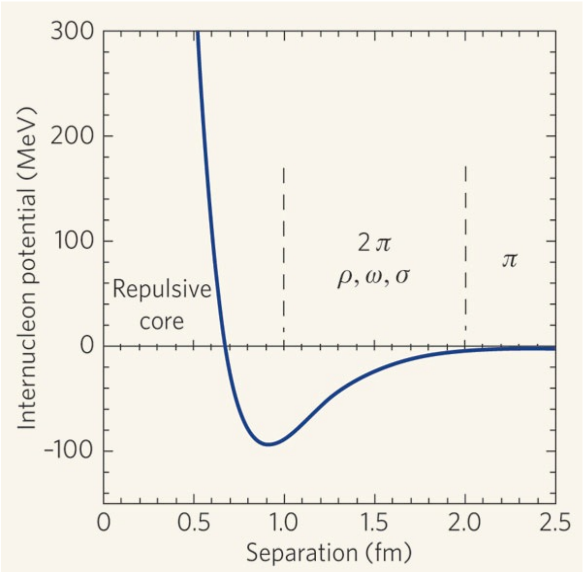


Covariant ChPT, explicit Δ

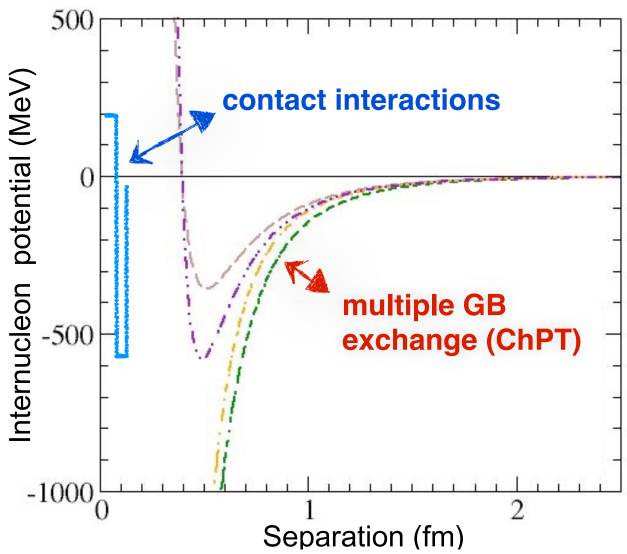


Chiral dynamics and nuclear forces

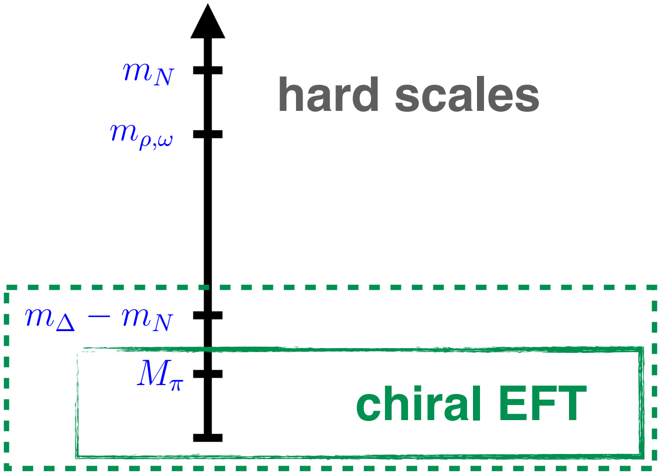
conventional picture



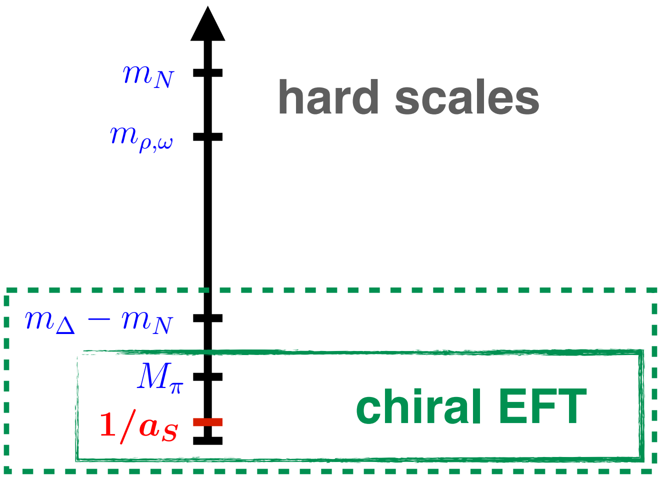
chiral EFT



Chiral EFT for nuclei



Chiral EFT for nuclei

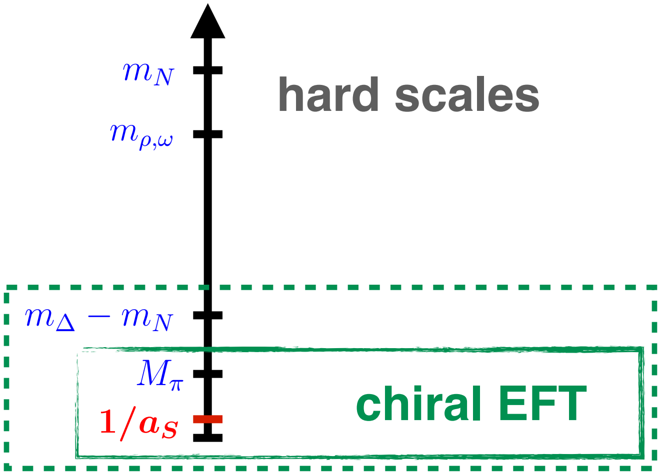


A new, soft scale associated with nuclear binding

$Q \sim 1/a_S \simeq 8.5$ MeV(36 MeV) in $^1S_0$ ($^3S_1$)

to be generated dynamically (need resummations...)

Chiral EFT for nuclei



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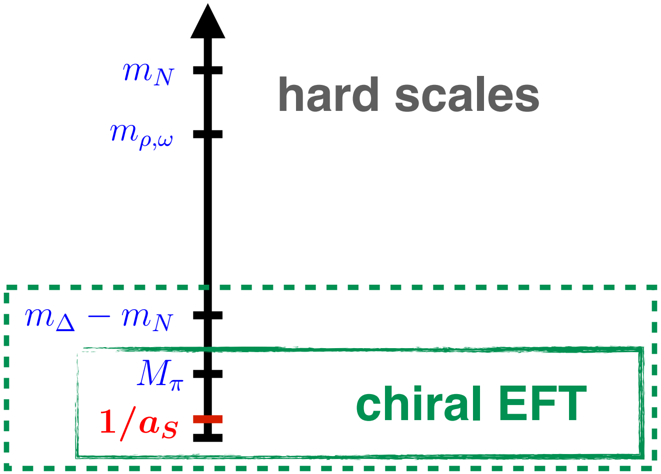
Pionless EFT (valid for $\sqrt{m_N E_B} \ll Q \ll M_\pi$)

- zero-range forces between nucleons
- for 2N equivalent to Effective Range Theory
- universality, Efimov physics, cold gases, halos,...

Chiral EFT for nuclei



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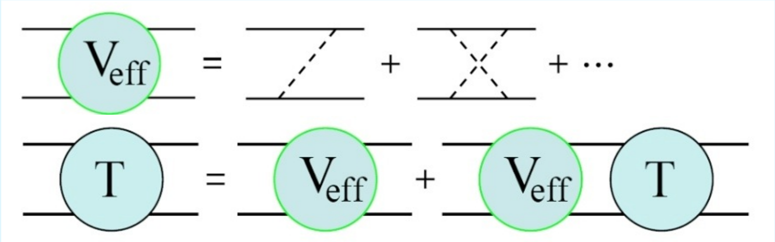
- zero-range forces between nucleons
- for 2N equivalent to Effective Range Theory
- universality, Efimov physics, cold gases, halos,...

Chiral EFT (valid for $Q \sim M_\pi$)

Weinberg, van Kolck, EE, Glöckle, Meißner, Machleidt, Entem...

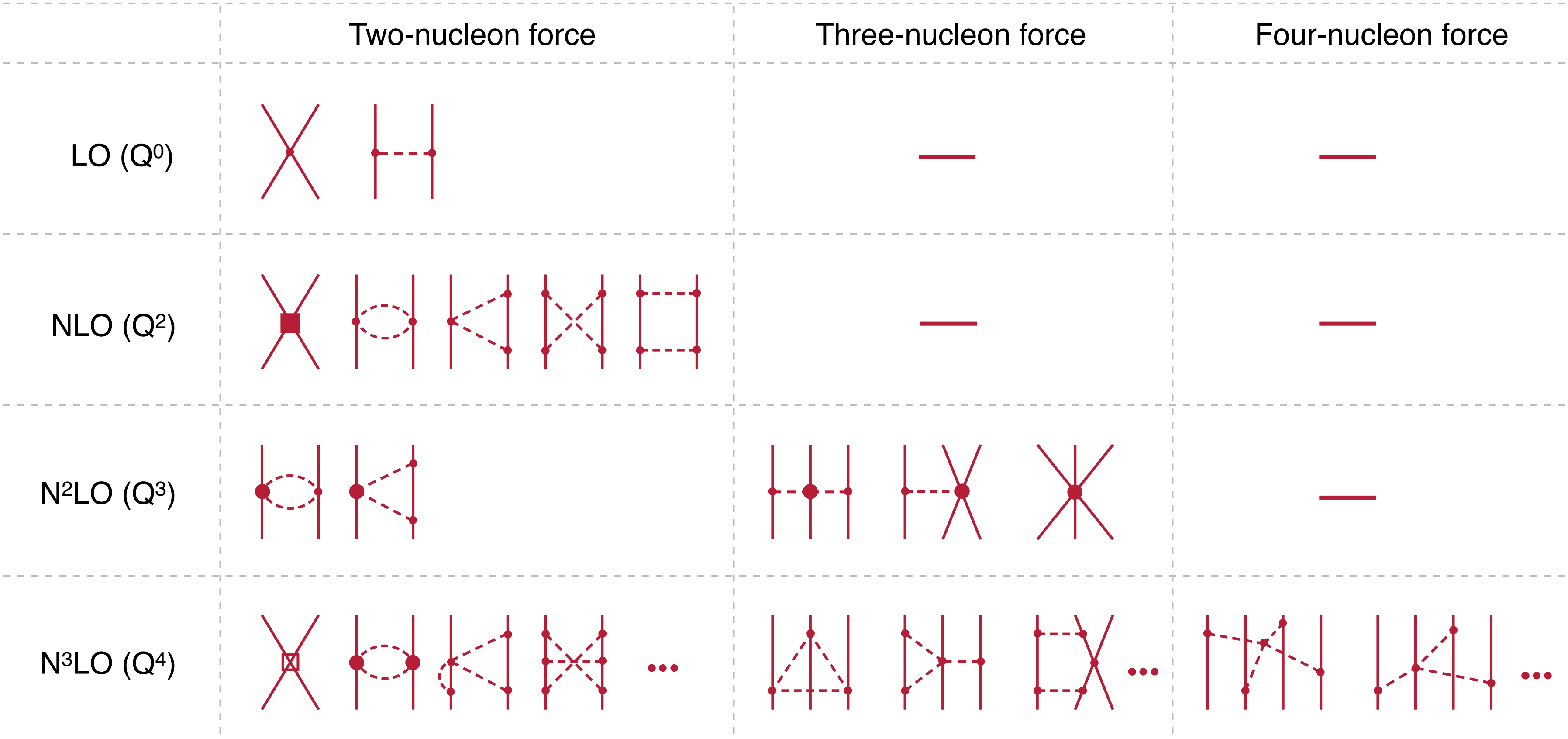
- Schrödinger equation for nucleons interacting via contact forces and long-range potentials (pion exchanges)



$$\left[\left(\sum_{i=1}^{A}\frac{-\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3})\right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \ldots}_{\text{derived in ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$

- access to heavier nuclei (ab initio few-/many-body methods)

Chiral expansion of nuclear forces

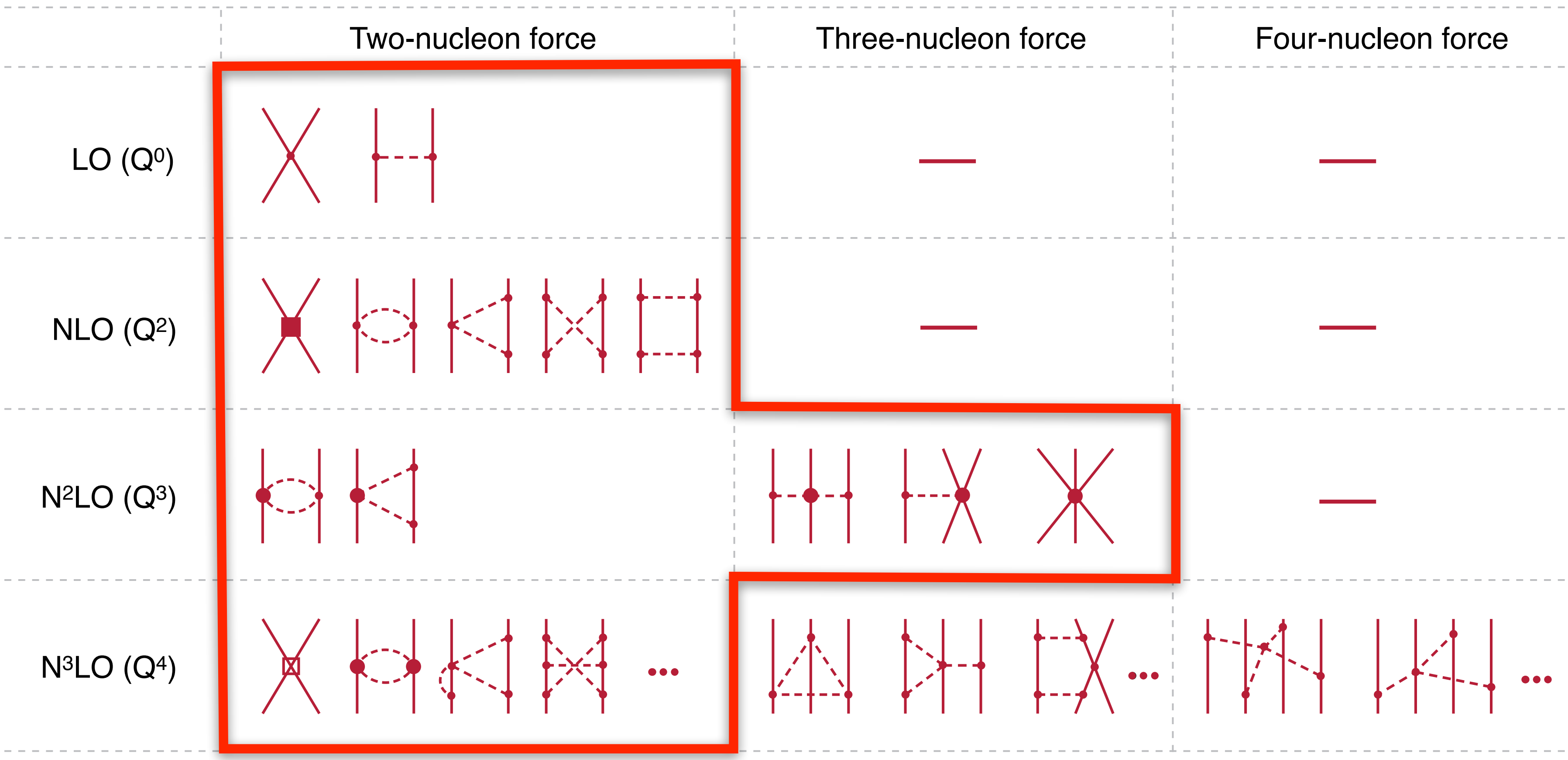


2N force: accurate N^{3}LO potentials are available Entem-Machleidt '03; EE-Glöckle-Meißner '04

3N force: N^{2}LO 3NF included in most calculations
N^{3}LO 3NF worked out Bernard, EE, Krebs Meißner '08,'11; (probably) not yet converged → higher orders
numerical PWD developed Golak, Skibinski, Krebs, Hebeler, …, first results available Witala et al.'13

4N force: leading (i.e. N^{3}LO) terms worked out EE '06; contrib. to ^{4}He BE ~ few 100 keV Rospedzik et al. '06

Chiral expansion of nuclear forces

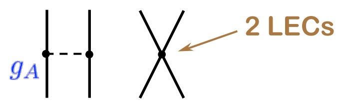


The „standard“ nuclear chiral Hamiltonian has been extensively tested in few- and many-body systems

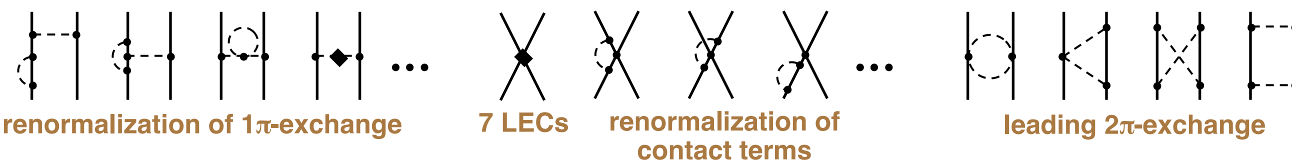
Nucleon-nucleon force up to N^{3}LO

Ordonez et al. '94; Friar & Coon '94; Kaiser et al. '97; E.E. et al. '98,'03; Kaiser '99-'01; Higa, Robilotta '03; …

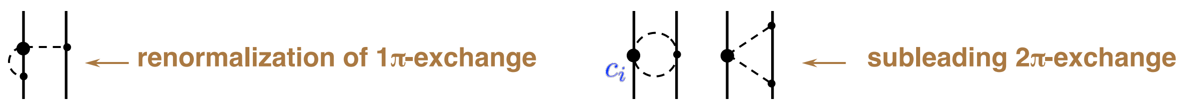
- **LO (Q^0):**



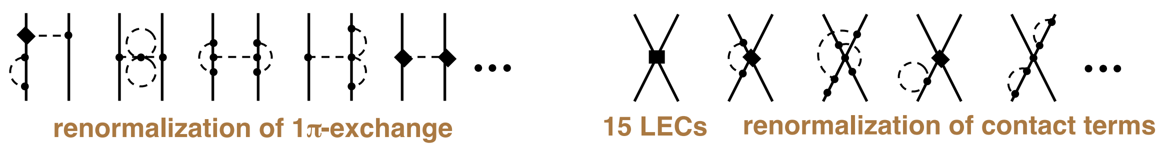
- **NLO (Q^2):**

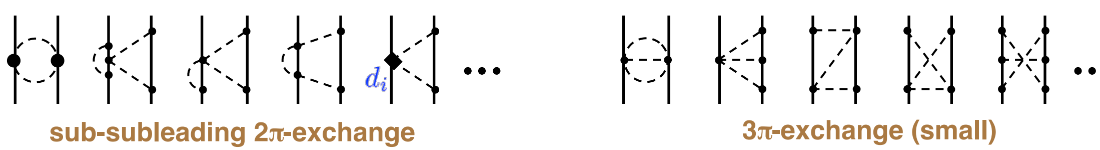


- **N^{2}LO (Q^3):**



- **N^{3}LO (Q^4):**





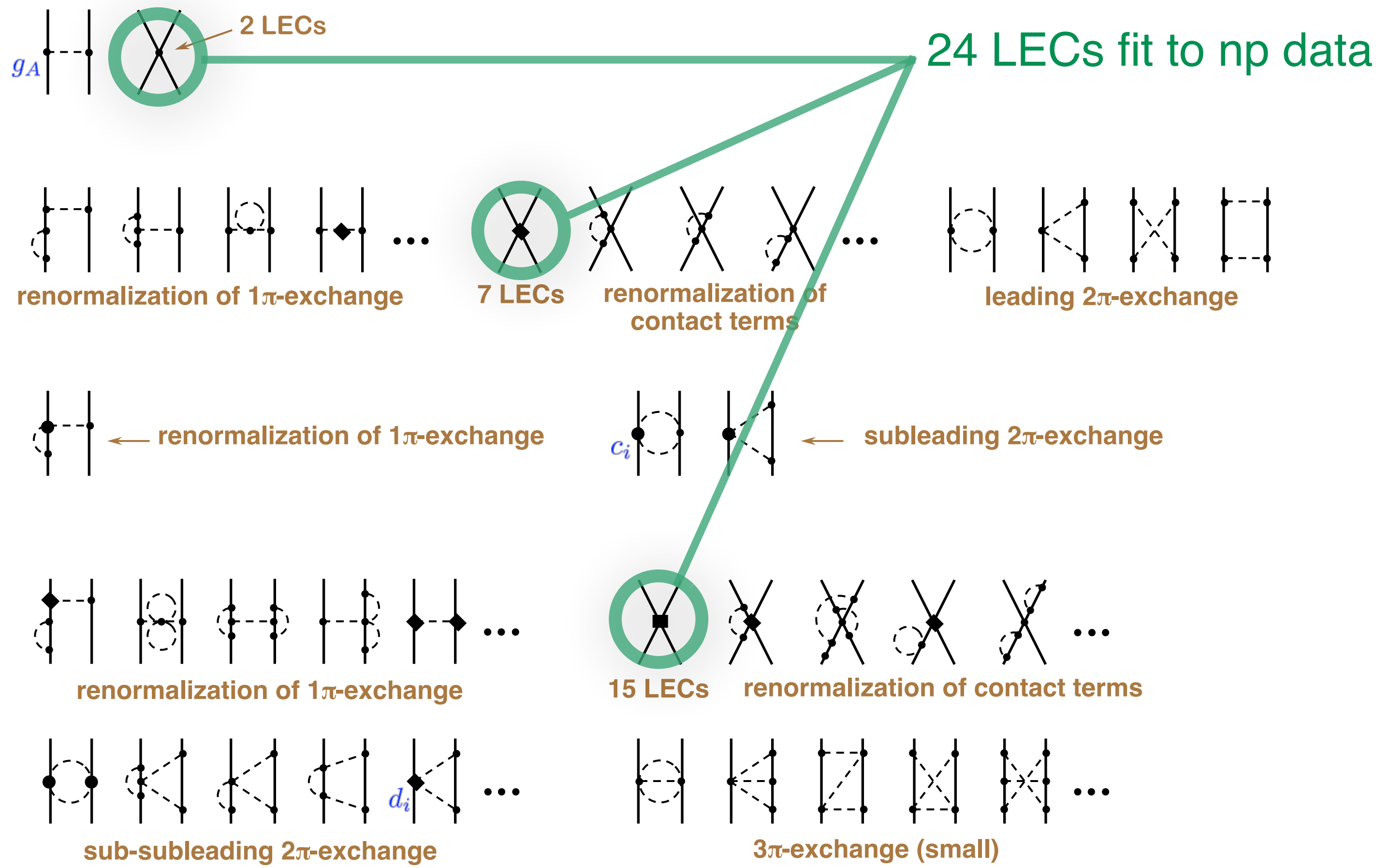
\+ isospin-breaking corrections…

van Kolck et al. '93, '96; Friar et al. '99, '03, '04; Niskanen '02; Kaiser '06; E.E. et al. '04,'05,'07; …

Nucleon-nucleon force up to N^{3}LO

Ordonez et al. ’94; Friar & Coon ’94; Kaiser et al. ’97; E.E. et al. ’98,‘03; Kaiser ’99-’01; Higa, Robilotta ’03; …

- **LO (Q^0):**
- **NLO (Q^2):**
- **N^{2}LO (Q^3):**
- **N^{3}LO (Q^4):**



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Nucleon-nucleon force up to N^{3}LO

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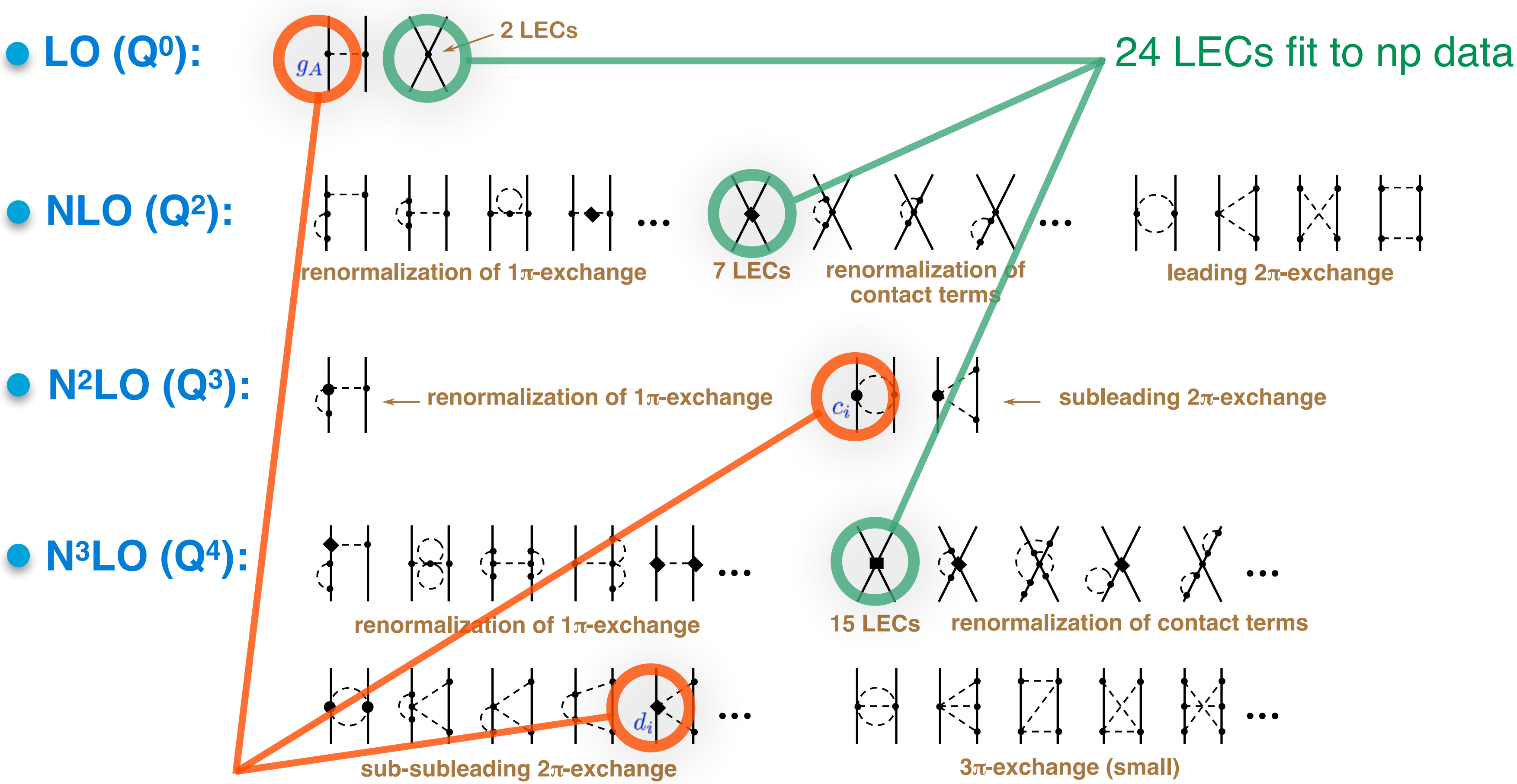


LECs fixed from πN

➙ long-range tail of the nuclear force fixed by chiral symmetry and exp. information on the πN system

+ isospin-breaking corrections…

van Kolck et al. '93, '96; Friar et al. '99, '03, '04; Niskanen '02; Kaiser '06; E.E. et al. '04,'05,'07; …

Chiral 2π exchange (upto N²LO)

$$\begin{aligned}\mathcal{V}_{NN} &= V_C(r) + \vec{\tau}_1 \cdot \vec{\tau}_2 W_C(r) \;+\; \left[V_S(r) + \vec{\tau}_1 \cdot \vec{\tau}_2 W_S(r)\right] \vec{\sigma}_1 \cdot \vec{\sigma}_2 \\ &+ \left[V_T(r) + \vec{\tau}_1 \cdot \vec{\tau}_2 W_T(r)\right] (3\vec{\sigma}_1 \cdot \hat{r}\vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2) \;+\; \left[V_{LS}(r) + \vec{\tau}_1 \cdot \vec{\tau}_2 W_{LS}(r)\right] \vec{L} \cdot \vec{S}\,,\end{aligned}$$

Chiral 2π exchange (upto N²LO)

$$\mathcal{V}_{NN} = V_C(r) + \vec{\tau}_1\cdot\vec{\tau}_2 W_C(r) + \left[V_S(r) + \vec{\tau}_1\cdot\vec{\tau}_2 W_S(r)\right]\vec{\sigma}_1\cdot\vec{\sigma}_2$$
$$+ \left[V_T(r) + \vec{\tau}_1\cdot\vec{\tau}_2 W_T(r)\right](3\vec{\sigma}_1\cdot\hat{r}\vec{\sigma}_2\cdot\hat{r} - \vec{\sigma}_1\cdot\vec{\sigma}_2) + \left[V_{LS}(r) + \vec{\tau}_1\cdot\vec{\tau}_2 W_{LS}(r)\right]\vec{L}\cdot\vec{S},$$

The profile functions (in Dimensional Regularization)

$$V_C^{TPE}(r) = \frac{3g^2m^6}{32\pi^2f^4}\frac{e^{-2x}}{x^6}\left\{\left(2c_1 + \frac{3g^2}{16M}\right)x^2(1+x)^2 + \frac{g^5x^5}{32M} + \left(c_3 + \frac{3g^2}{16M}\right)(6+12x+10x^2+4x^3+x^4)\right\}$$

$$W_T^{TPE}(r) = \frac{g^2m^6}{48\pi^2f^4}\frac{e^{-2x}}{x^6}\left\{-\left(c_4 + \frac{1}{4M}\right)(1+x)(3+3x+x^2) + \frac{g^2}{32M}(36+72x+52x^2+17x^3+2x^4)\right\},$$

$$V_T^{TPE}(r) = \frac{g^4m^5}{128\pi^3f^4x^4}\left\{-12K_0(2x) - (15+4x^2)K_1(2x) + \frac{3\pi m e^{-2x}}{8Mx}(12x^{-1}+24+20x+9x^2+2x^3)\right\},$$

$$W_C^{TPE}(r) = \frac{g^4m^5}{128\pi^3f^4x^4}\left\{\left[1+2g^2(5+2x^2) - g^4(23+12x^2)\right]K_1(2x) + x\left[1+10g^2 - g^4(23+4x^2)\right]K_0(2x),\right.$$
$$\left. + \frac{g^2m\pi e^{-2x}}{4Mx}\left[2(3g^2-2)(6x^{-1}+12+10x+4x^2+x^3)\right] + g^2x(2+4x+2x^2+3x^2)\right\},$$

$$V_S^{TPE}(r) = \frac{g^4m^5}{32\pi^3f^4}\left\{3xK_0(2x) + (3+2x^2)K_1(2x) - \frac{3\pi m e^{-2x}}{16Mx}(6x^{-1}+12+11x+6x^2+2x^3)\right\},$$

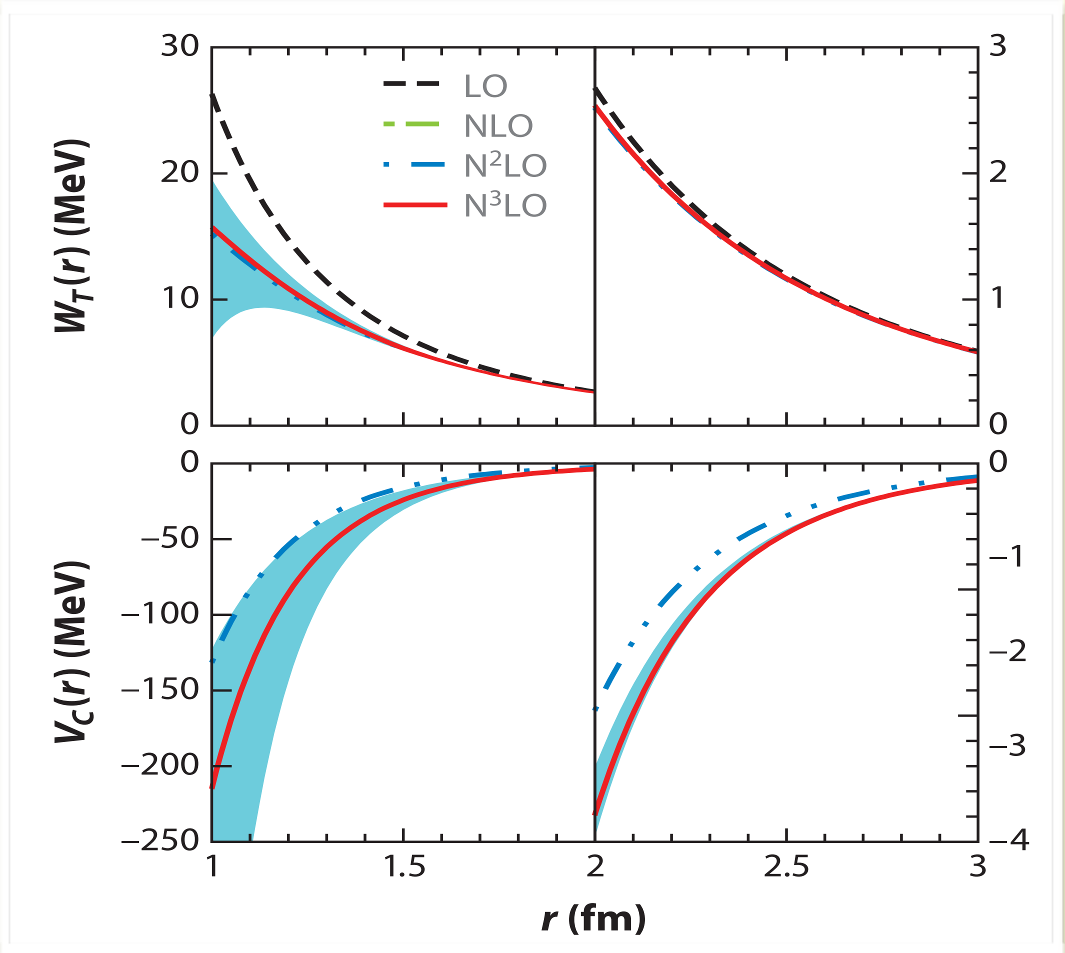
$$W_S^{TPE}(r) = \frac{g^2m^6}{48\pi^2f^4}\frac{e^{-2x}}{x^6}\left\{\left(c_4 + \frac{1}{4M}\right)(1+x)(3+3x+2x^2) - \frac{g^2}{16M}(18+36x+31x^2+14x^3+2x^4)\right\},$$

$$V_{LS}^{TPE}(r) = -\frac{3g^4m^6}{64\pi^2Mf^4}\frac{e^{-2x}}{x^6}(1+x)(2+2x+x^2),$$

$$W_{LS}^{TPE}(r) = \frac{g^2(g^2-1)m^6}{32\pi^2Mf^4}\frac{e^{-2x}}{x^6}(1+x)^2,$$

Chiral 2π exchange (upto N²LO)

$$\mathcal{V}_{NN} = V_C(r) + \vec{\tau}_1 \cdot \vec{\tau}_2 W_C(r) + \left[V_S(r) + \vec{\tau}_1 \cdot \vec{\tau}_2 W_S(r)\right] \vec{\sigma}_1 \cdot \vec{\sigma}_2$$
$$+ \left[V_T(r) + \vec{\tau}_1 \cdot \vec{\tau}_2 W_T(r)\right] (3\vec{\sigma}_1 \cdot \hat{r} \vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2) + \left[V_{LS}(r) + \vec{\tau}_1 \cdot \vec{\tau}_2 W_{LS}(r)\right] \vec{L} \cdot \vec{S},$$

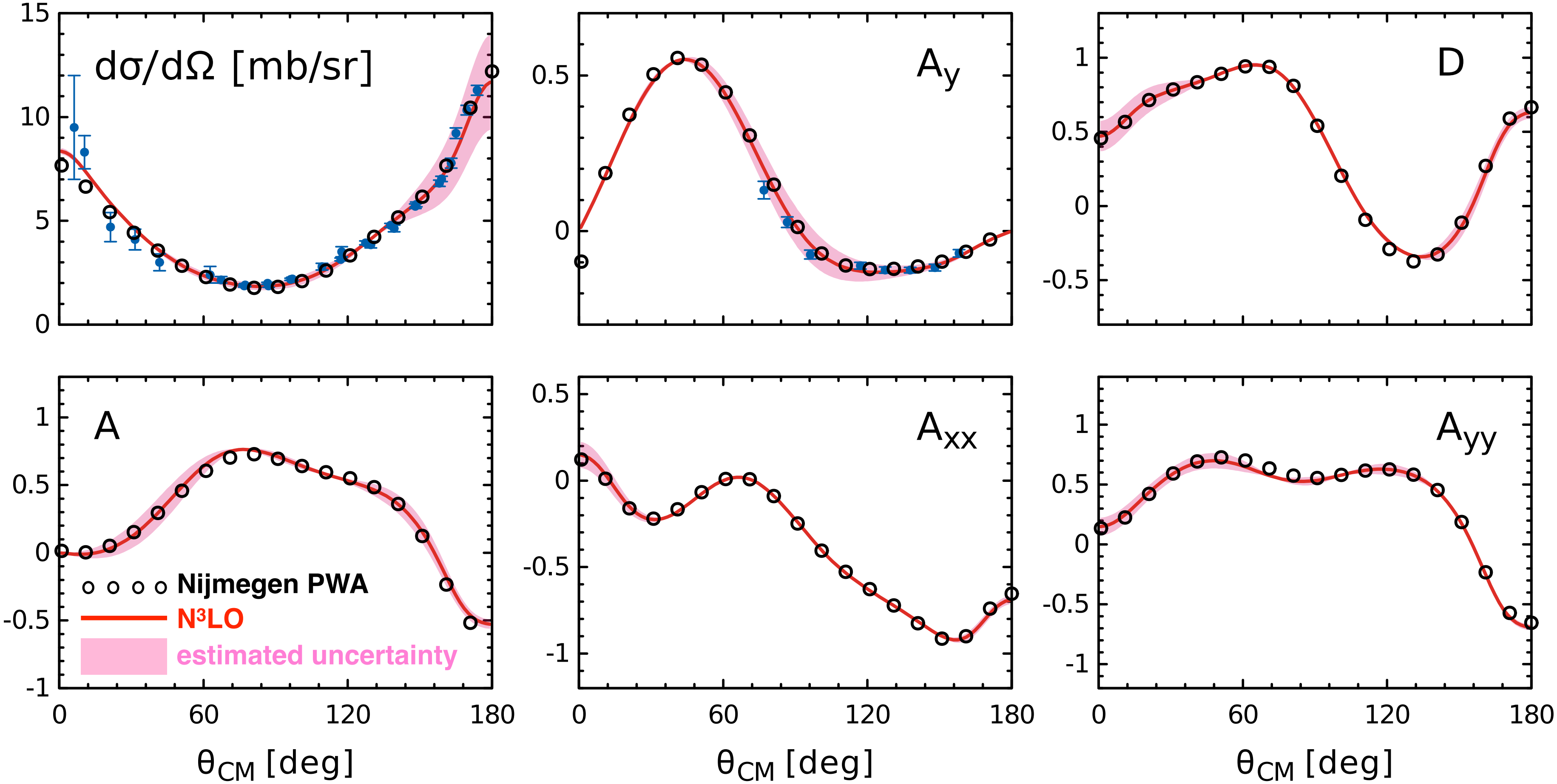


Effects of the chiral 2π exchange are clearly visible in NN phases

Rentmeester et al.’99,‘03
Birse, McGovern ’06

Nucleon-nucleon scattering

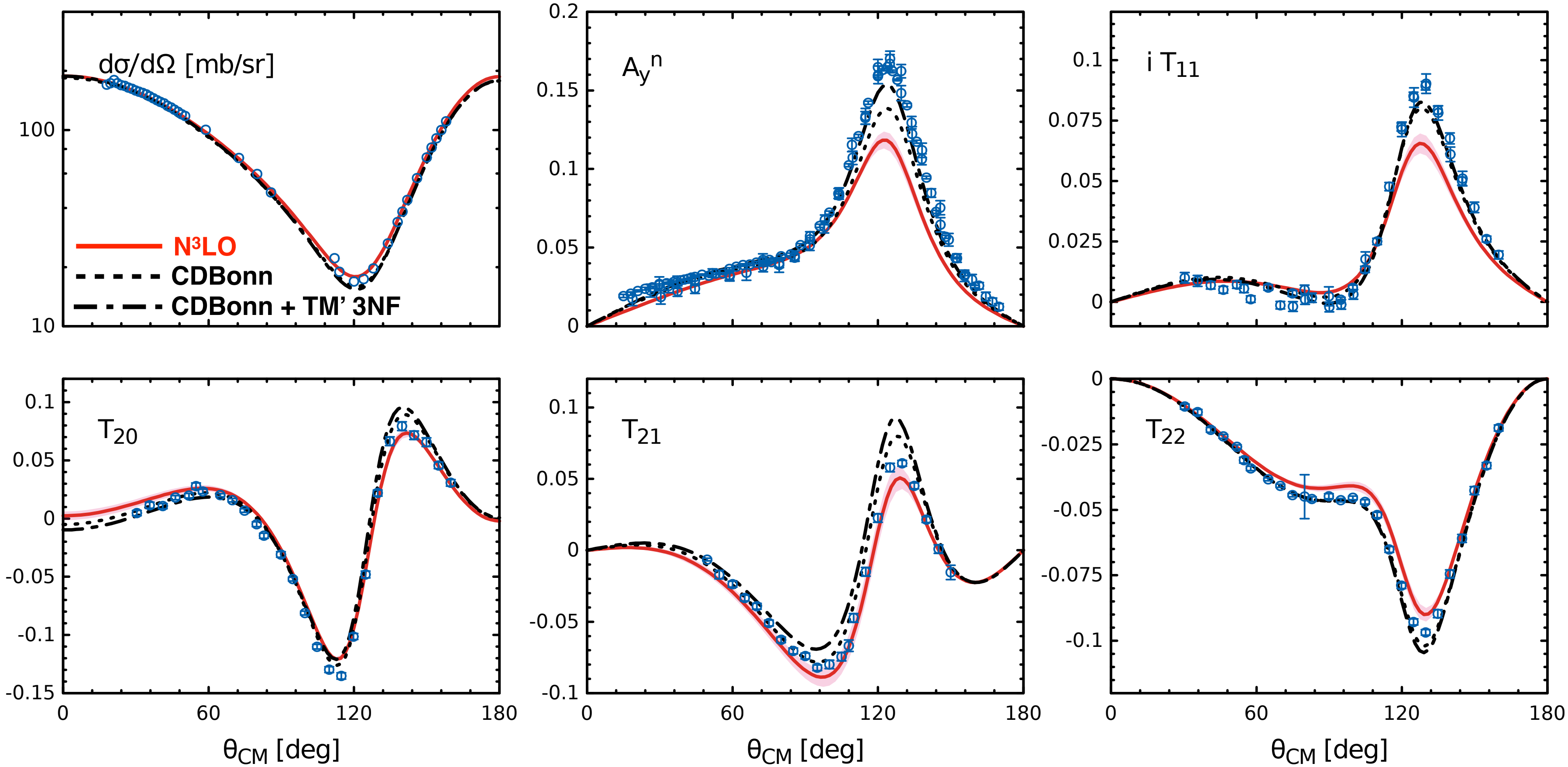
Selected neutron-proton scattering observables at E_{lab} = 200 MeV
(preliminary results with $i_{improved}$-chiral N^3LO potential)



At N^3LO, 2N observables are accurately described up to at least E_{lab} ~ 200 MeV

Elastic Nd scattering with N³LO 2NF

Selected neutron-deuteron scattering observables at E_{lab} = 10 MeV
(preliminary results with $i_{mproved}$-chiral N³LO potential)

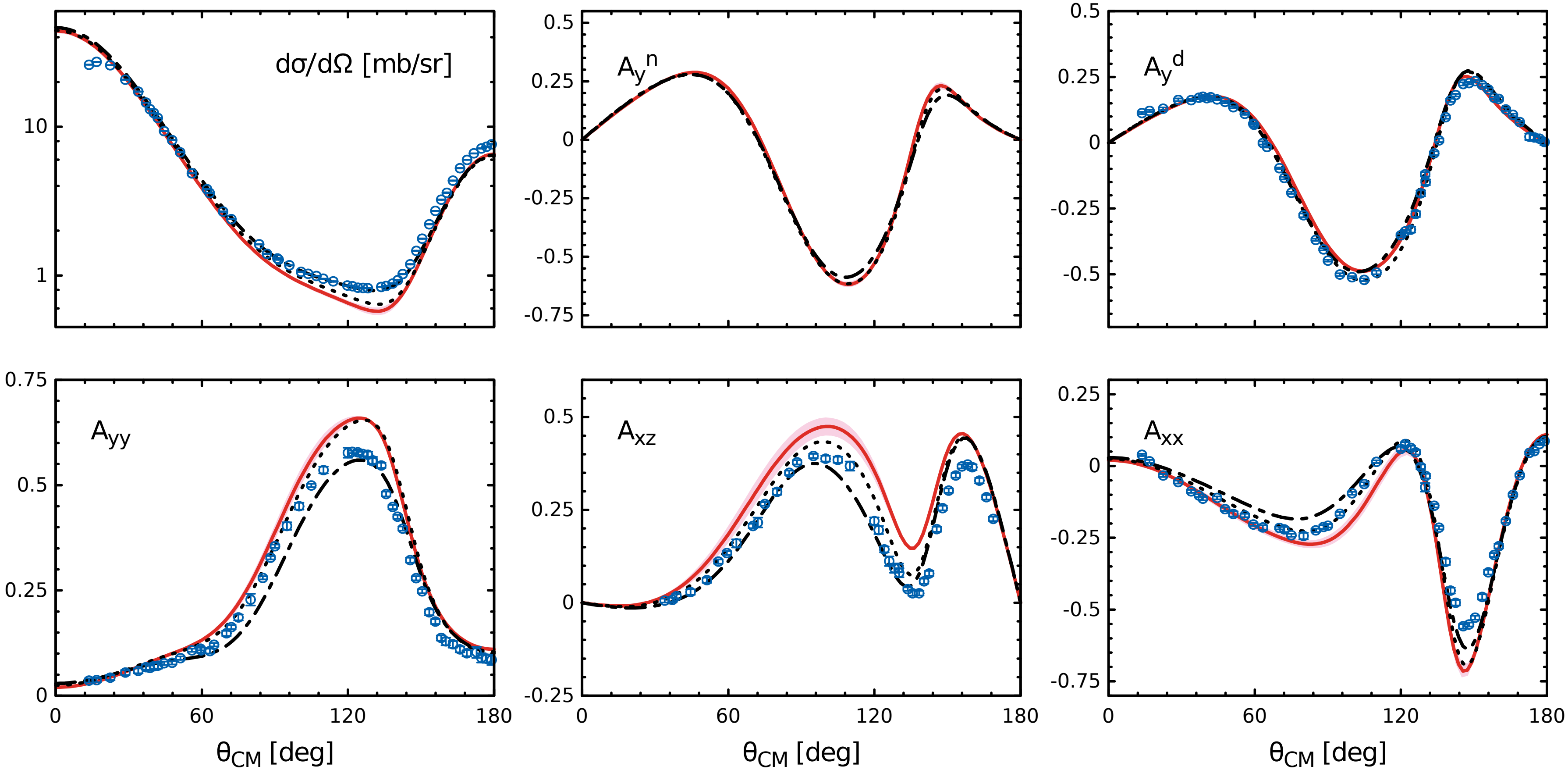


Clear room for 3NF effects in A_y and iT_{11}

Notice: most of the data are Coulomb-corrected pd data

Elastic Nd scattering with N^3LO 2NF

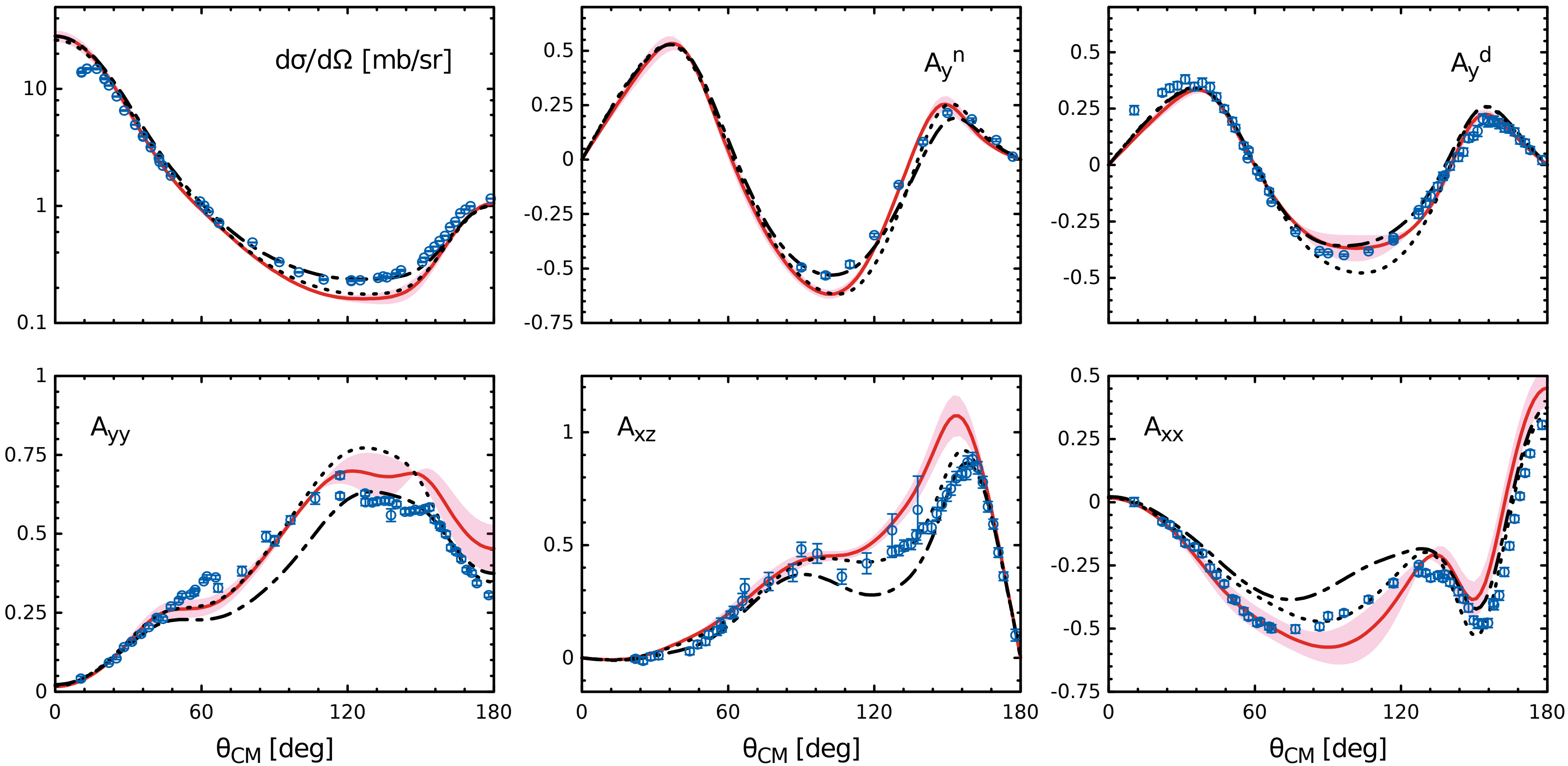
Selected neutron-deuteron scattering observables at E_{lab} = 70 MeV
(preliminary results with $i_{improved}$-chiral N^3LO potential)



Clear room for 3NF effects in the cross section and A_{ij}

Elastic Nd scattering with N³LO 2NF

Selected neutron-deuteron scattering observables at E_{lab} = 135 MeV
(preliminary results with $i_{mproved}$-chiral N³LO potential)



Clear room for 3NF effects

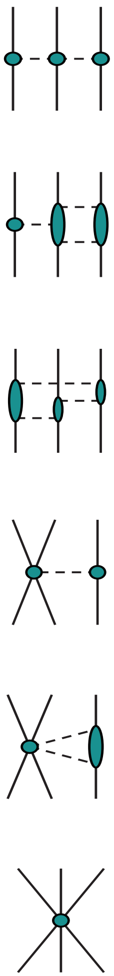
Three-nucleon force:
Status and ongoing developments

Chiral expansion of the 3NF (Δ-less EFT)

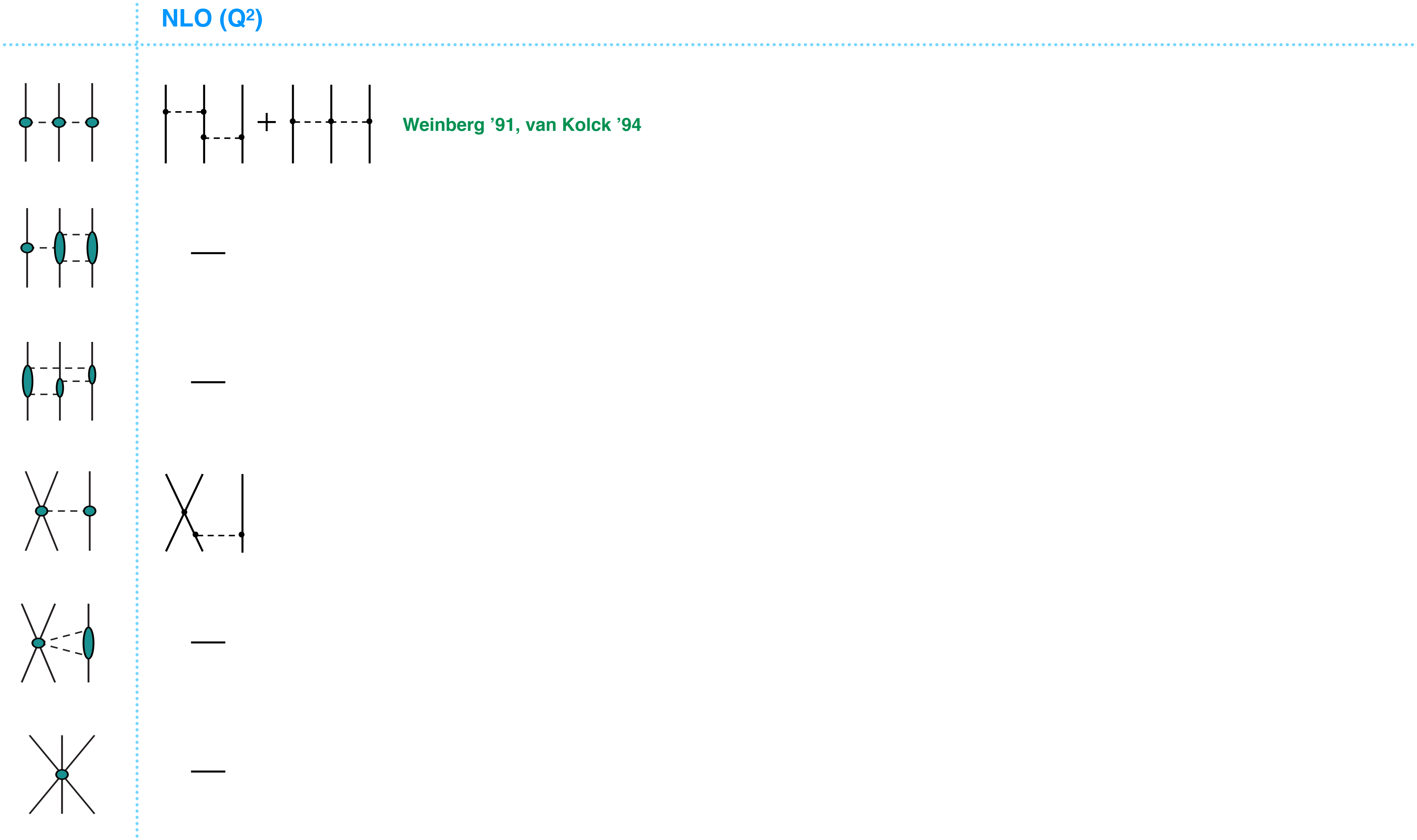
Chiral expansion of the 3NF (Δ-less EFT)

3NF structure functions at large distance are
model-independent and parameter-free predictions
based on χ symmetry of QCD + exp. information on πN system

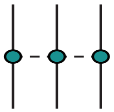
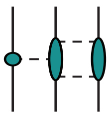
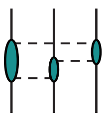
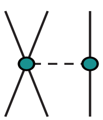
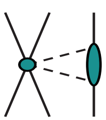
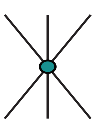
Chiral expansion of the 3NF (Δ-less EFT)



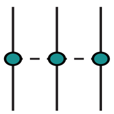
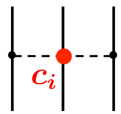
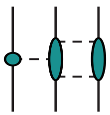
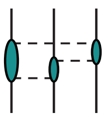
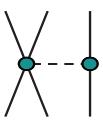
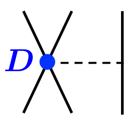
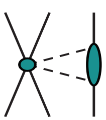
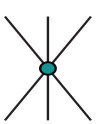
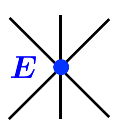
Chiral expansion of the 3NF (Δ-less EFT)



Chiral expansion of the 3NF (Δ-less EFT)

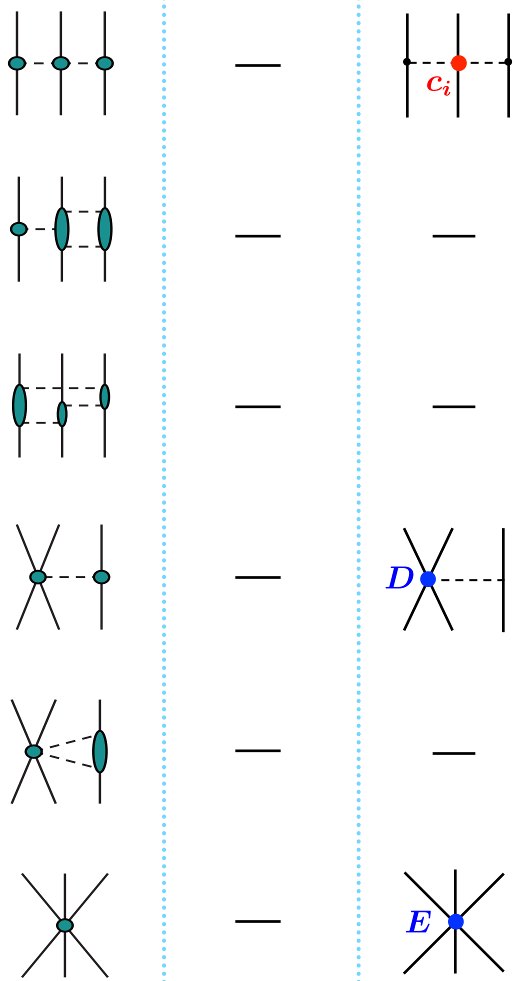
	NLO (Q^2)
	—
	—
	—
	—
	—
	—

Chiral expansion of the 3NF (Δ-less EFT)

	NLO (Q^2)	N^2LO (Q^3)
	—	
	—	—
	—	—
	—	
	—	—
	—	

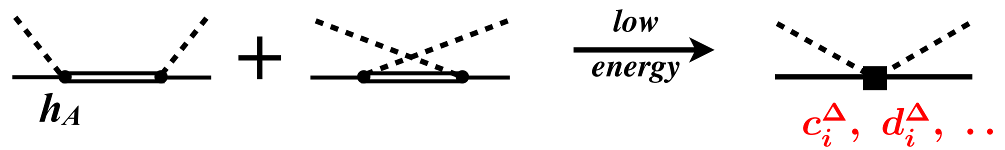
Chiral expansion of the 3NF (Δ-less EFT)

NLO (Q^2) | N^{2}LO (Q^3)



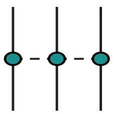
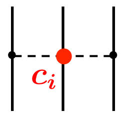
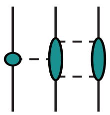
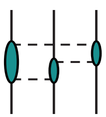
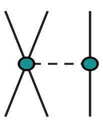
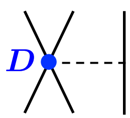
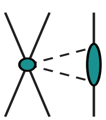
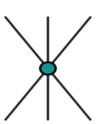
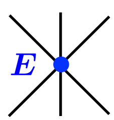
Notice: c_i receive large Δ(1232) contributions

Bernard, Kaiser, Meißner '97

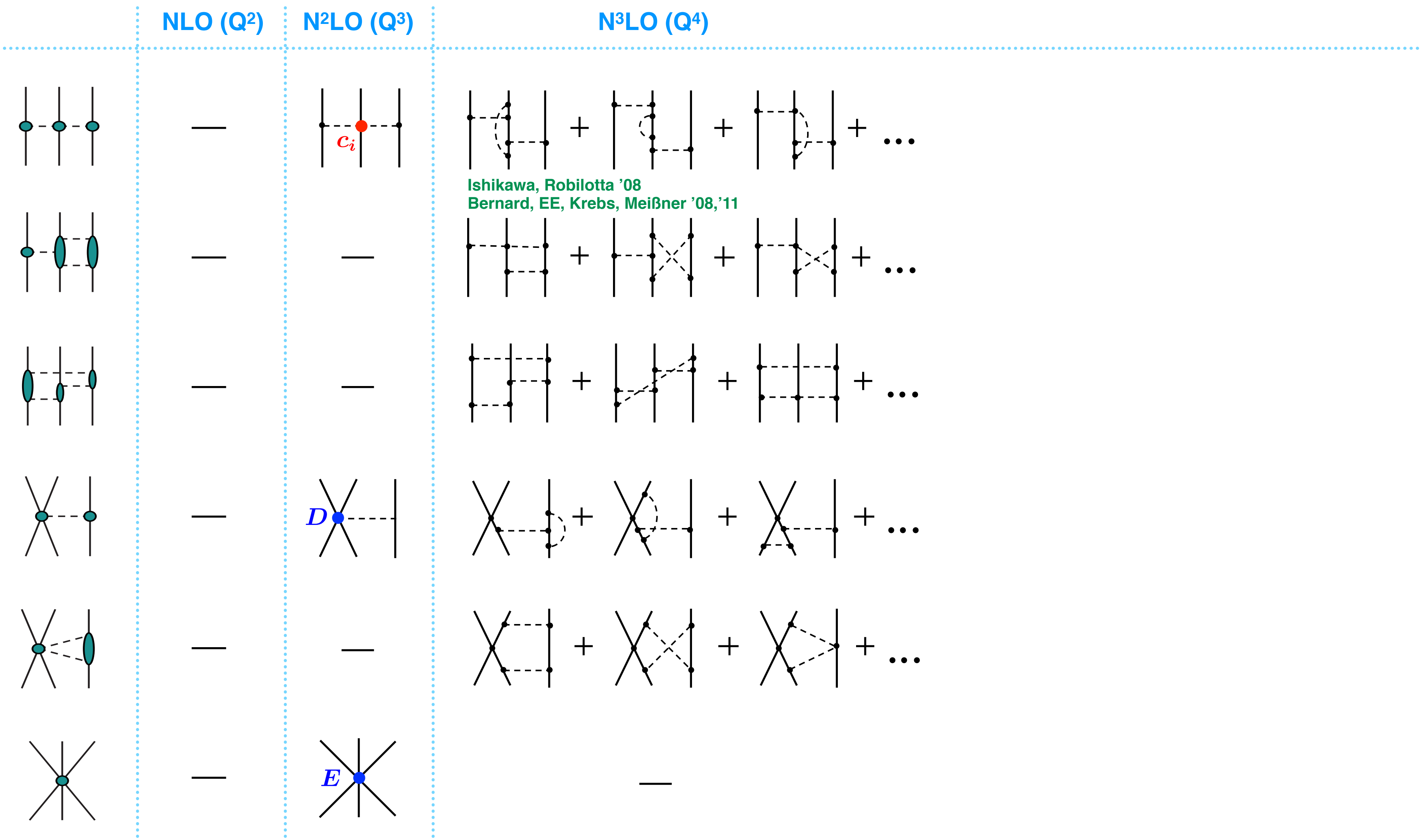


$$c_2^\Delta = -c_3^\Delta = 2c_4^\Delta = \frac{4h_A^2}{9(m_\Delta - m_N)} \simeq 2.8\,\mathrm{GeV}^{-1}$$

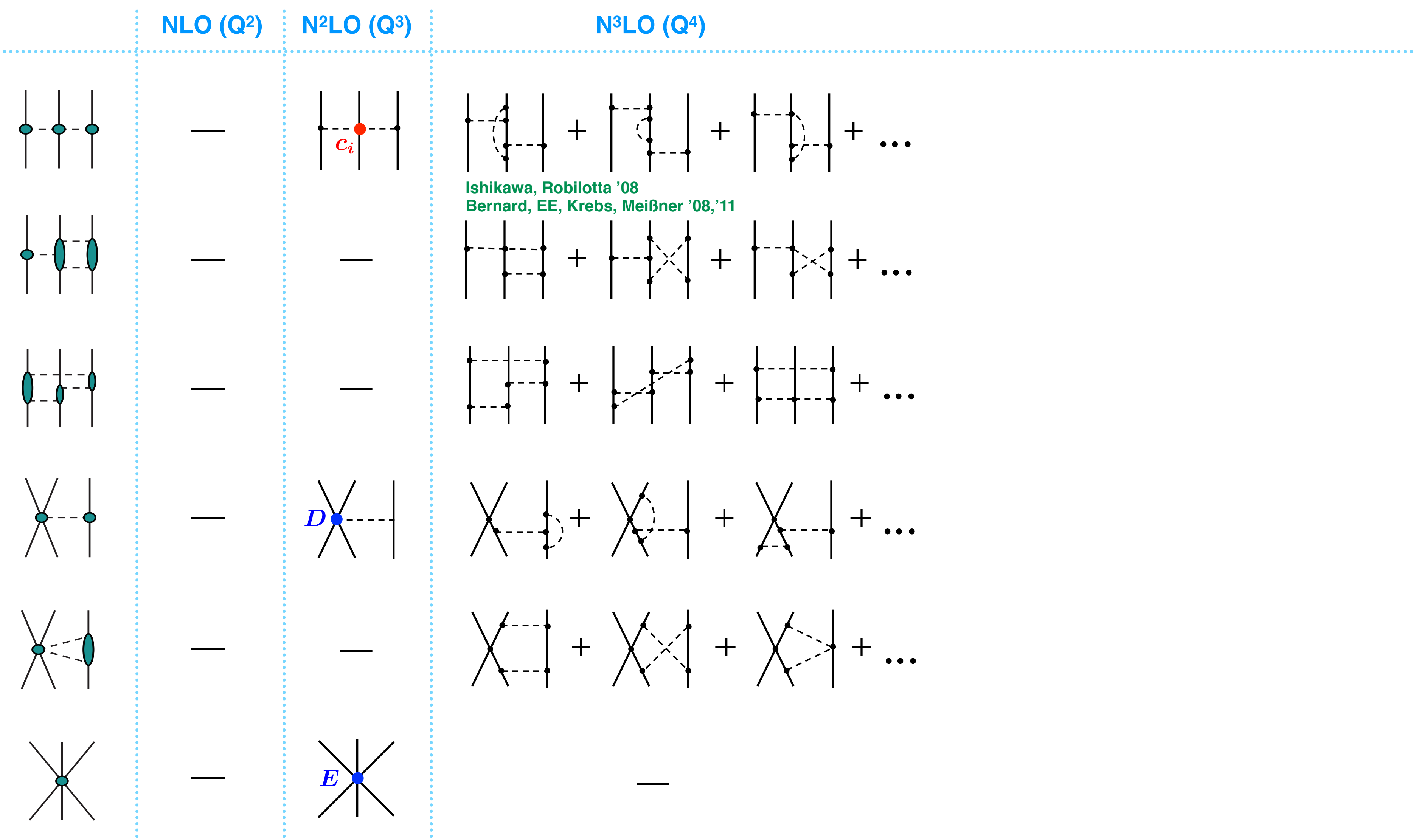
Chiral expansion of the 3NF (Δ-less EFT)

	NLO (Q^2)	N^2LO (Q^3)
	—	
	—	—
	—	—
	—	
	—	—
	—	

Chiral expansion of the 3NF (Δ-less EFT)

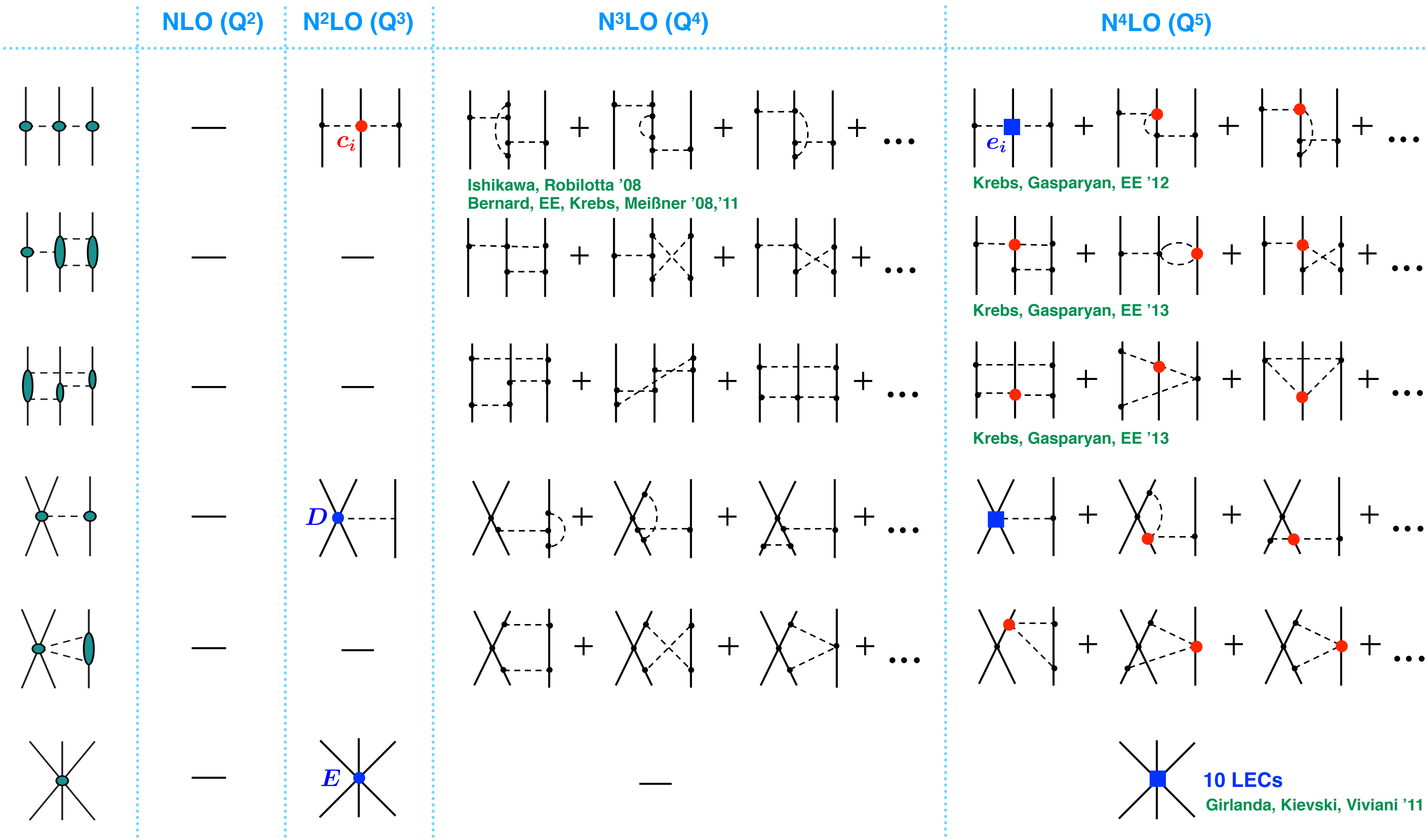


Chiral expansion of the 3NF (Δ-less EFT)



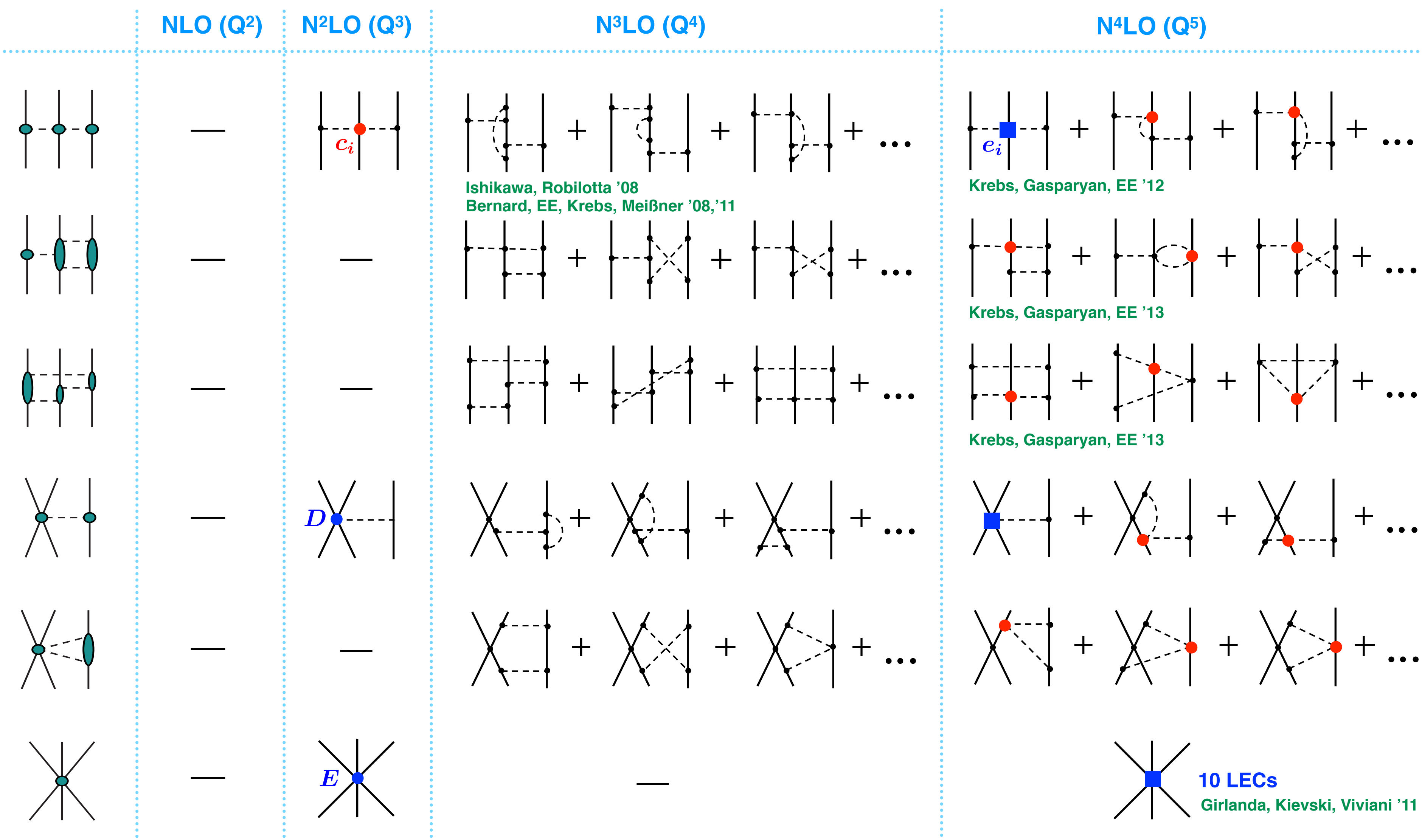
- parameter-free!
- Δ-effects are missing (except for the 2π 3NF) → expect large N^{4}LO corrections

Chiral expansion of the 3NF (Δ-less EFT)



- parameter-free!
- Δ-effects are missing (except for the 2π 3NF) → expect large N^{4}LO corrections

Chiral expansion of the 3NF (Δ-less EFT)

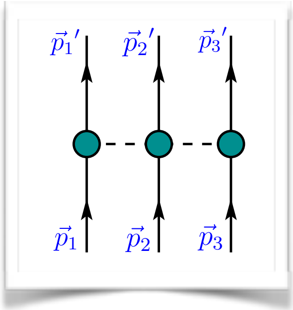


- parameter-free!
- Δ-effects are missing (except for the 2π 3NF) → expect large N^{4}LO corrections

- long range parameter-free (after determination of LECs in πN)
- converged?? (graphs ~ c_i^2, c_i^3…)

Longest-range 3NF up to N⁴LO

The TPE 3NF has the form (modulo 1/m-terms):

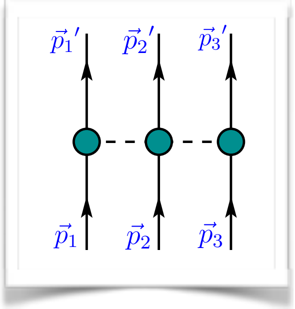


$$V_{2\pi} = \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2]\,[q_3^2 + M_\pi^2]} \Big(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \mathcal{A}(q_2) + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \, \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2 \, \mathcal{B}(q_2) \Big)$$

Longest-range 3NF up to N^{4}LO

The TPE 3NF has the form (modulo 1/m-terms):

$$V_{2\pi} = \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2]\,[q_3^2 + M_\pi^2]} \Big(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \mathcal{A}(q_2) + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \, \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2 \, \mathcal{B}(q_2) \Big)$$

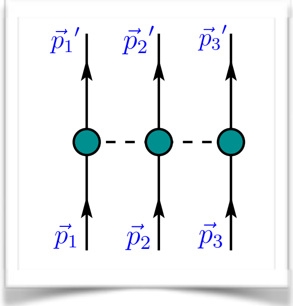


- N^{2}LO [Q^3]: $\mathcal{A}^{(3)}(q_2) = \frac{g_A^2}{8F_\pi^4}\Big((2c_3 - 4c_1)M_\pi^2 + c_3 q_2^2\Big)\,, \qquad \mathcal{B}^{(3)}(q_2) = \frac{g_A^2 c_4}{8F_\pi^4}$

  van Kolck '94

Longest-range 3NF up to N⁴LO

The TPE 3NF has the form (modulo 1/m-terms):



$$V_{2\pi} = \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2]\,[q_3^2 + M_\pi^2]} \Big(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \mathcal{A}(q_2) + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \, \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2 \, \mathcal{B}(q_2) \Big)$$

- N²LO [Q³]: van Kolck '94

$$\mathcal{A}^{(3)}(q_2) = \frac{g_A^2}{8F_\pi^4}\Big((2c_3 - 4c_1)M_\pi^2 + c_3 q_2^2\Big)\,, \qquad \mathcal{B}^{(3)}(q_2) = \frac{g_A^2 c_4}{8F_\pi^4}$$

- N³LO [Q⁴]:

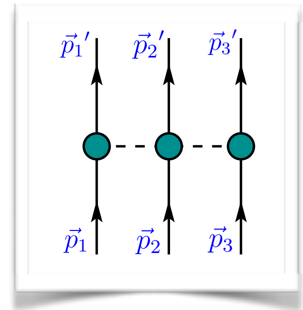
$$\mathcal{A}^{(4)}(q_2) = \frac{g_A^4}{256\pi F_\pi^6}\Big[A(q_2)\left(2M_\pi^4 + 5M_\pi^2 q_2^2 + 2q_2^4\right) + \left(4g_A^2 + 1\right) M_\pi^3 + 2\left(g_A^2 + 1\right) M_\pi q_2^2\Big]\,,$$

$$\mathcal{B}^{(4)}(q_2) = -\frac{g_A^4}{256\pi F_\pi^6}\Big[A(q_2)\left(4M_\pi^2 + q_2^2\right) + (2g_A^2 + 1)M_\pi\Big]$$

Ishikawa, Robilotta '07
Bernard, EE, Krebs, Meißner '08

Longest-range 3NF up to N^{4}LO

The TPE 3NF has the form (modulo 1/m-terms):



$$V_{2\pi} = \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2]\,[q_3^2 + M_\pi^2]} \Big(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \mathcal{A}(q_2) + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \, \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2 \, \mathcal{B}(q_2) \Big)$$

- N^{2}LO [Q^3]: **van Kolck '94**

$$\mathcal{A}^{(3)}(q_2) = \frac{g_A^2}{8F_\pi^4} \Big((2c_3 - 4c_1) M_\pi^2 + c_3 q_2^2 \Big) , \qquad \mathcal{B}^{(3)}(q_2) = \frac{g_A^2 c_4}{8F_\pi^4}$$

- N^{3}LO [Q^4]:

$$\mathcal{A}^{(4)}(q_2) = \frac{g_A^4}{256\pi F_\pi^6} \Big[A(q_2) \left(2M_\pi^4 + 5M_\pi^2 q_2^2 + 2q_2^4 \right) + \left(4g_A^2 + 1 \right) M_\pi^3 + 2 \left(g_A^2 + 1 \right) M_\pi q_2^2 \Big] ,$$

$$\mathcal{B}^{(4)}(q_2) = -\frac{g_A^4}{256\pi F_\pi^6} \Big[A(q_2) \left(4M_\pi^2 + q_2^2 \right) + (2g_A^2 + 1) M_\pi \Big]$$

Ishikawa, Robilotta '07
Bernard, EE, Krebs, Meißner '08

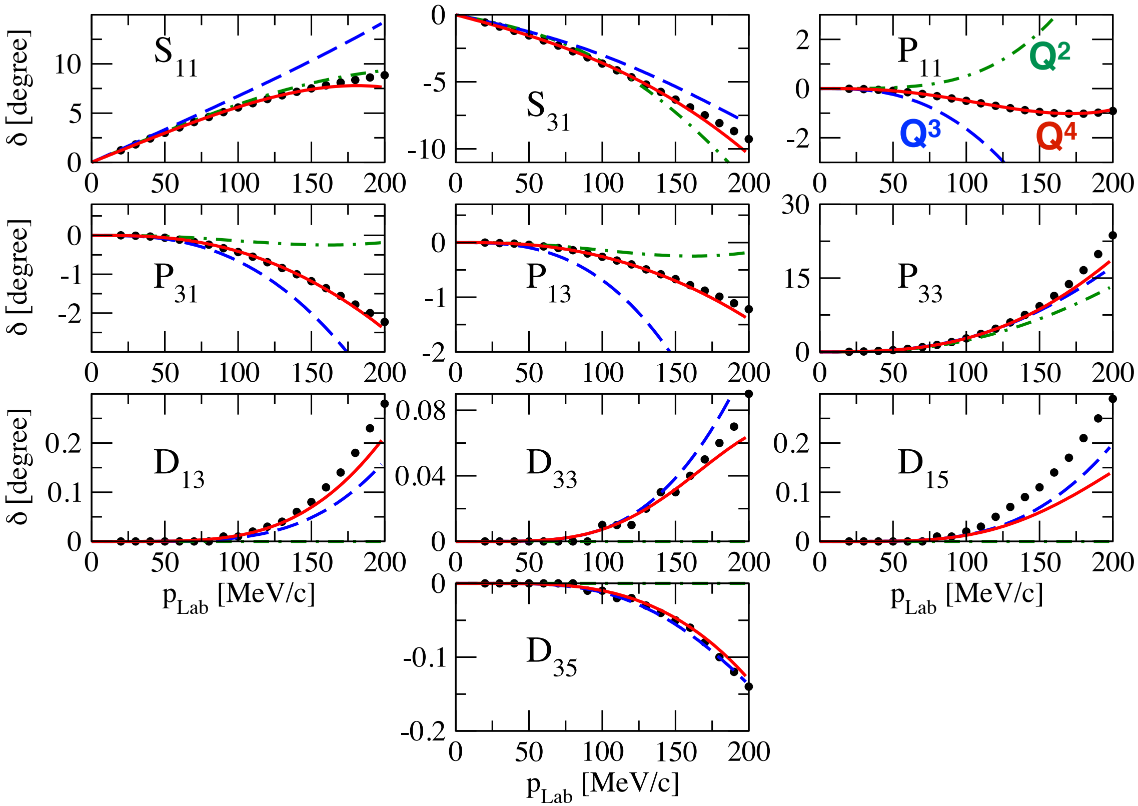
- N^{4}LO [Q^5]: **Krebs, Gasparyan, EE '12**

$$\begin{aligned}
\mathcal{A}^{(5)}(q_2) &= \frac{g_A}{4608\pi^2 F_\pi^6} \Big[M_\pi^2 q_2^2 (F_\pi^2 \left(2304\pi^2 g_A (4\bar{e}_{14} + 2\bar{e}_{19} - \bar{e}_{22} - \bar{e}_{36}) - 2304\pi^2 \bar{d}_{18} c_3 \right) \\
&+ g_A (144 c_1 - 53 c_2 - 90 c_3)) + M_\pi^4 \left(F_\pi^2 \left(4608\pi^2 \bar{d}_{18} (2c_1 - c_3) + 4608\pi^2 g_A (2\bar{e}_{14} + 2\bar{e}_{19} - \bar{e}_{36} - 4\bar{e}_{38}) \right) \right. \\
&+ \left. g_A \left(72 \left(64\pi^2 \bar{l}_3 + 1 \right) c_1 - 24 c_2 - 36 c_3 \right) \right) + q_2^4 \left(2304\pi^2 \bar{e}_{14} F_\pi^2 g_A - 2g_A (5c_2 + 18c_3) \right) \Big] \\
&- \frac{g_A^2}{768\pi^2 F_\pi^6} L(q_2) \left(M_\pi^2 + 2q_2^2 \right) \left(4M_\pi^2 (6c_1 - c_2 - 3c_3) + q_2^2 (-c_2 - 6c_3) \right) , \\
\mathcal{B}^{(5)}(q_2) &= -\frac{g_A}{2304\pi^2 F_\pi^6} \Big[M_\pi^2 \left(F_\pi^2 \left(1152\pi^2 \bar{d}_{18} c_4 - 1152\pi^2 g_A (2\bar{e}_{17} + 2\bar{e}_{21} - \bar{e}_{37}) \right) + 108 g_A^3 c_4 + 24 g_A c_4 \right) \\
&+ q_2^2 \left(5 g_A c_4 - 1152\pi^2 \bar{e}_{17} F_\pi^2 g_A \right) \Big] + \frac{g_A^2 c_4}{384\pi^2 F_\pi^6} L(q_2) \left(4M_\pi^2 + q_2^2 \right)
\end{aligned}$$

Longest-range 3NF up to N^{4}LO

Krebs, Gasparyan, EE ’12

πN phase shifts in HB ChPT up to Q^4 (KH PWA)



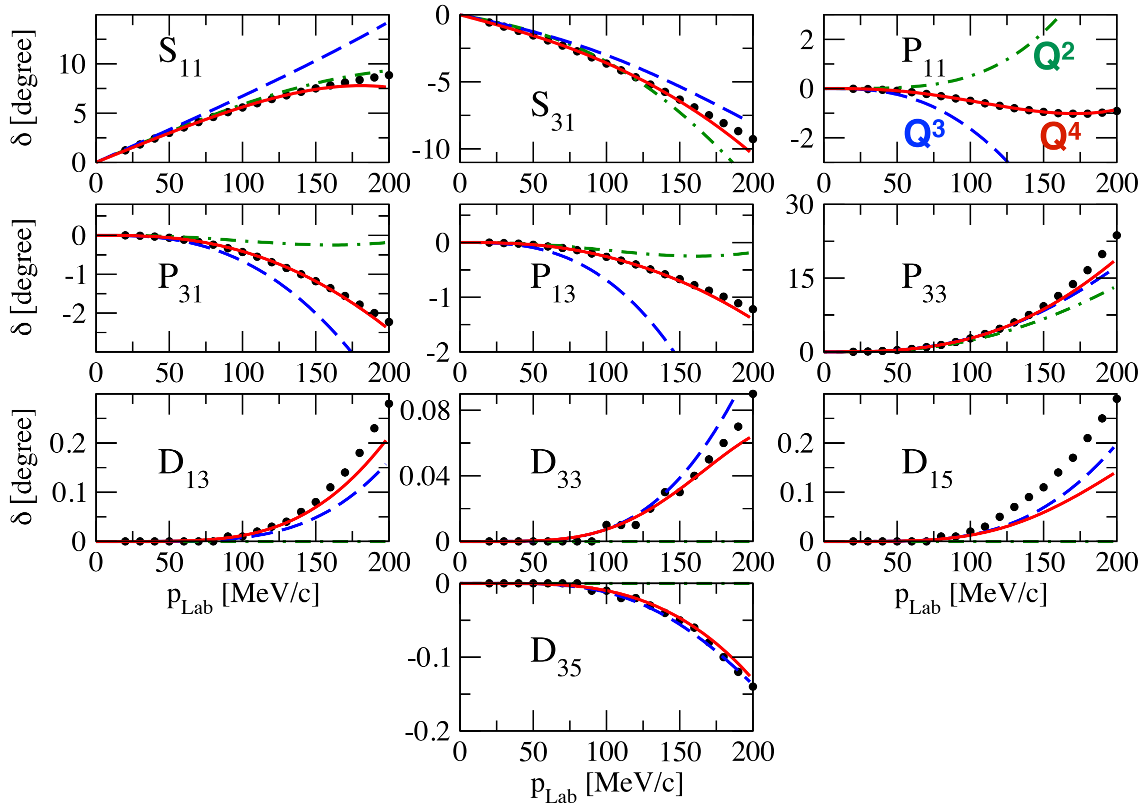
The determined values of LECs

	c_1	c_2	c_3	c_4	$\bar{d}_1+\bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14}-\bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{15}$	$\bar{e}_{16}$	$\bar{e}_{17}$	$\bar{e}_{18}$
Q^4 fit to GW	-1.13	3.69	-5.51	3.71	5.57	-5.35	0.02	-10.26	1.75	-5.80	1.76	-0.58	0.96
Q^4 fit to KH	-0.75	3.49	-4.77	3.34	6.21	-6.83	0.78	-12.02	1.52	-10.41	6.08	-0.37	3.26

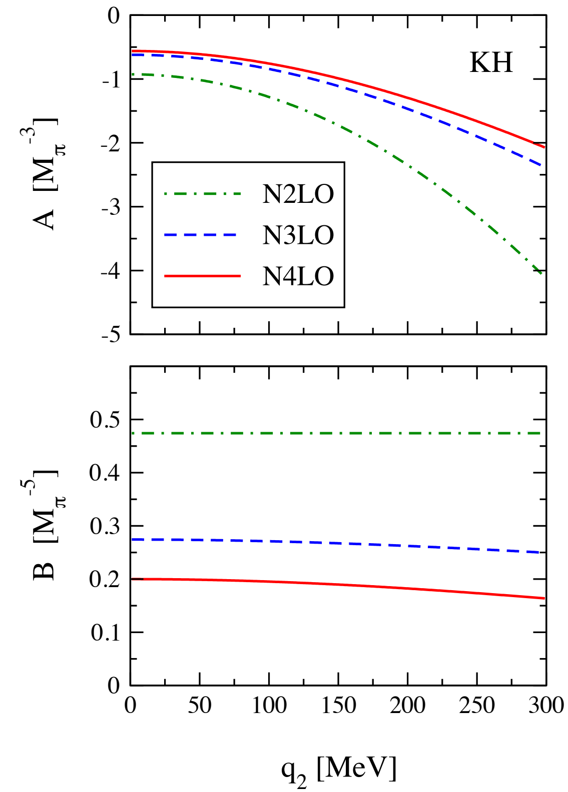
Longest-range 3NF up to N^{4}LO

Krebs, Gasparyan, EE ’12

πN phase shifts in HB ChPT up to Q^4 (KH PWA)



3NF „structure functions“



The determined values of LECs

	c_1	c_2	c_3	c_4	$\bar{d}_1 + \bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14} - \bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{15}$	$\bar{e}_{16}$	$\bar{e}_{17}$	$\bar{e}_{18}$
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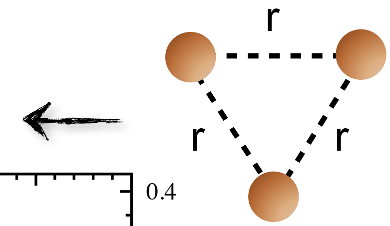
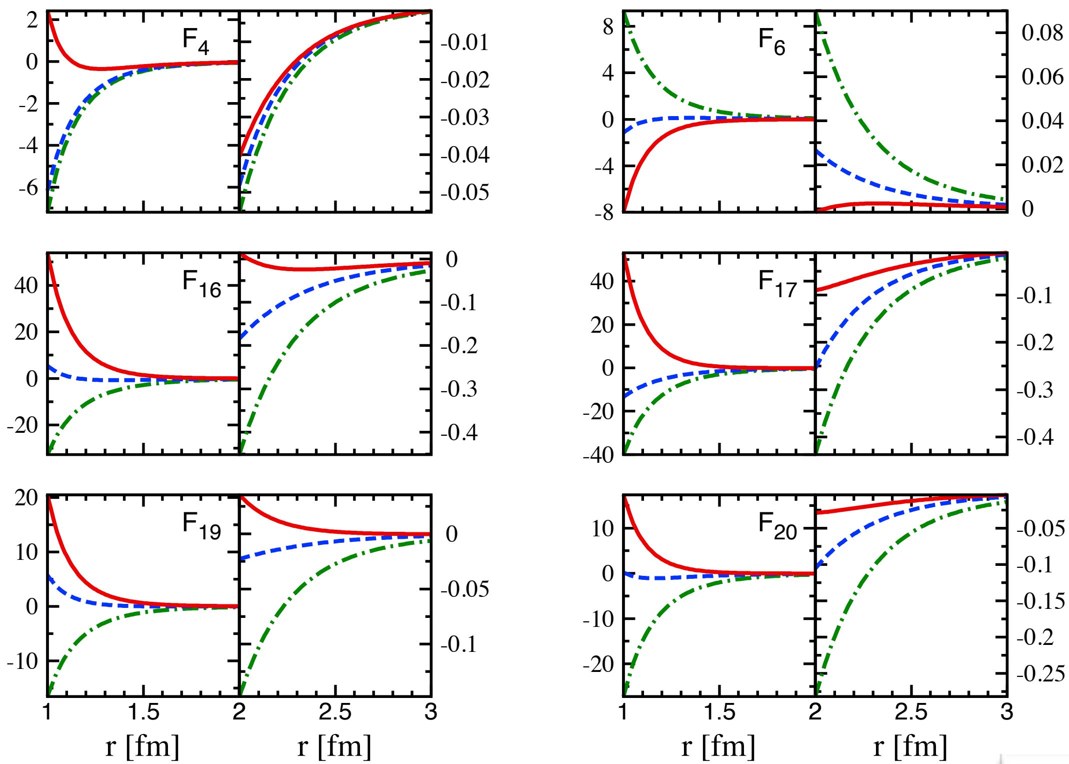
Long-range 3NF up to N⁴LO (preliminary)

EE, Gasparyan, Krebs, Schat, in preparation

Chiral expansion of TPE „structure functions“ F_i (in MeV) in the equilateral-triangle configuration





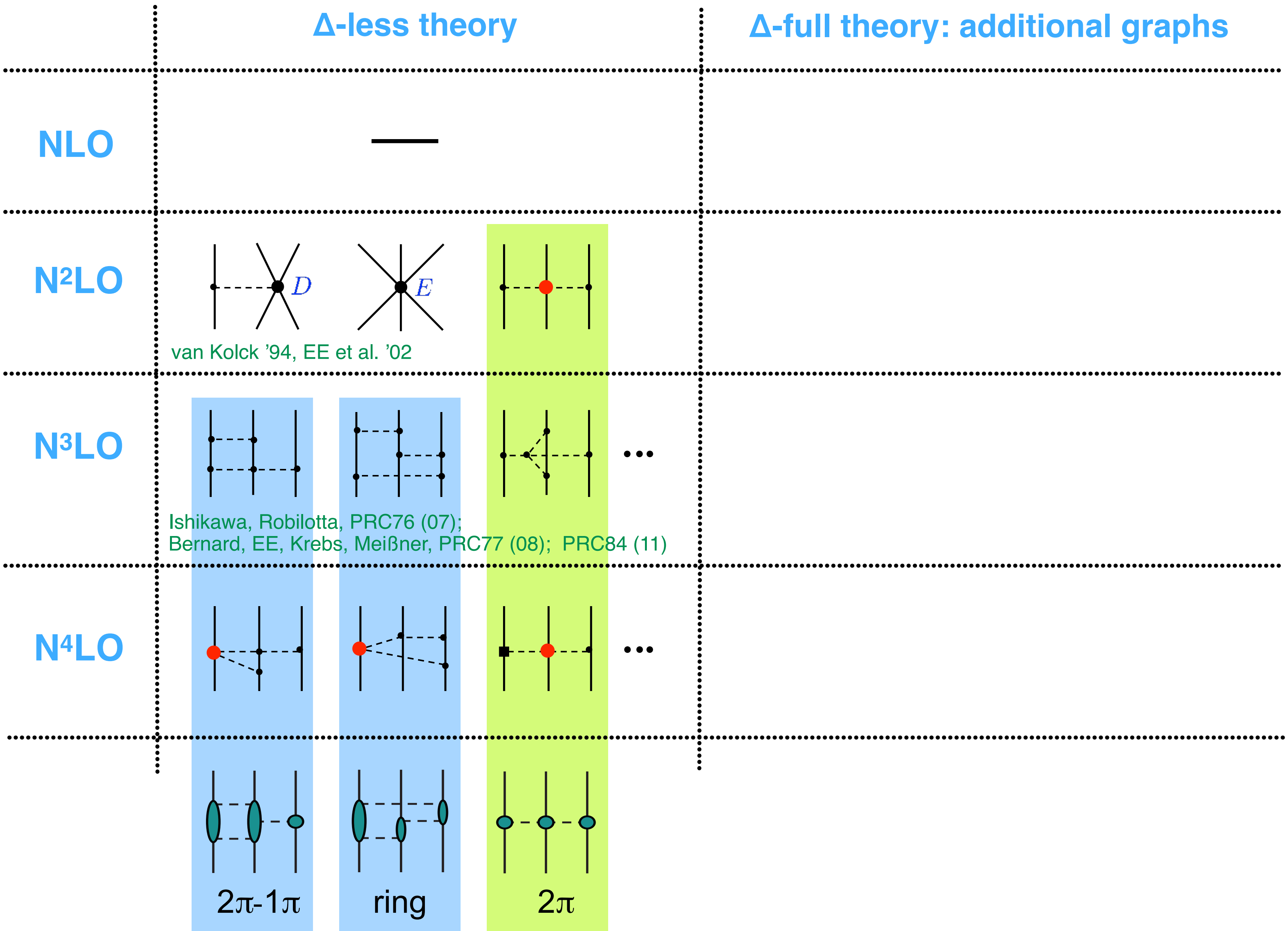
tree level (N^2LO)

+ N^3LO

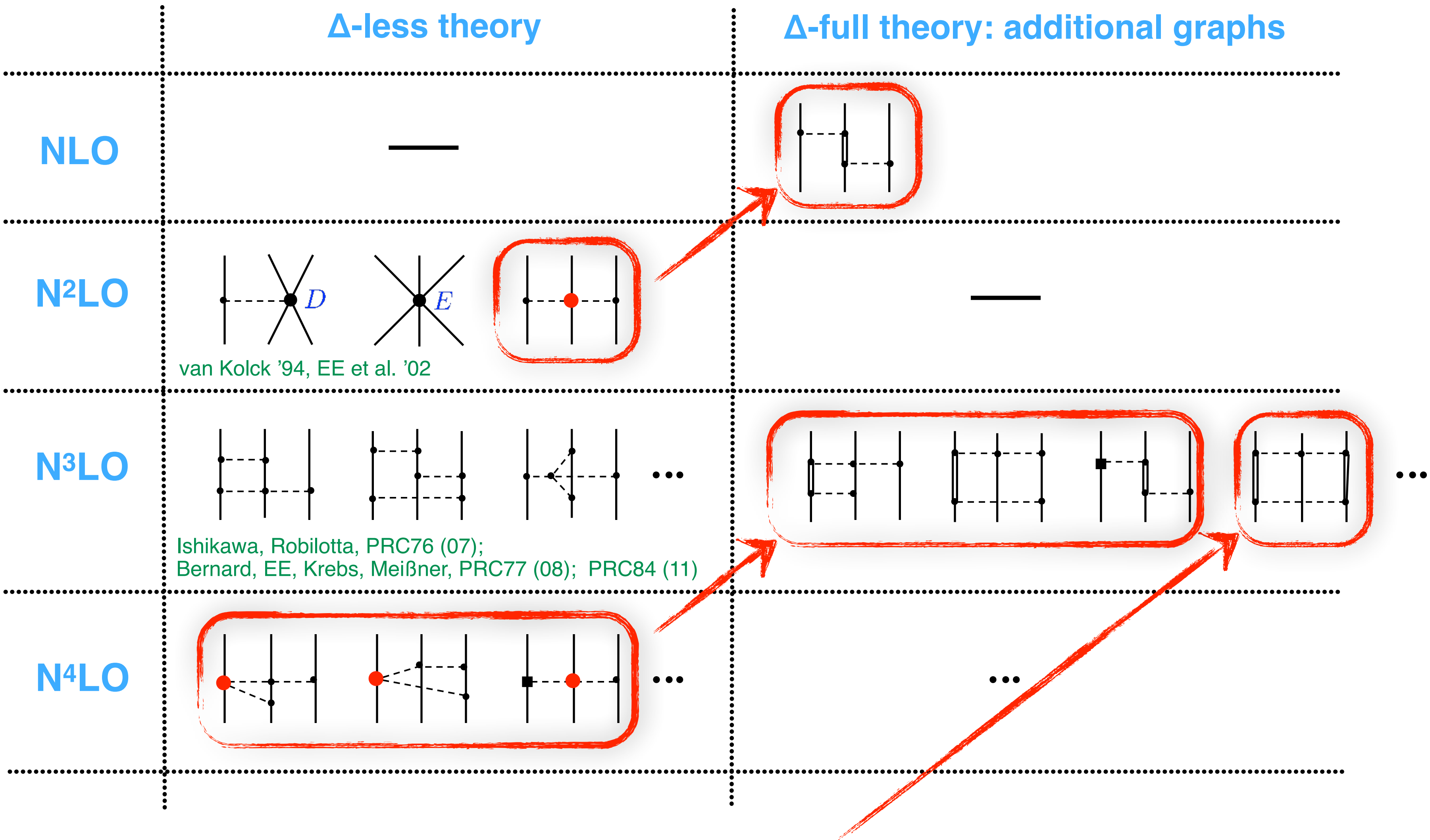
+ N^4LO

- 8 structures out of 20
- N^4LO corrections are large, seem to converge only at very large r

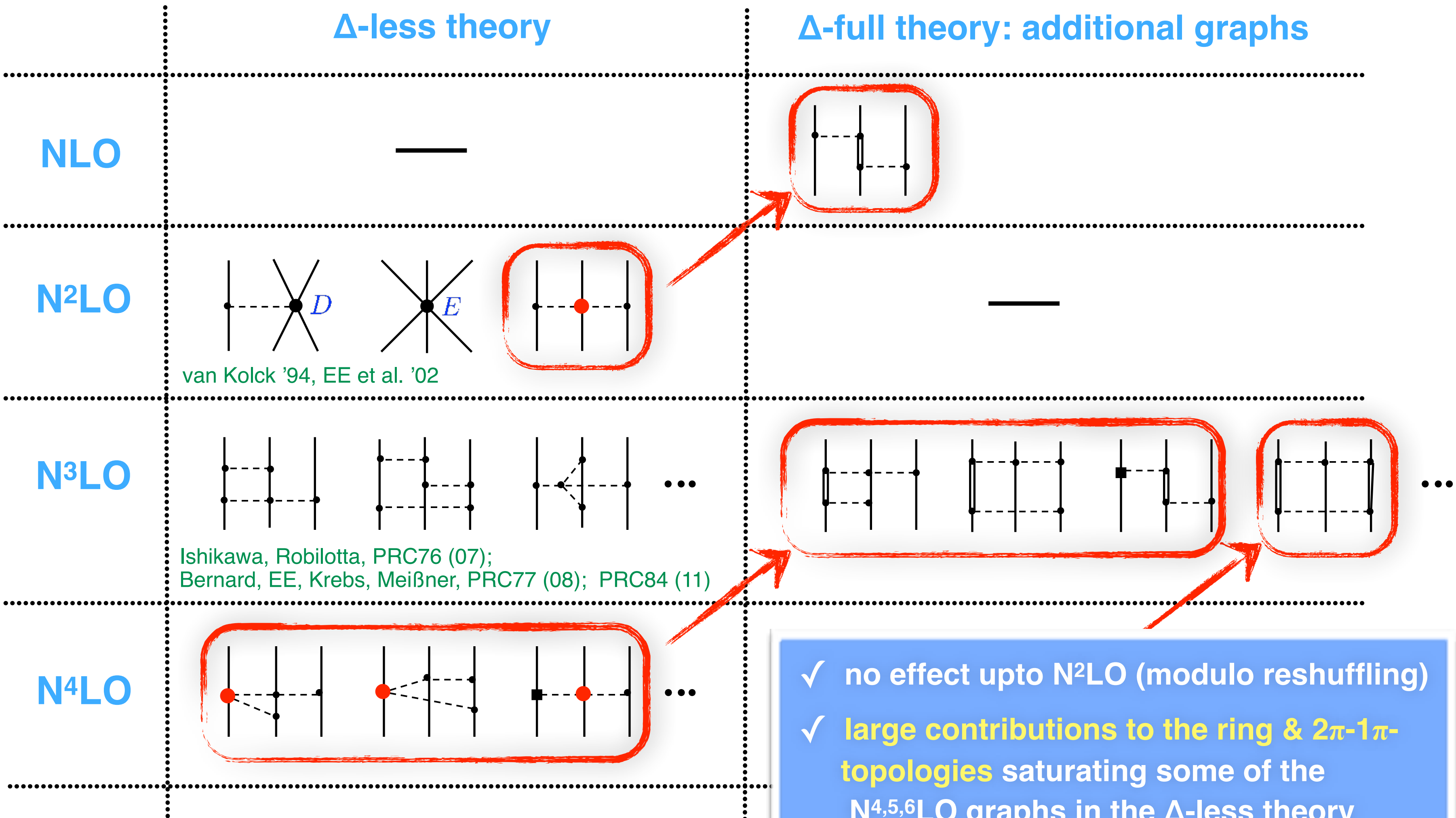
Chiral expansion of the 3NF



Chiral expansion of the 3NF



Chiral expansion of the 3NF

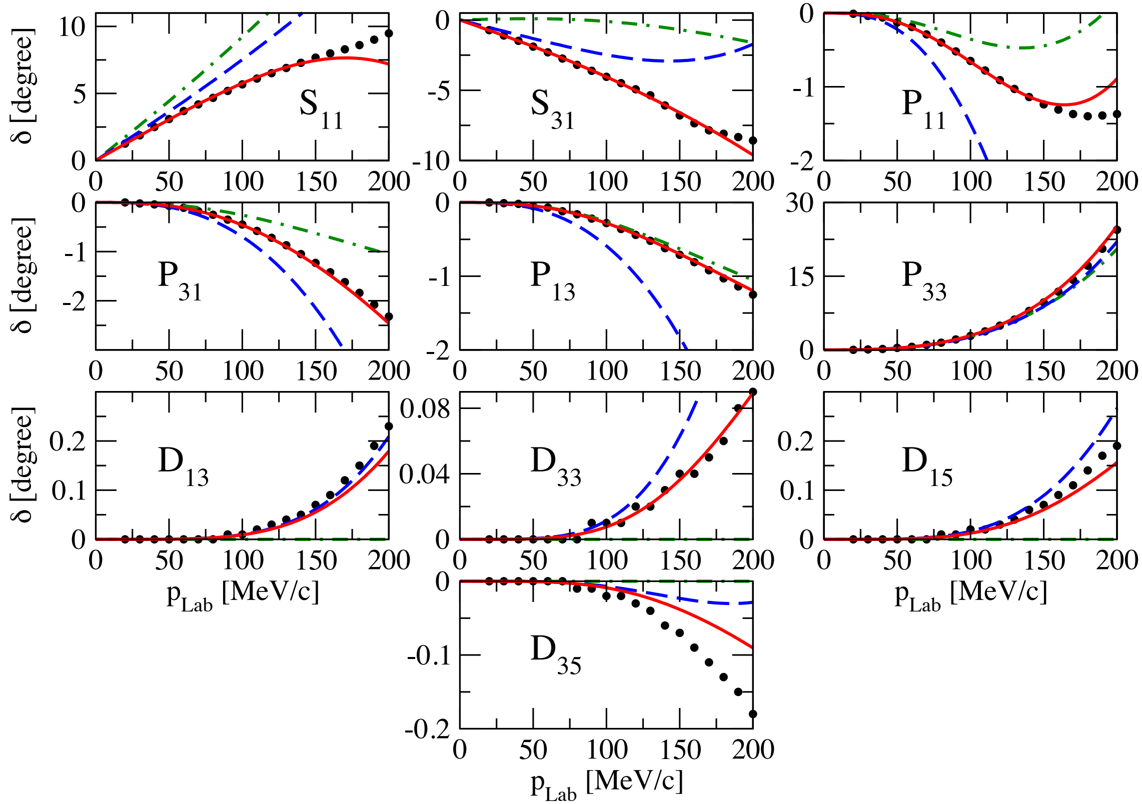


- ✓ no effect upto N^{2}LO (modulo reshuffling)
- ✓ **large contributions to the ring & 2π-1π-topologies** saturating some of the N^{4,5,6}LO graphs in the Δ-less theory
- ✓ What is more efficient: **Δ-less N^{4}LO (and beyond?) vs Δ-full N^{3}LO ??**

Pion-nucleon system in Δ-full EFT up to Q^4

Krebs, Gasparyan, EE, to appear

πN phase shifts in HB ChPT up to Q^4 (KH PWA)

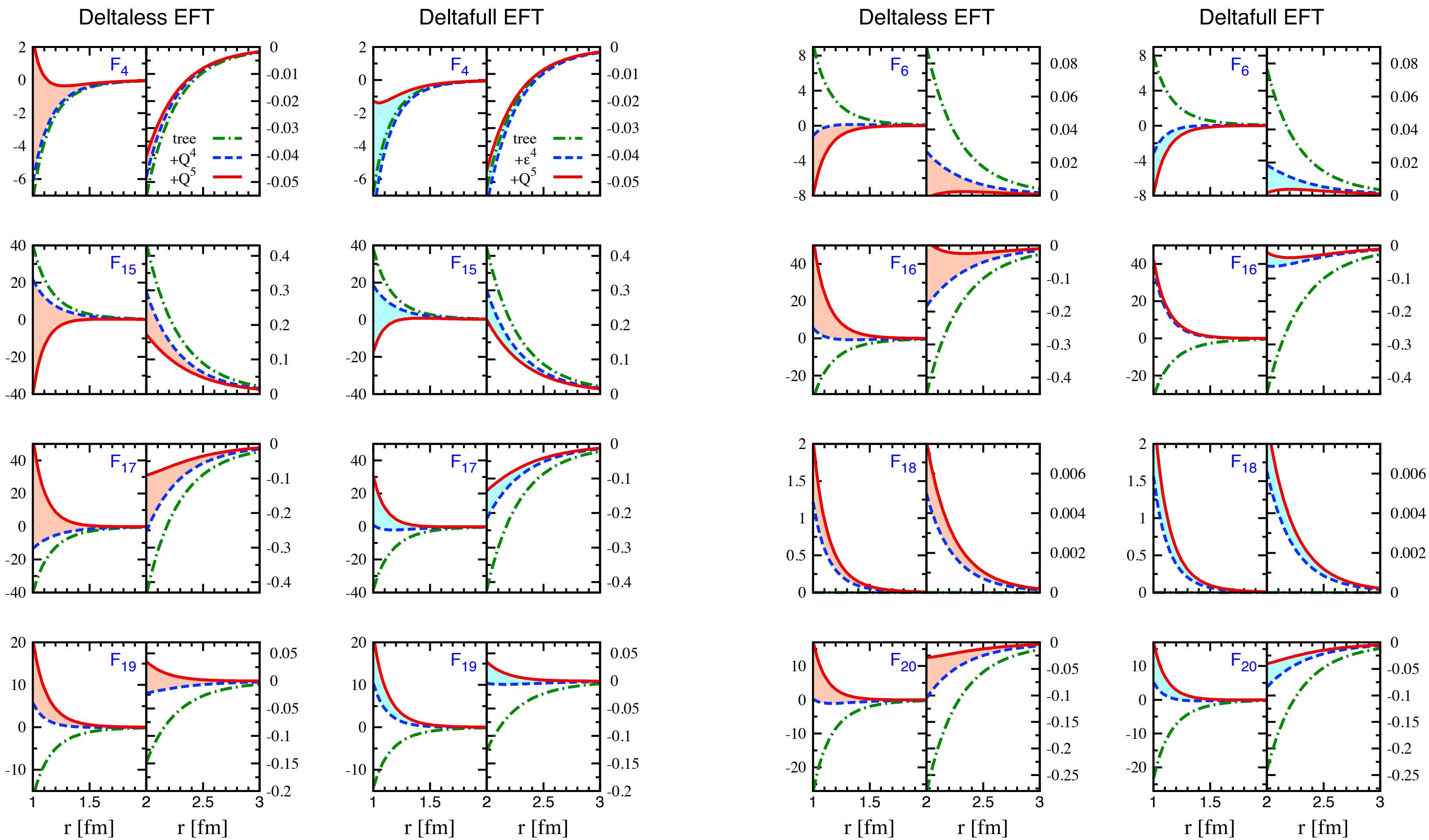


LECs from pion-nucleon scattering (HB ChPT) in units of GeV^{-n} (fit to KH PWA)

	c_1	c_2	c_3	c_4	$\bar{d}_1 + \bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14} - \bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{15}$	$\bar{e}_{16}$	$\bar{e}_{17}$	$\bar{e}_{18}$
Δ-less approach	−0.75	3.49	−4.77	3.34	6.21	−6.83	0.78	−12.02	1.52	−10.41	6.08	−0.37	3.26
Δ-full approach	−0.95	1.90	−1.78	1.50	2.40	−3.87	1.21	−5.25	−0.24	−6.35	2.34	−0.39	2.81
Δ-contribution	0	2.81	−2.81	1.40	2.39	−2.39	0	−4.77	1.87	−4.15	4.15	−0.17	1.32

2π-exchange 3NF: Δ-full vs Δ-less EFT

Krebs, Gasparyan, EE, to appear

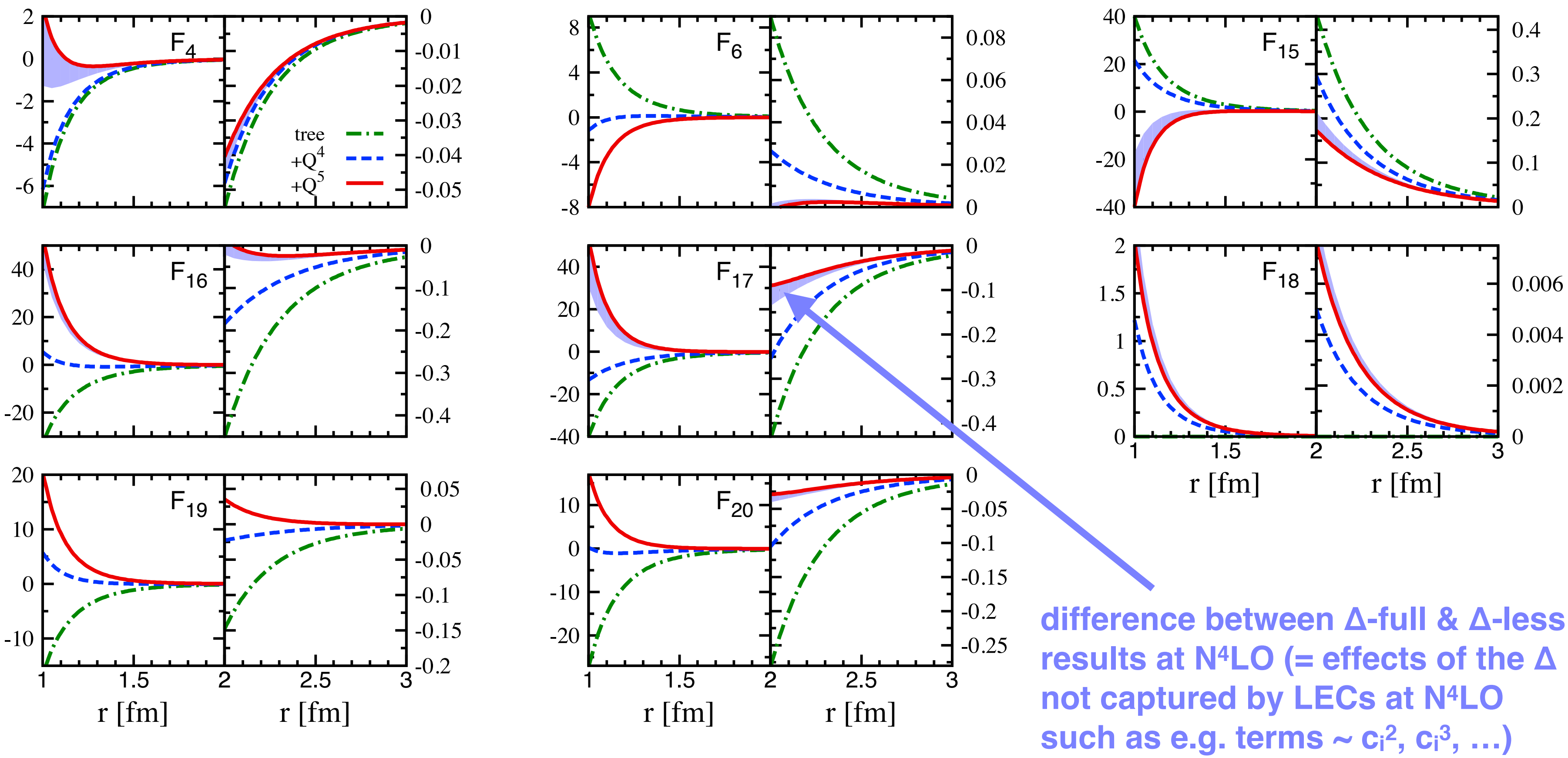


- Δ-full and Δ-less EFT predictions agree well with each other
- Δ-full approach shows clearly a superior convergence
- remarkably, the final 2π 3NF turns out to be rather weak at large distances…

2π-exchange 3NF at N⁴LO in Δ-less EFT

Krebs, Gasparyan, EE, to appear

Chiral expansion of TPE „structure functions" F_i (in MeV) in the equilateral-triangle configuration (Δ-less EFT)



For intermediate-range topologies, the effects of the Δ appear to be more pronounced (work in progress)

Partial wave decomposition of the 3NF

Low Energy Nuclear Physics International Collaboration (LENPIC)

Bochum-Bonn-Cracow-Darmstadt-Iowa-Jülich-Kyushu-Ohio-Orsay

Faddeev equation is solved in the partial wave basis: $|p,q,\alpha\rangle \equiv |pq(ls)j(\lambda\frac{1}{2})I(jI)JM_J\rangle\,|(t\frac{1}{2})TM_T\rangle$

Too many terms for doing PWD "manually" ⟶ let computer do the job…

Golak et al. EPJA 43 (2010) 241

$$\underbrace{\langle p'q'\alpha'|V|pq\alpha\rangle}_{\text{matrix} \sim 10^5 \times 10^5} = \int \underbrace{d\hat{p}'\,d\hat{q}'\,d\hat{p}\,d\hat{q}}_{\text{can be reduced to 5 dim. integral}} \sum_{m_l,\dots} \Big(\text{CG coeffs.}\Big)\Big(Y_{l,m_l}(\hat{p})\,Y_{l',m_l'}(\hat{p}')\,\dots\Big)\underbrace{\langle m_{s_1}'m_{s_2}'m_{s_3}'|V|m_{s_1}m_{s_2}m_{s_3}\rangle}_{\text{depends on } \vec{p},\,\vec{q},\,\vec{p}\,',\,\vec{q}\,',\text{ spin \& isospin}}$$

⟶ feasible task but requires a few 10 MCPU hours for the N^{3}LO 3NF…

It is possible to reduce the number of integrations to 2 exploiting locality of the 3NF Krebs, Hebeler

⟶ no need for supercomputers!

Current status

- PW matrix elements of the 3NF without regulator and with the old nonlocal regulator are available
- PWD of r-space regularized 3NF consistent with the new **i$_{\text{chiral}}$-potential** in progress

Summary and outlook

Improved chiral NN potential up to N^3LO

- Better performance at high energies, no fine tuning in πN LECs, no need for additional spectral function regularization, careful error estimation…
- Application to elastic Nd scattering shows clearly the need for 3NF (most striking at energies of $E_{lab} \sim 70 \ldots 150$ MeV)

Chiral three-nucleon force

- Worked out completely at N^3LO and at N^4LO for 2π, 2π-1π and ring graphs. The N^4LO contributions are driven by the Δ and are large (as expected).
- Alternatively, calculations in EFT with explicit Δ are being performed. For 2π 3NF, both approaches lead to comparable results (with the Δ-full approach showing superior convergence). Δ-contributions to 2π-1π and ring topologies have also been worked out, short-range terms in progress…
- Very good progress on the PWD of the 3NF

Future plans: completing derivation of the 3NF at N^4LO and Δ contributions at N^3LO; PWD of the locally regularized 3NF; 3NF effects in 3N scattering and spectra of light nuclei...

Two nucleons à la Weinberg

How to renormalize the Schrödinger equation

Lepage, nucl-th/9697929

1. Introduce a ***finite*** cutoff $M_\pi \ll \Lambda \sim \Lambda_{\rm hard}$
   All symmetries can be preserved — Slavnov '71; Djukanovic et al.'05, Hall, Pascalutsa '12
2. Tune $C_i(\Lambda)$ to low-energy observables ← (implicit) renormalization
3. Check self-consistency by means of error-plots (Lepage-plots)

Predictive power easily understood in terms of Modified Effective Range Theory...

How <u>not</u> to renormalize the Schrödinger equation: an infinite cutoff limit

Removing Λ by taking the limit $\Lambda \to \infty$ may yield finite results for the amplitude but does not qualify for a consistent renormalization in the EFT sense. It is only justified if all necessary counterterms are included... EE, Gegelia, EPJA 41 (2009) 341

$$T = \frac{\alpha_1 + \alpha_2 \Lambda + \alpha_3 \Lambda^2}{\beta_1 + \beta_2 \Lambda + \beta_3 \Lambda^2}$$

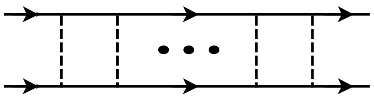
$$\xrightarrow{\Lambda \to \infty} \quad T = \frac{\alpha_3}{\beta_3}$$

$$\xrightarrow{\text{renormalization}} \quad T = \frac{\alpha_1 + \alpha_2 \mu + \alpha_3 \mu^2}{\beta_1 + \beta_2 \mu + \beta_3 \mu^2}$$

The cutoff issue

Why cutoff?

$$T = \underbrace{V}_{\text{truncated at a given order in the expansion}} + VG_0T = \underbrace{V + VG_0V + VG_0VG_0V + \dots}_{\text{increasingly UV divergent integrals are generated through iterations}}$$



Ideally, would like to calculate the integrals

$$T^{(n)} = V[G_0V]^{n-1} = m^{n-1}\int_0^\Lambda d^3l_1 \dots d^3l_{n-1}\, \frac{V(\vec{p}\,', \vec{l}_1)\, V(\vec{l}_1, \vec{l}_2) \dots V(\vec{l}_{n-1}, \vec{p})}{[p^2 - l_1^2 + i\epsilon] \dots [p^2 - l_{n-1}^2 + i\epsilon]}$$

subtract the UV divergences ($\sim \log\Lambda;\ \Lambda;\ \Lambda^2; \dots$) and take the limit $\Lambda \to \infty$. This is not possible in practice (except for pionless EFT) ⟶ let Λ finite and adjust bare C_i to exp data (= implicit renormalization). Λ should be not taken too high [Lepage, EE, Meißner, Gegelia], in practice $\max[\Lambda] \sim 600$ MeV (otherwise spurious BS...)

Price to pay: finite cutoff artifacts (i.e. terms $\sim 1/\Lambda;\ 1/\Lambda^2;\ 1/\Lambda^3; \dots$), may become an issue at higher energies (e.g. $E_{lab} \sim 200$ MeV corresponds to $p \sim 310$ MeV/c)

Is it possible to eliminate or at least reduce finite cutoff artifacts?