New Insights into Nuclear Clustering

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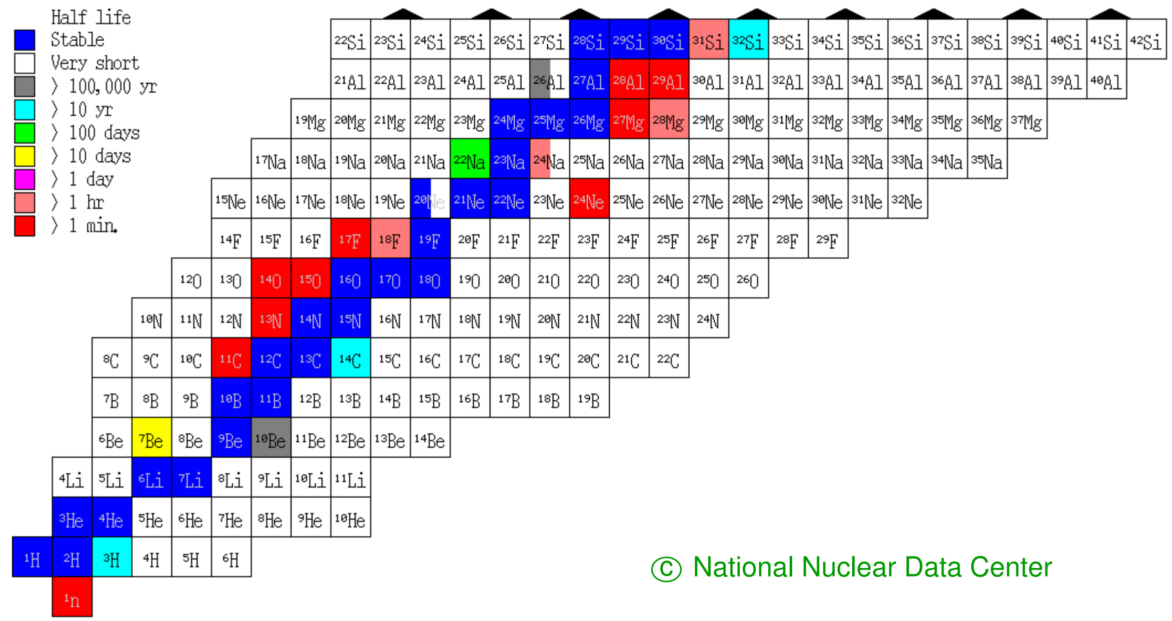
- Very brief introduction
- Basics of nuclear lattice simulations
- Results from nuclear lattice simulations
- New insights into nuclear clustering
- Summary & outlook

Very brief introduction

AB INITIO NUCLEAR STRUCTURE and SCATTERING

- Nuclear structure:
  - ⋆ limits of stability
  - ⋆ 3-nucleon forces
  - ⋆ alpha-clustering
  - ⋮
  - → this talk



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- Nuclear scattering: processes relevant for nuclear astrophysics
  - ⋆ alpha-particle scattering: ^{4}He + ^{4}He → ^{4}He + ^{4}He → Dean Lee's talk
  - ⋆ triple-alpha reaction: ^{4}He + ^{4}He + ^{4}He → ^{12}C + γ
  - ⋆ alpha-capture on carbon: ^{4}He + ^{12}C → ^{16}O + γ
  - ⋮

Basics of nuclear lattice simulations

for an easy intro, see: UGM, Nucl. Phys. News **24** (2014) 11

for an early review, see: D. Lee, Prog. Part. Nucl. Phys. **63** (2009) 117

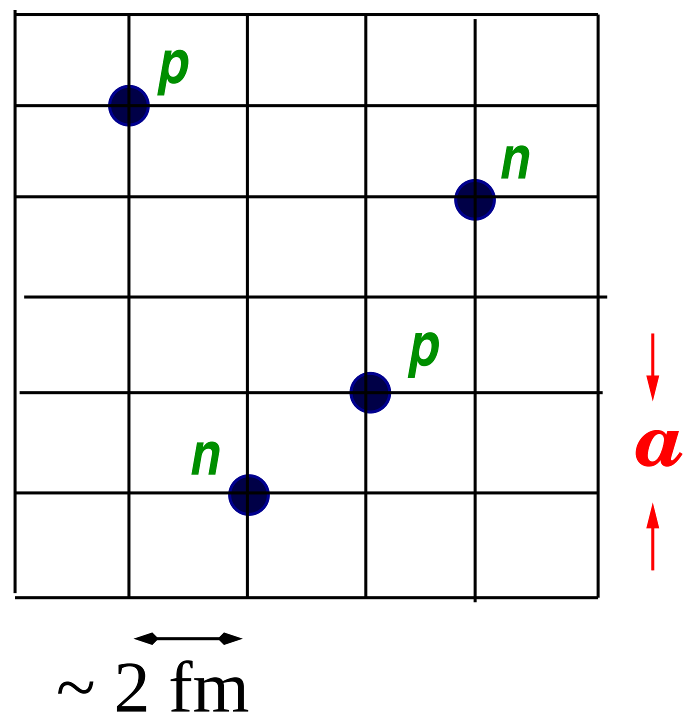
NUCLEAR LATTICE EFFECTIVE FIELD THEORY

Frank, Brockmann (1992), Koonin, Müller, Seki, van Kolck (2000) , Lee, Schäfer (2004), . . .
Borasoy, Krebs, Lee, UGM, Nucl. Phys. **A768** (2006) 179; Borasoy, Epelbaum, Krebs, Lee, UGM, Eur. Phys. J. **A31** (2007) 105

- *new method* to tackle the nuclear many-body problem
- discretize space-time $V = L_s \times L_s \times L_s \times L_t$: nucleons are point-like particles on the sites
- discretized chiral potential w/ pion exchanges and contact interactions + Coulomb

$\rightarrow$ see Epelbaum, Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773

- typical lattice parameters



$$p_{\max} = \frac{\pi}{a} \simeq 314 \text{ MeV [UV cutoff]}$$

- strong suppression of sign oscillations due to approximate Wigner SU(4) symmetry

E. Wigner, Phys. Rev. **51** (1937) 106; T. Mehen et al., Phys. Rev. Lett. **83** (1999) 931; J. W. Chen et al., Phys. Rev. Lett. **93** (2004) 242302

- physics independent of the lattice spacing for $a = 1 \ldots 2$ fm

J. Alarcon et al., EPJA **53** (2017) 83

TRANSFER MATRIX METHOD

- Correlation–function for A nucleons: $Z_A(\tau) = \langle \Psi_A | \exp(-\tau H) | \Psi_A \rangle$

  with Ψ_A a Slater determinant for A free nucleons

  [or a more sophisticated (correlated) initial/final state]

Euclidean time

- Transient energy

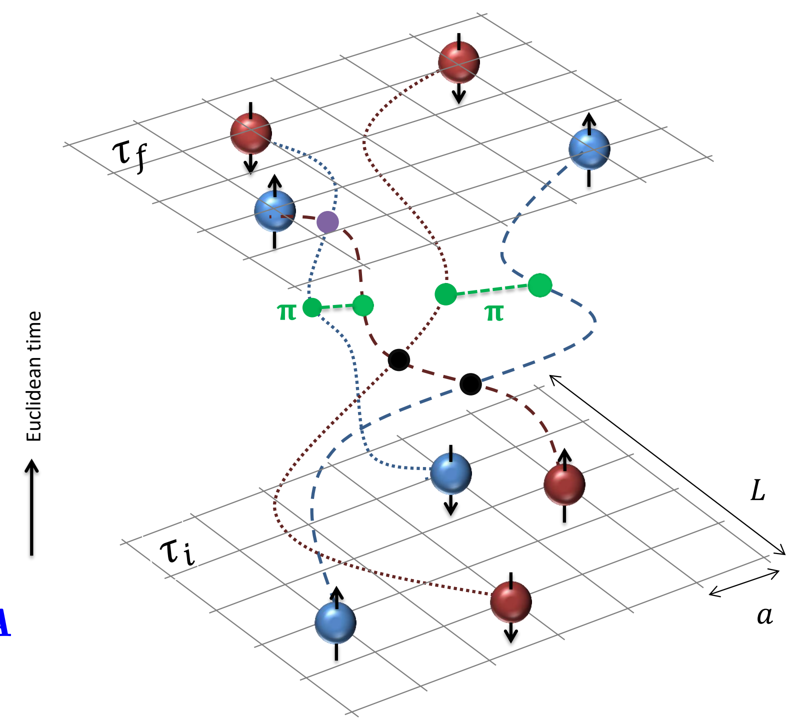
$$E_A(\tau) = -\frac{d}{d\tau} \ln Z_A(\tau)$$

$\rightarrow$ ground state: $E_A^0 = \lim_{\tau \to \infty} E_A(\tau)$

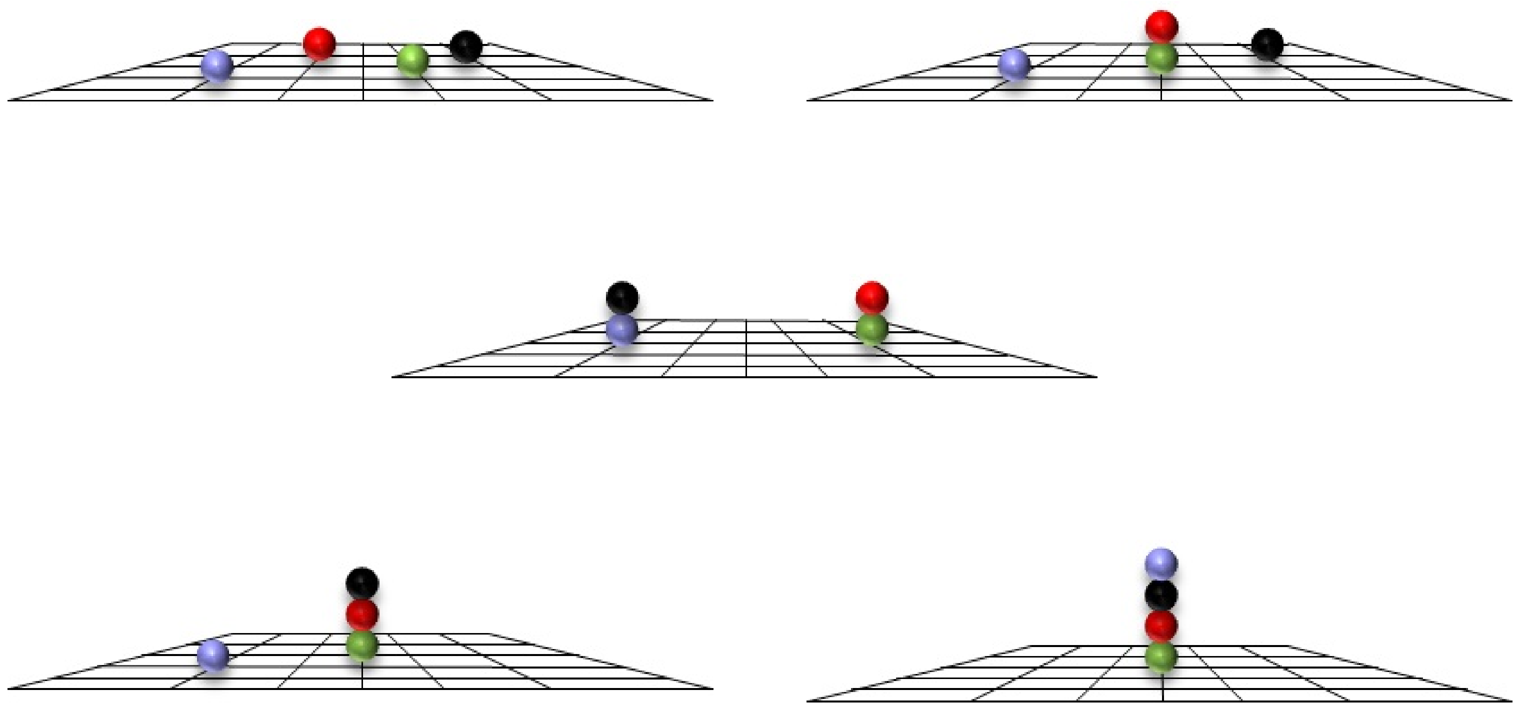
- Exp. value of any normal–ordered operator $\mathcal{O}$

$$Z_A^{\mathcal{O}} = \langle \Psi_A | \exp(-\tau H/2)\, \mathcal{O}\, \exp(-\tau H/2)\, | \Psi_A$$

$$\lim_{\tau \to \infty} \frac{Z_A^{\mathcal{O}}(\tau)}{Z_A(\tau)} = \langle \Psi_A | \mathcal{O} | \Psi_A \rangle$$



CONFIGURATIONS



⇒ all *possible* configurations are sampled

⇒ preparation of *all possible* initial/final states

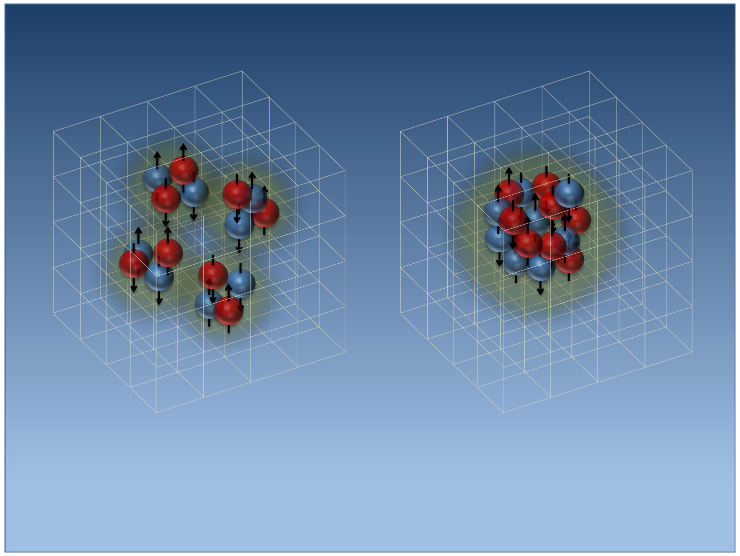
⇒ *clustering* emerges *naturally*

COMPUTATIONAL EQUIPMENT

- Present = JUQUEEN (BlueGene/Q)

6 Pflops

Lattice: some results



NLEFT

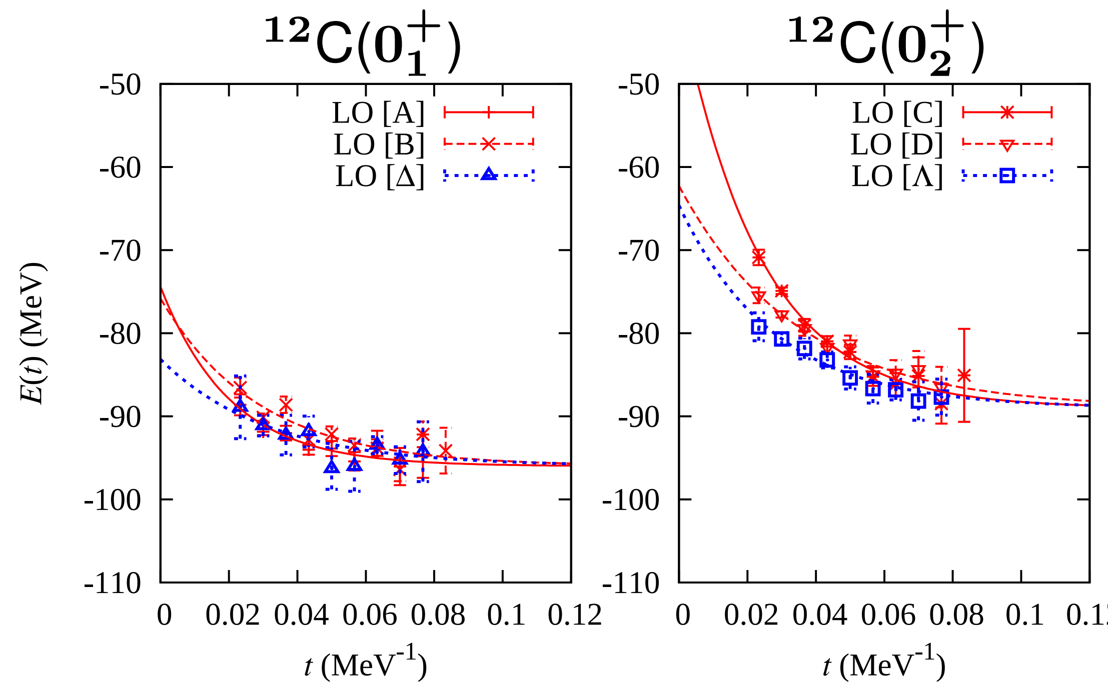
Epelbaum, Krebs, Lähde, Lee, Luu, UGM, Rupak + post-docs + students

FIXING PARAMETERS and FIRST RESULTS

Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. **104** (2010) 142501; Eur. Phys. J. A **45** (2010) 335; . . .

- some groundstate energies and differences [NNLO, 11+2 LECs]

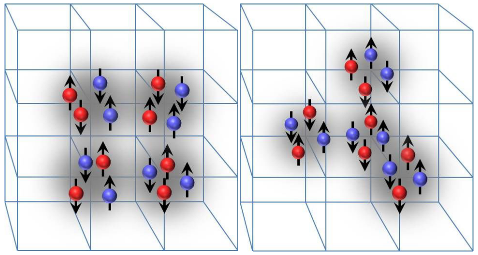
	E [MeV]	NLEFT	Exp.
old algorithm	^{3}He - ^{3}H	0.78(5)	0.76
	^{4}He	−28.3(6)	−28.3
	^{8}Be	−55(2)	−56.5
	^{12}C	−92(3)	−92.2
new algorithm	^{16}O	−131(1)	−127.6
	^{20}Ne	−166(1)	−160.6
	^{24}Mg	−198(2)	−198.3
	^{28}Si	−234(3)	−236.5



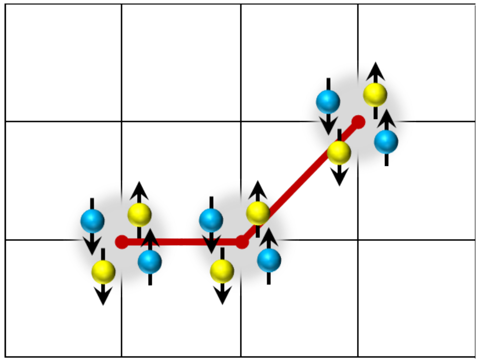
- promising results ⇒ uncertainties down to the 1% level

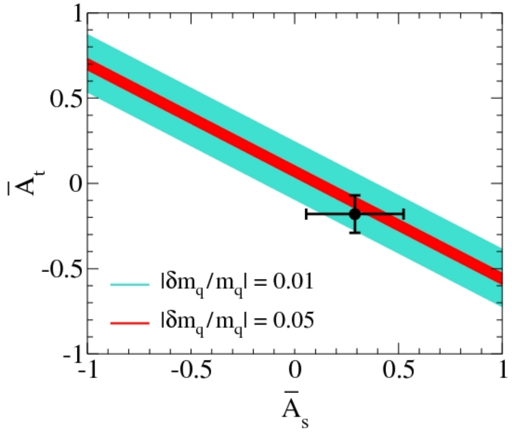
- excited states more difficult ⇒ projection MC method + triangulation

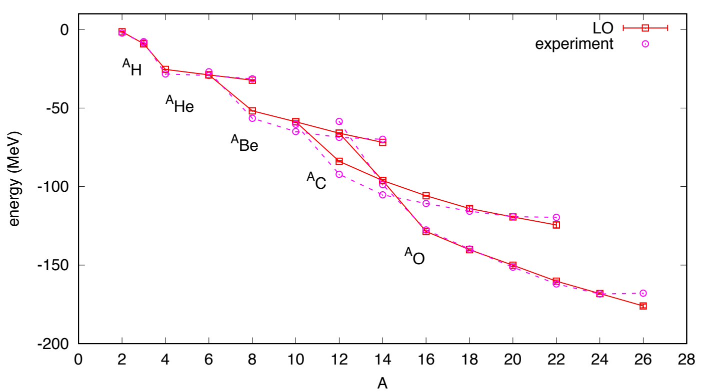
RESULTS from LATTICE NUCLEAR EFT

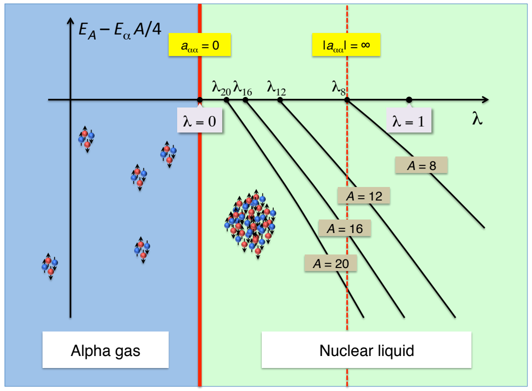


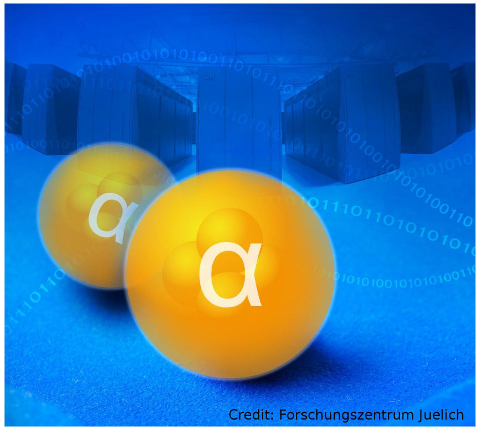
- ☐ Lattice EFT calculations for A=3,4,6,12 nuclei, PRL 104 (2010) 142501
- ☐ *Ab initio* calculation of the Hoyle state, PRL 106 (2011) 192501
- ☐ Structure and rotations of the Hoyle state, PRL 109 (2012) 142501
- ☐ Validity of Carbon-Based Life as a Function of the Light Quark Mass PRL 110 (2013) 142501
- ☐ *Ab initio* calculation of the Spectrum and Structure of ^{16}O, PRL 112 (2014) 142501
- ☐ *Ab initio* alpha-alpha scattering, Nature 528 (2015) 111 → Dean Lee's talk
- ☐ Nuclear Binding Near a Quantum Phase Transition, PRL 117 (2016) 132501
- ☐ *Ab initio* calculations of the isotopic dependence of nuclear clustering, arXiv:1702.05177 → this talk











Ab initio calculations of the isotopic dependence of nuclear clustering

Elhatisari, Epelbaum, Krebs, Lähde, Lee, Li, Lu, UGM, Rupak
[arXiv:1702.05117]

EARLIER RESULTS on NUCLEAR CLUSTERING

- Already a number of intriguing results on clustering:

  Ab initio calculation of the spectrum and structure of ^{12}C (esp. the Hoyle state)

  Ab initio calculation of the spectrum and structure of ^{16}O

  Ground state energies of α-type nuclei up to ^{28}Si within 1%

  Ab initio calculation of α-α scattering

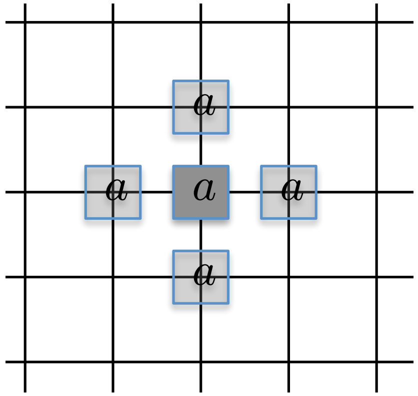
  Quantum phase transition from Bose gas of α's to nuclear liquid for α-type nuclei

- However: when adding extra neutrons/protons, the precision quickly deteriorates due to sign oscillations

- New LO action with smeared SU(4) local+non-local symmetric contact interactions & smeared one-pion exchange

$$a_{\rm NL}(\mathbf{n}) = a(\mathbf{n}) + s_{\rm NL} \sum_{\langle \mathbf{n}'\, \mathbf{n}\rangle} a(\mathbf{n}')$$

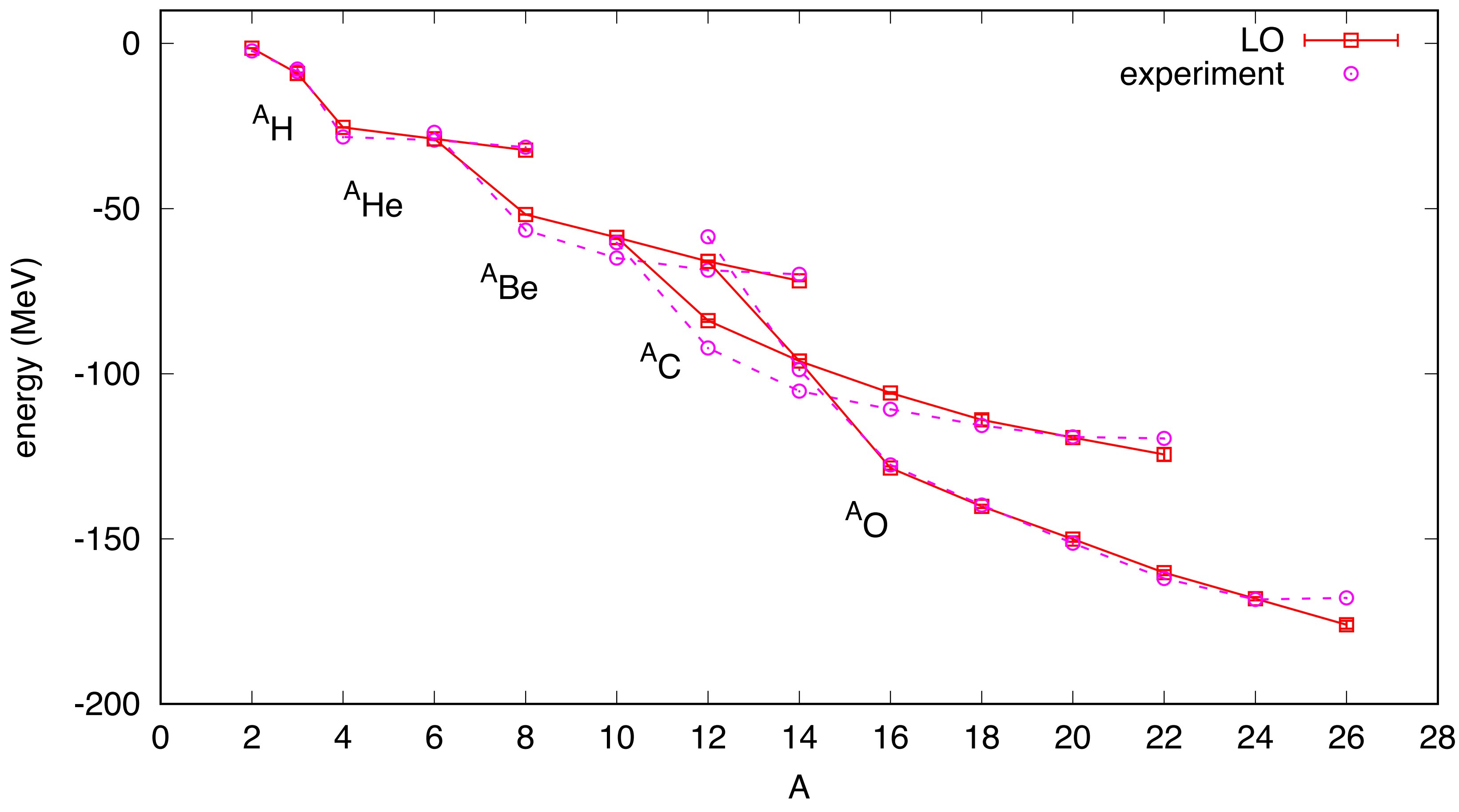
$$a^\dagger_{\rm NL}(\mathbf{n}) = a^\dagger(\mathbf{n}) + s_{\rm NL} \sum_{\langle \mathbf{n}'\, \mathbf{n}\rangle} a^\dagger(\mathbf{n}')$$



GROUND STATE ENERGIES

- Fit parameters to average NN S-wave scattering length and effective range and α-α S-wave scattering length

$\rightarrow$ predict g.s. energies of H, He, Be, C and O isotopes $\rightarrow$ quite accurate (LO)



PROBING NUCLEAR CLUSTERING

- Local densities on the lattice: $\rho(\mathbf{n})\,,\quad \rho_p(\mathbf{n})\,,\quad \rho_n(\mathbf{n})$

- Probe of alpha clusters: $\rho_4 = \sum_{\mathbf{n}} :\rho^4(\mathbf{n})/4!:$

- Another probe for $Z = N =$ even nuclei: $\rho_3 = \sum_{\mathbf{n}} :\rho^3(\mathbf{n})/3!:$

- ρ_4 couples to the center of the α-cluster while ρ_3 gets contributions from a wider portion of the alpha-particle wave function

- Both ρ_3 and ρ_4 depend on the regulator, a, but not on the nucleus

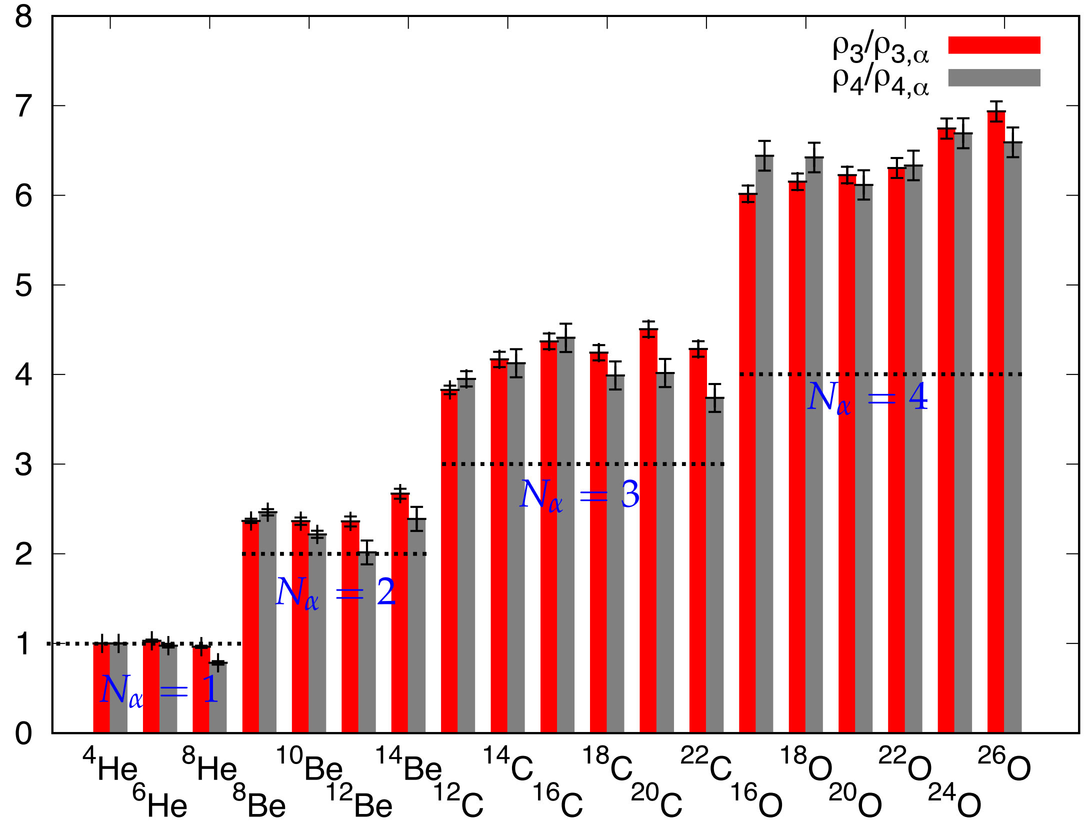
- The ratios $\rho_3/\rho_{3,\alpha}$ and $\rho_4/\rho_{4,\alpha}$ free of short-distance ambiguities and model-independent

- $\rho_3/\rho_{3,\alpha}$ measures the effective number of alpha-cluster N_α

$\Rightarrow$ Any deviation from $N_\alpha =$ integer measures the entanglement of the α-clusters in a given nucleus

PROBING NUCLEAR CLUSTERING

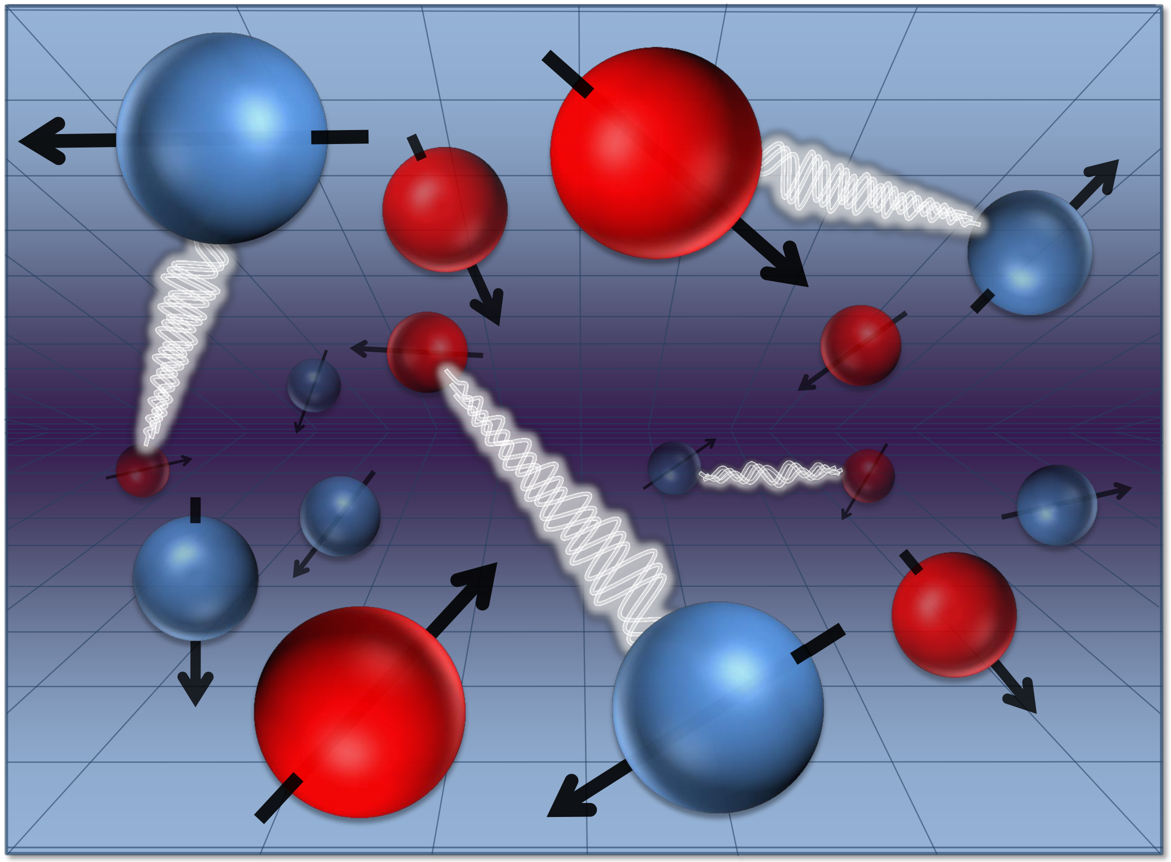
- ρ_3-entanglement of the α-clusters: $$\frac{\Delta_\alpha^{\rho_3}}{N_\alpha} = \frac{\rho_3/\rho_{3,\alpha}}{N_\alpha} - 1$$



Nucleus	^{4,6,8}He	^{8,10,12,14}Be	^{12,14,16,18,20,22}C	^{16,18,20,22,24,26}O
$\Delta_\alpha^{\rho_3}/N_\alpha$	0.00 - 0.03	0.20 - 0.35	0.25 - 0.50	0.50 - 0.75

PROBING NUCLEAR CLUSTERING

- The transition from cluster-like states in light systems to nuclear liquid-like states in heavier systems should not be viewed as a simple suppression of multi-nucleon short-distance correlations, but rather as an increasing *entanglement* of the nucleons involved in the multi-nucleon correlations.



PINHOLE ALGORITHM

- AFQMC calculations involve states that are superpositions of many different center-of-mass positions

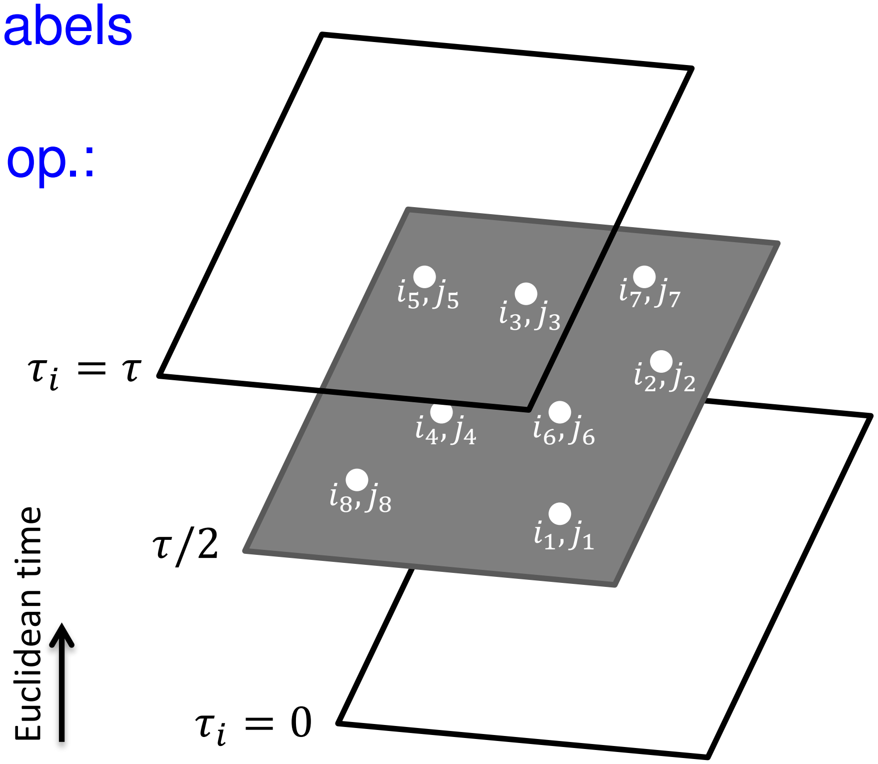
$\longrightarrow$ density distributions of nucleons can not be computed directly

- Insert a screen with pinholes with spin & isospin labels that allows nucleons with corresponding spin & isospin to pass = insertion of the A-body density op.:

$$\rho_{i_1,j_1,\cdots i_A,j_A}(\mathbf{n}_1,\cdots\mathbf{n}_A) \\ = :\rho_{i_1,j_1}(\mathbf{n}_1)\cdots\rho_{i_A,j_A}(\mathbf{n}_A):$$

- MC sampling of the amplitude:

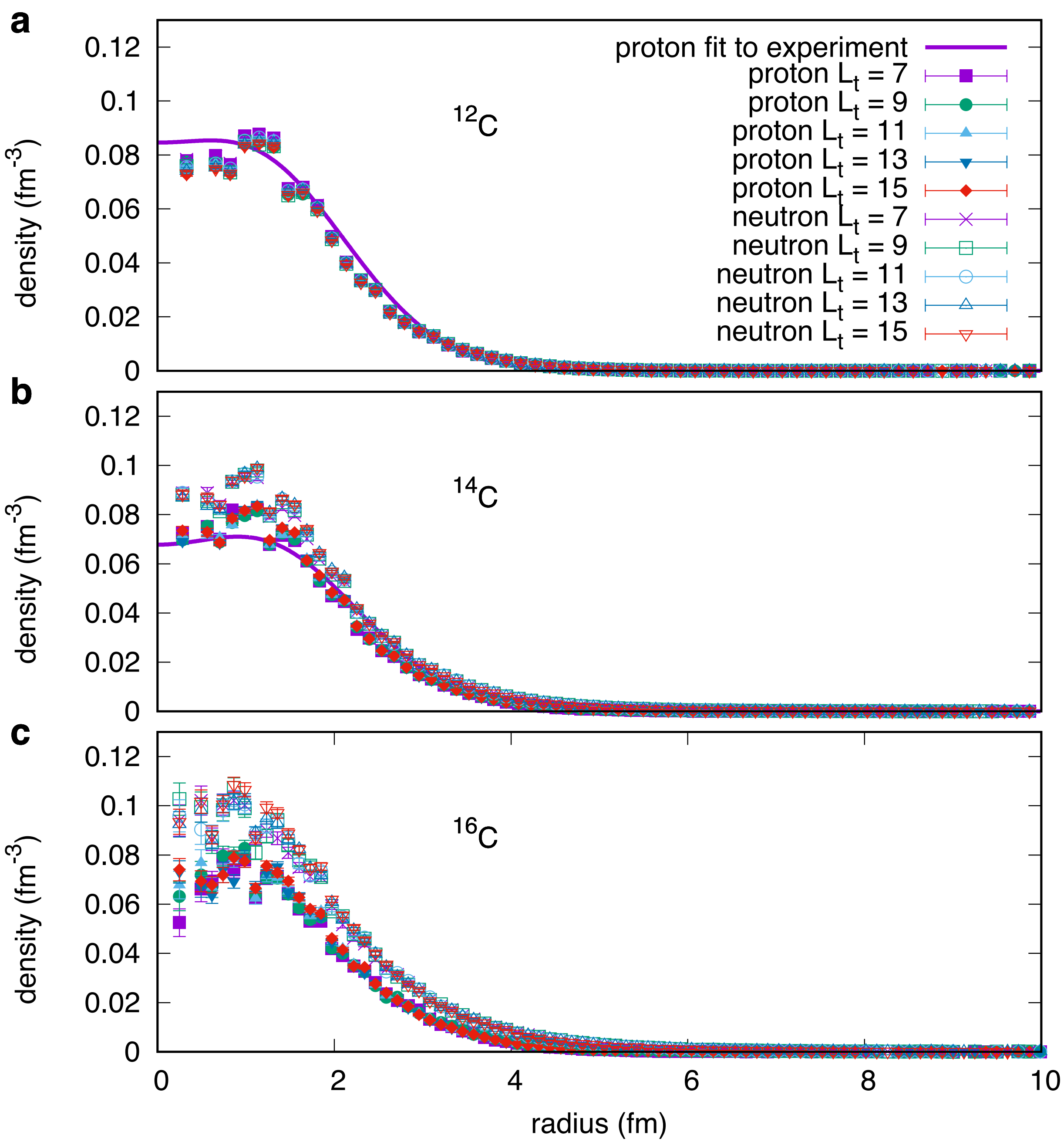
$$A_{i_1,j_1,\cdots i_A,j_A}(\mathbf{n}_1,\ldots,\mathbf{n}_A,L_t) \\ = \langle\psi(\tau/2)|\rho_{i_1,j_1,\cdots i_A,j_A}(\mathbf{n}_1,\ldots,\mathbf{n}_A)|\psi(\tau/2)\rangle$$



- Allows to measure proton and neutron distributions

- Resolution scale $\sim a/A$ as cm position $\mathbf{r}_{\rm cm}$ is an integer $\mathbf{n}_{\rm cm}$ times a/A

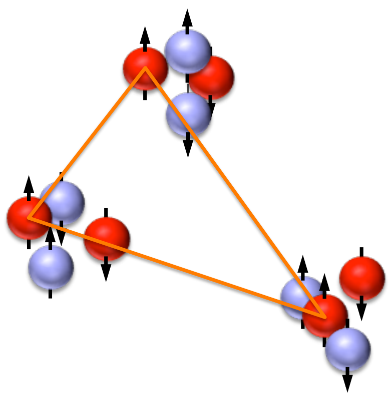
PROTON and NEUTRON DENSITIES in CARBON



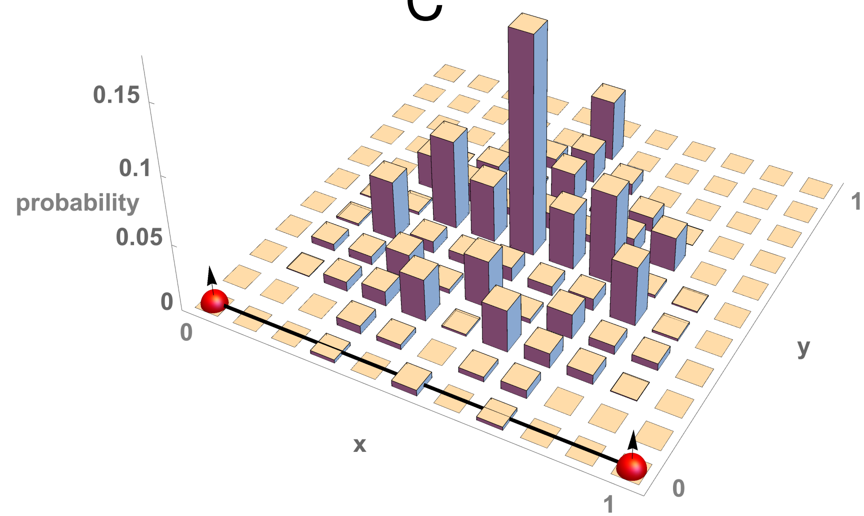
- open symbols: neutron
- closed symbols: proton
- proton size accounted for
- asymptotic properties of the distributions from the volume dependence of N-body bound states
  König, Lee, [arXiv:1701.00279]
- consistent with data
- fit to data from
  Kline et al., NPA209 (1973) 381

ALPHA CLUSTER GEOMETRY

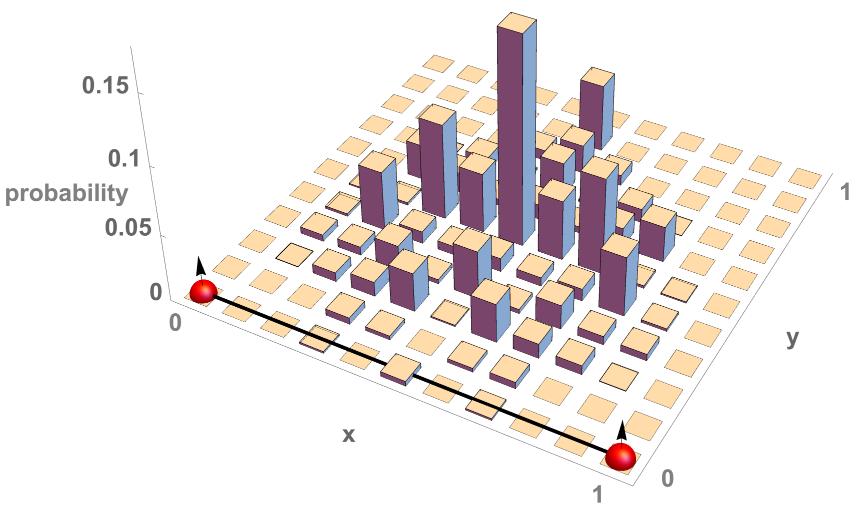
- Measuring the three spin-up protons by considering triangular shapes



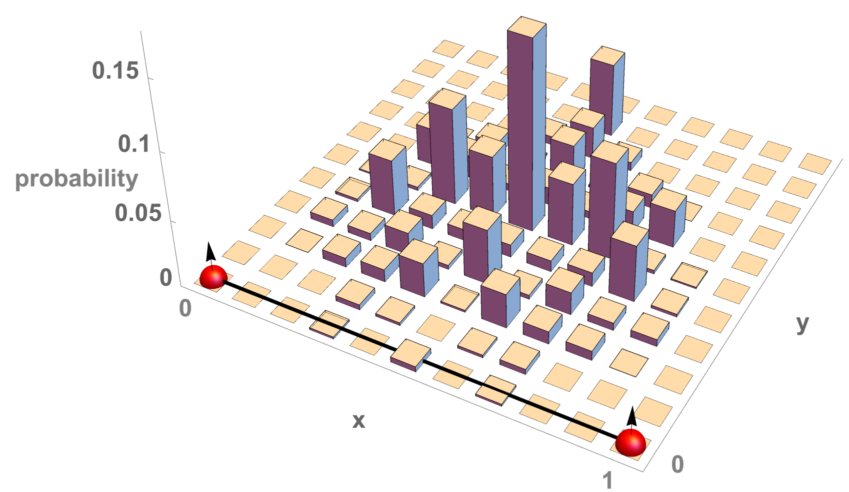
^{12}C



^{14}C



^{16}C



SUMMARY & OUTLOOK

- Nuclear lattice simulations: a new quantum many-body approach

  $\rightarrow$ based on the successful continuum nuclear chiral EFT

  $\rightarrow$ a number of highly visible results already obtained

- New developments

  $\rightarrow$ new highly smeared LO action

  $\hookrightarrow$ good description of isotopic chains of H, He, Be, C and O

  $\rightarrow$ new probes of nuclear clustering: ρ_3 and ρ_4

  $\hookrightarrow$ increasing entanglement between α-clusters with increasing A

  $\rightarrow$ new pinhole algorithm

  $\hookrightarrow$ calculation of the proton and neutron distributions in ^{12,14,16}C

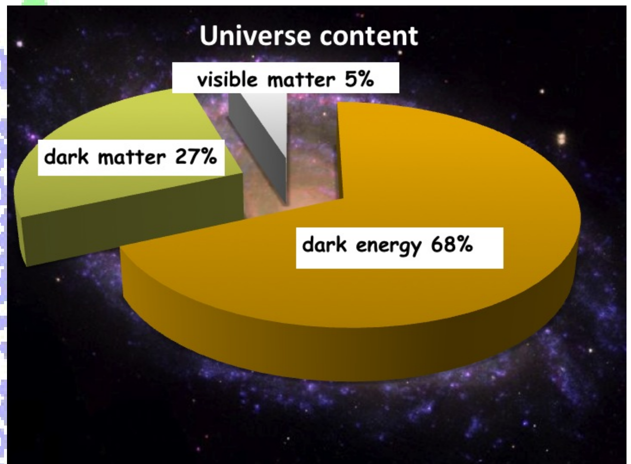
  $\hookrightarrow$ new method of measuring cluster geometries

- Recent review: Freer, Horiuchi, Kanada-En'yo, Lee, UGM, "Microscopic Clustering in Nuclei", arXiv:1705.06192

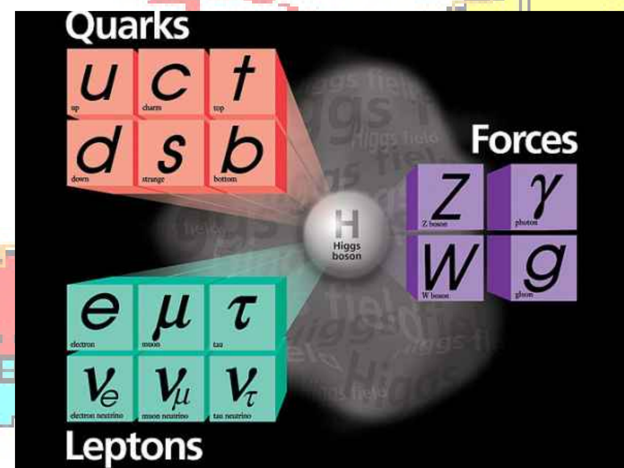
The BIG Picture

WHY NUCLEAR PHYSICS?

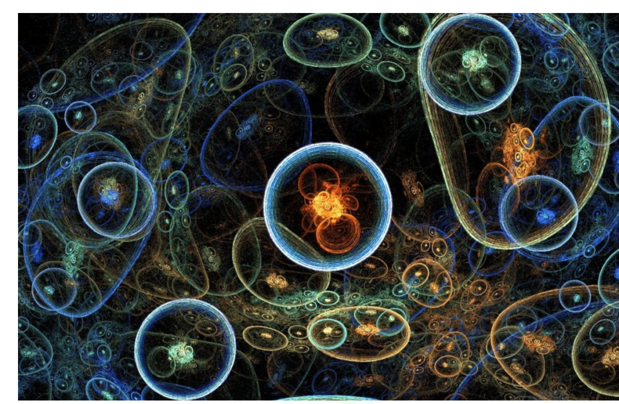
- **The matter we are made off**



- **The last frontier of the SM**



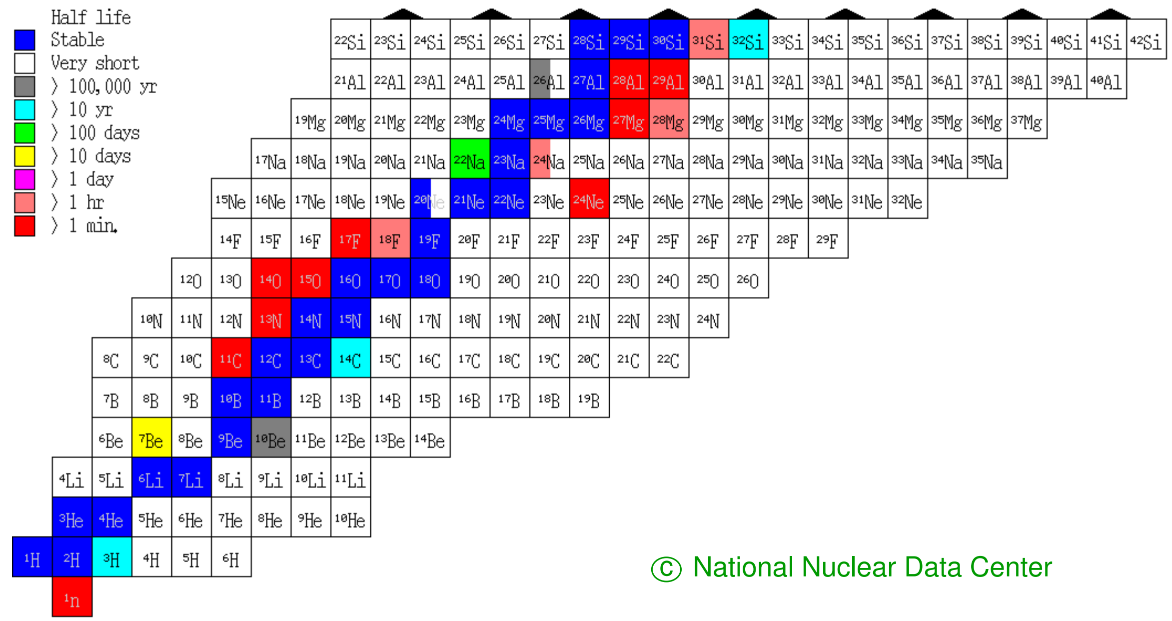
- **Access to the Multiverse**



AB INITIO NUCLEAR STRUCTURE and SCATTERING

- Nuclear structure:
  - ⋆ 3-nucleon forces
  - ⋆ limits of stability
  - ⋆ alpha-clustering

    ⋮



- Nuclear scattering: processes relevant for nuclear astrophysics
  - ⋆ alpha-particle scattering: ^{4}He + ^{4}He $\to$ ^{4}He + ^{4}He
  - ⋆ triple-alpha reaction: ^{4}He + ^{4}He + ^{4}He $\to$ ^{12}C + γ
  - ⋆ alpha-capture on carbon: ^{4}He + ^{12}C $\to$ ^{16}O + γ

    ⋮

SPARES

THE NUCLEAR LANDSCAPE: AIMS & METHODS

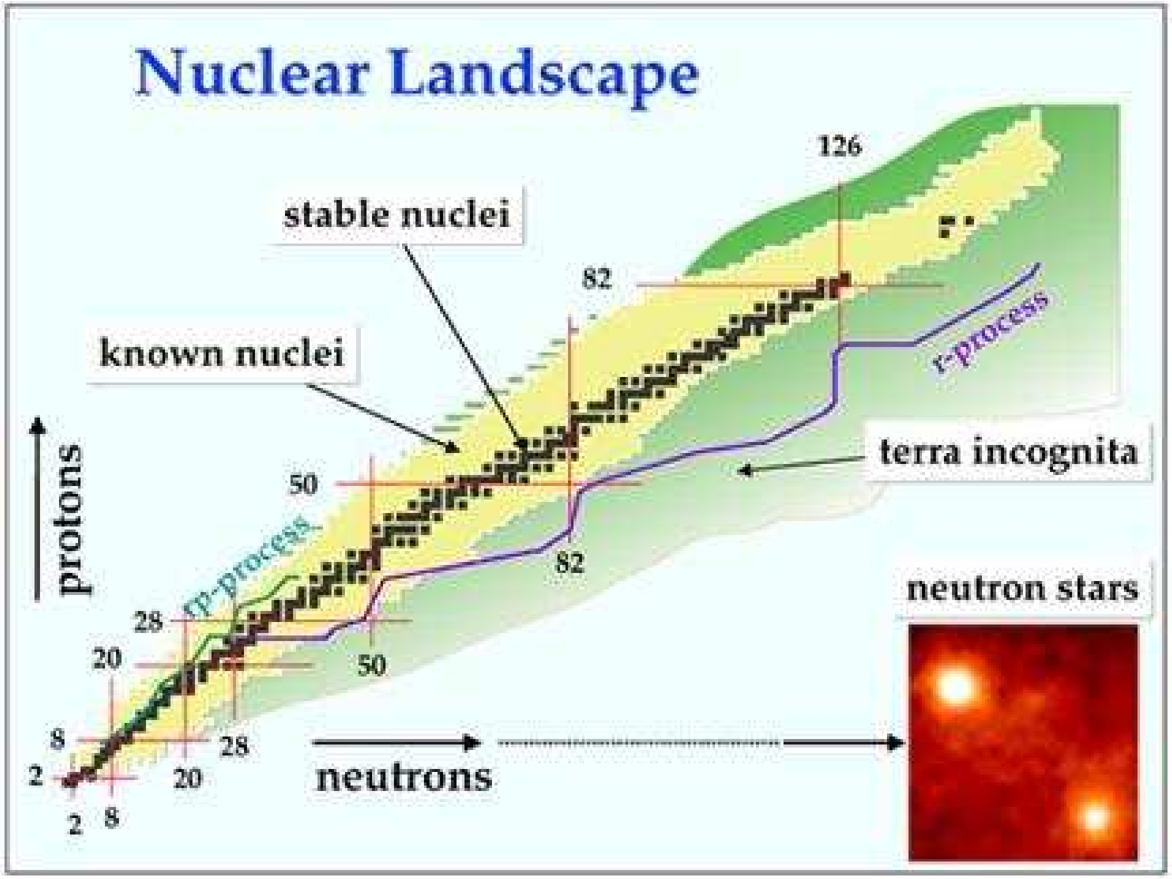
- Theoretical methods:

– Lattice QCD: *A = 0,1,2, . . .*

– NCSM, Faddeev-Yakubowsky, GFMC, ... : *A = 3 - 16*

– coupled cluster, . . .: *A = 16 -100*

– density functional theory, . . .: $A \geq 100$



- Chiral EFT:

– provides **accurate 2N, 3N and 4N forces**

– successfully applied in light nuclei with *A = 2, 3, 4*

– combine with simulations to get to larger A

⇒ Chiral Nuclear Lattice Effective Field Theory

MANY–BODY APPROACHES

- nuclear physics = notoriously difficult problem: strongly interacting fermions

- define *ab initio*: combine the precise and well-founded forces from chiral EFT with a many-body approach

- two different approaches followed in the literature:

  ⋆ combine chiral NN(N) forces with standard many-body techniques

  Dean, Hagen, Navratil, Nogga, Papenbrock, Schwenk, . . .

  $\longrightarrow$ successful, but problems with cluster states (SM, NCSM, CC,...)

  ⋆ combine chiral forces and lattice simulations methods

  $\longrightarrow$ this new method is called *nuclear lattice simulations* (NLEFT)

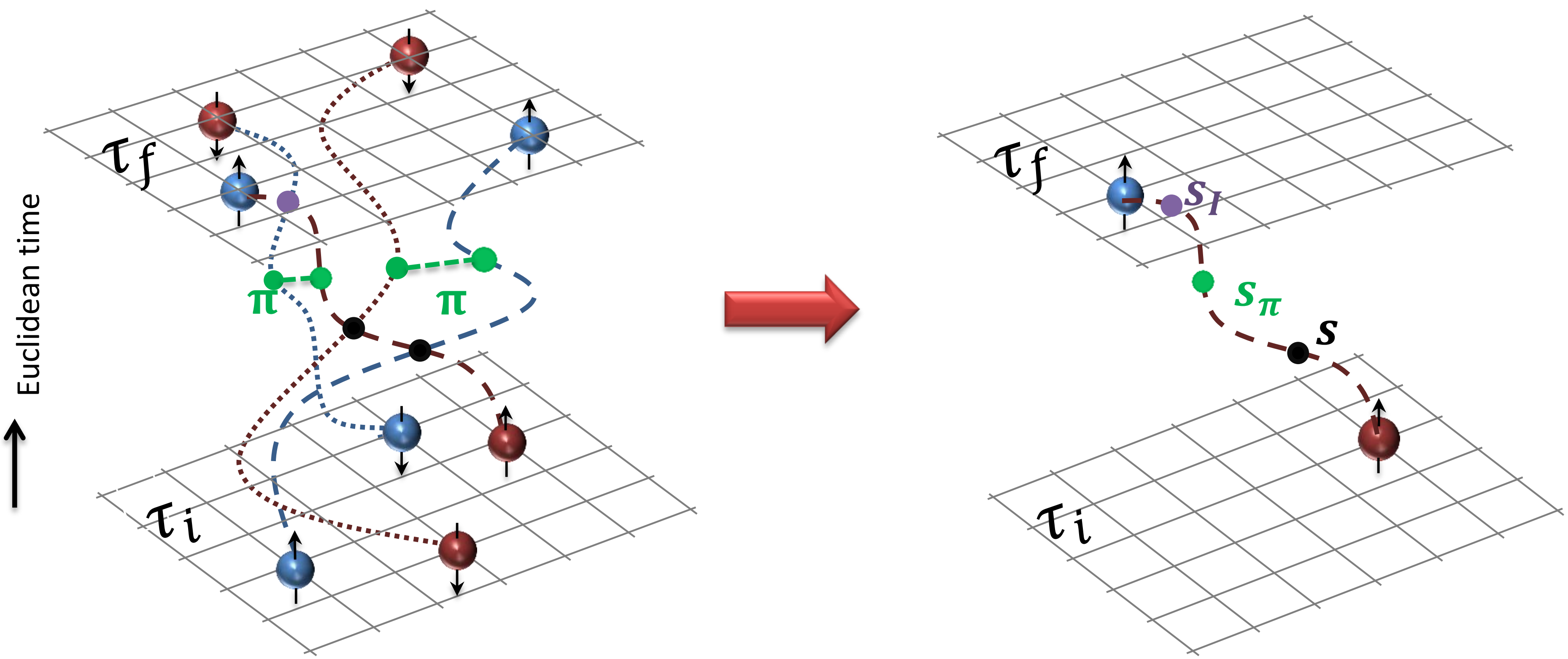
  Borasoy, Epelbaum, Krebs, Lee, Lähde, UGM, Rupak, . . .

  $\longrightarrow$ rest of the talk

AUXILIARY FIELD METHOD

- Represent interactions by auxiliary fields:

$$\exp\left[-\frac{C}{2}\left(N^\dagger N\right)^2\right] = \sqrt{\frac{1}{2\pi}}\int ds \exp\left[-\frac{s^2}{2} + \sqrt{C}\; s\left(N^\dagger N\right)\right]$$



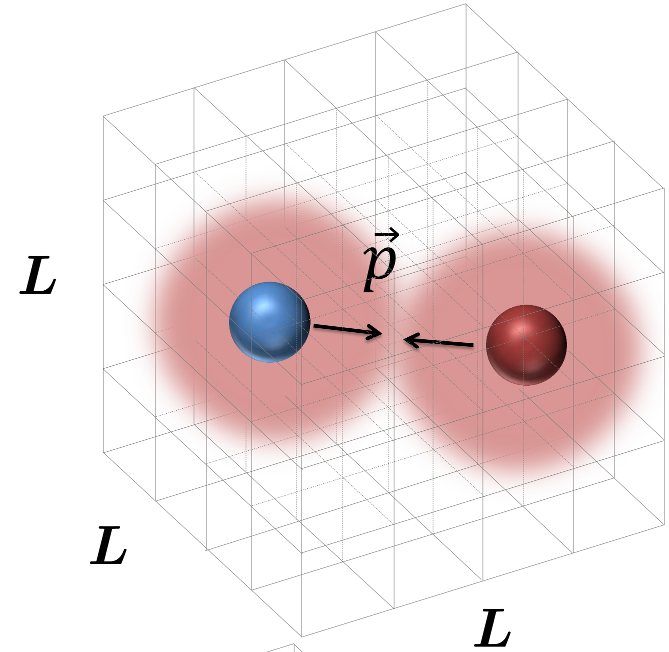
EXTRACTING PHASE SHIFTS on the LATTICE

- Lüscher's method:

  Two-body energy levels below the inelastic threshold on a periodic lattice are related to the phase shifts in the continuum

  Lüscher, Comm. Math. Phys. **105** (1986) 153

  Lüscher, Nucl. Phys. B **354** (1991) 531



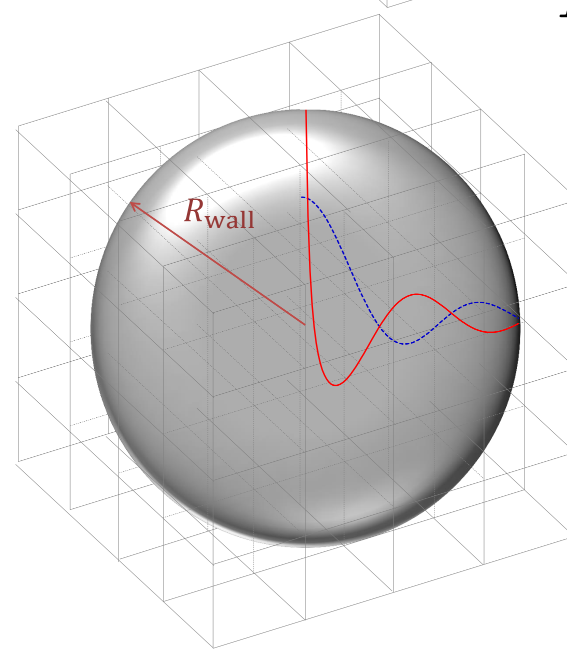
- Spherical wall method:

  Impose a hard wall on the lattice and use the fact that the wave function vanishes for $r = R_{\rm wall}$:

$$\psi_\ell(r) \sim [\cos\delta_\ell(p)F_\ell(pr) + \sin\delta_\ell(p)G_\ell(pr)]$$

  Borasoy, Epelbaum, Krebs, Lee, UGM,
  EPJA **34** (2007) 185

  Carlson, Pandharipande, Wiringa,
  NPA **424** (1984) 47



NUCLEAR FORCES: OPEN ENDS

- Why is there this hierarchy $\boxed{V_{2N} \gg V_{3N} \gg V_{4N}}$?
- Gauge and chiral symmetries difficult to include (meson-exchange currents)

Brown, Riska, Gari, . . .

- Connection to QCD ?

most models have one-pion-exchange, but not necessarily respect chiral symmetry

some models have two-pion exchange reconstructed via dispersion relations from $\pi N \to \pi N$

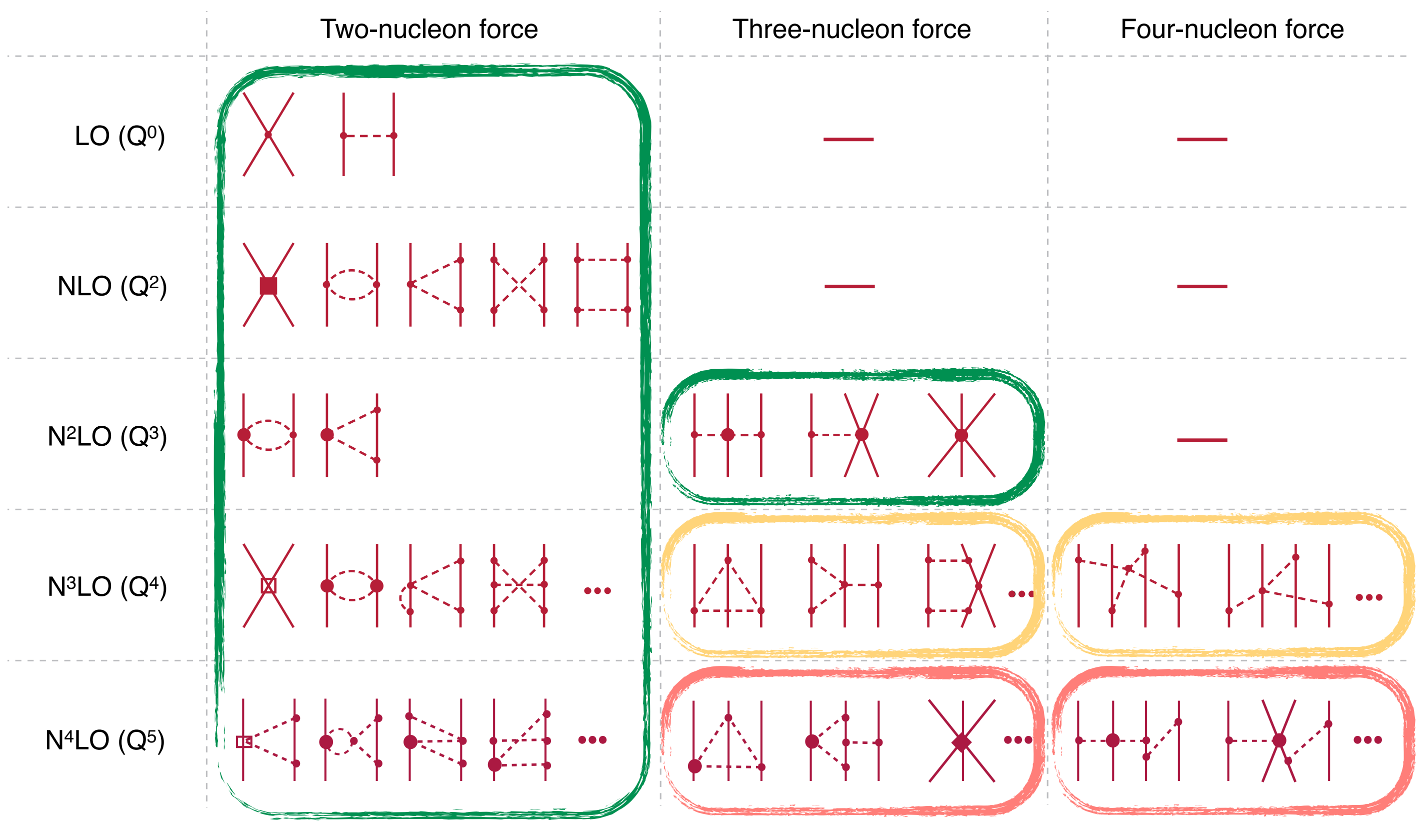
⇒ We want an approach that

– is linked to QCD via its symmetries
– allows for systematic calc's with a controlled theoretical error
– explains the observed hierarchy of the nuclear forces
– matches nucleon structure to nuclear dynamics
– allows for a lattice formulation / chiral extrapolations
– puts nuclear physics on a sound basis

NUCLEAR FORCES in CHIRAL NUCLEAR EFT

- expansion of the potential in powers of Q [small parameter]: $\{p/\Lambda_b,\, M_\pi/\Lambda_b\}$
- explains observed hierarchy of the nuclear forces
- extremely successful in few-nucleon systems

Epelbaum, Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773

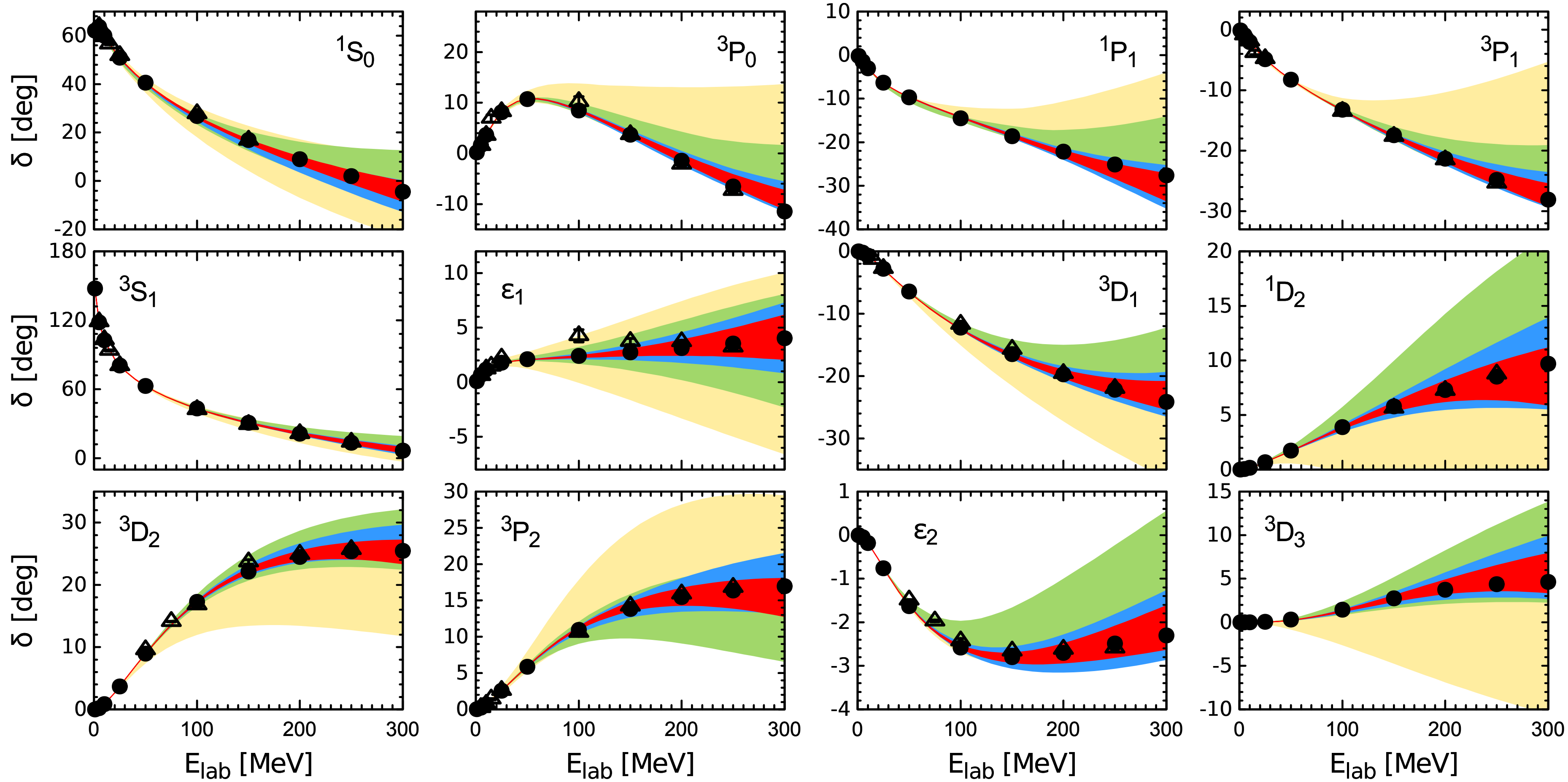


worked out and applied   worked out and to be applied   calculations in progress

PHASE SHIFTS at N4LO

$\Rightarrow$ Precision phase shifts with small uncertainties up to $E_{\rm lab} = 300$ MeV

Epelbaum, Krebs, UGM, Phys. Rev. Lett. **115** (2015) 122301



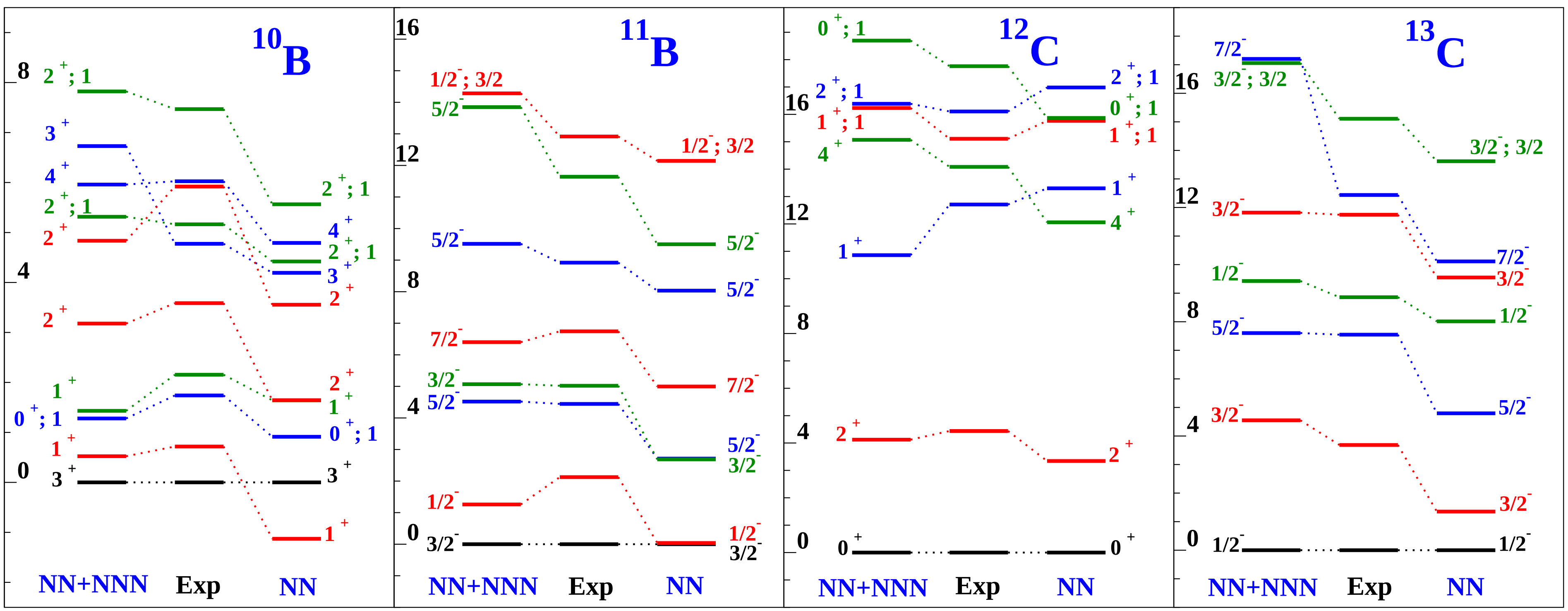
NLO N2LO N3LO N4LO

NO-CORE-SHELL MODEL: p-SHELL NUCLEI

- No-core-shell-model calculation Navratil *et al.*, Phys. Rev. Lett. **99**, 042501 (2007)

- NN interaction at N^3LO and NNN interaction at N^2LO

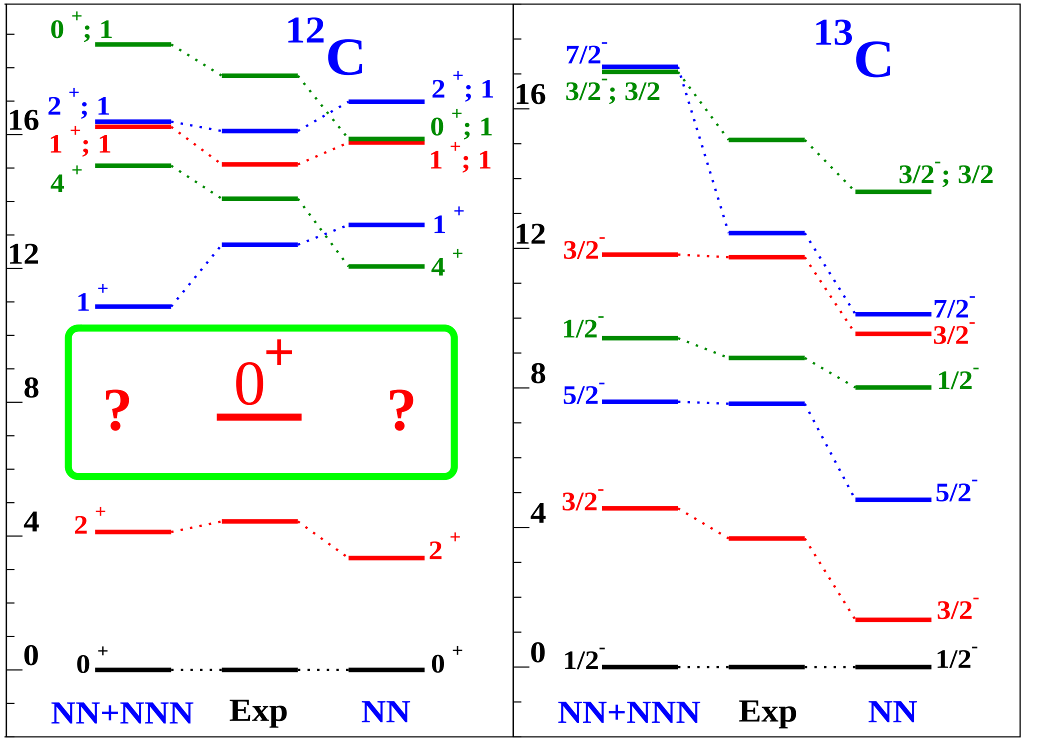
- Fix $D\&E$ from BE of ^{3}H and level structure of ^{4}He, ^{6}Li, ^{10,11}B and ^{12,13}C



MODERN MANY-BODY THEORY and the HOYLE STATE

- one of the most sophisticated many-body theories (No-Core-Shell-Model)
- excellent description of p-shell nuclei from ^{6}Li to ^{13}C

P. Navratil et al., Phys. Rev. Lett. **99** (2007) 042501 + updates

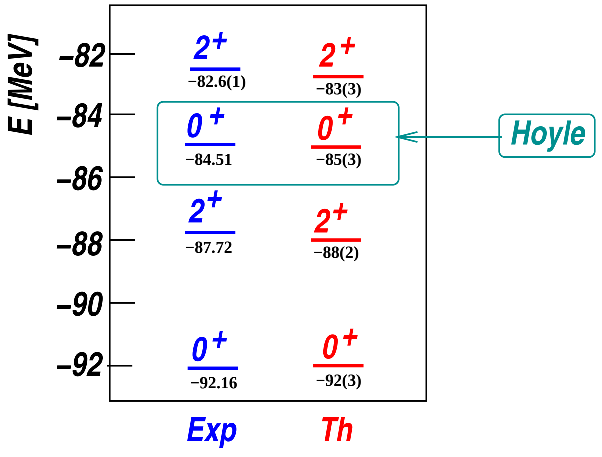


$\Rightarrow$ NO signal of the Hoyle state (i.g. α-cluster states)

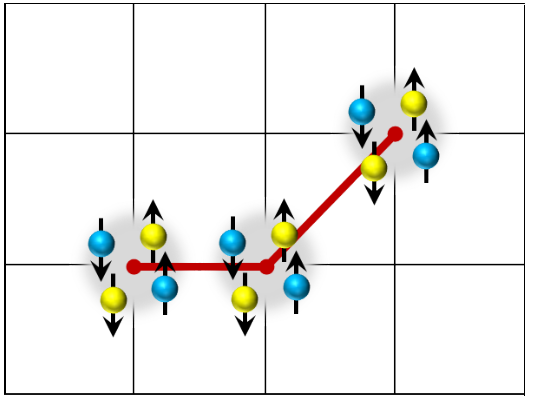
$\Rightarrow$ must develop a better method

RESULTS from LATTICE NUCLEAR EFT

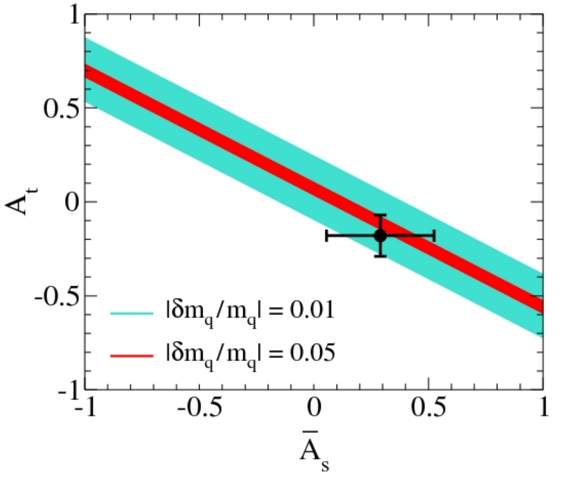
- Hoyle state in ^{12}C
PRL **106** (2011)



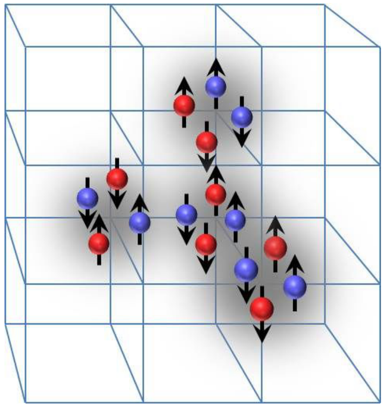
- Structure of the Hoyle state
PRL **109** (2012)



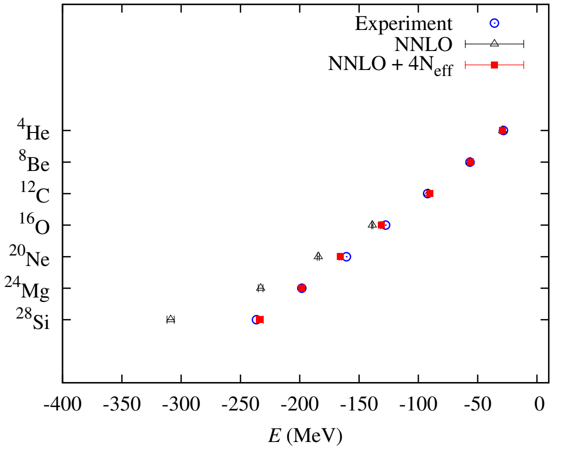
- Fate of carbon-based life
PRL **110** (2013), EPJA **49** (2013)



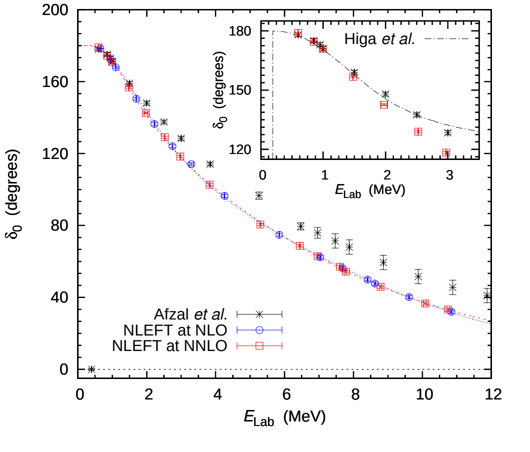
- Spectrum of ^{16}O
PRL **112** (2014)



- Going up the α-chain
PLB **732** (2014)



- Ab initio α-α scattering
Nature **528** (2015)

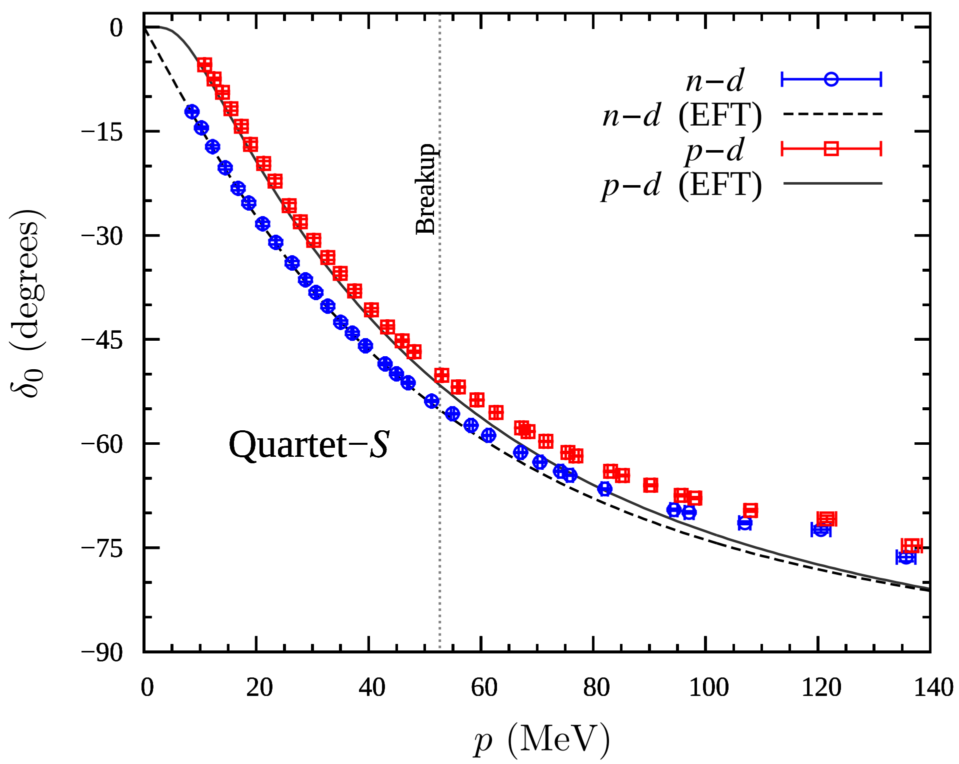


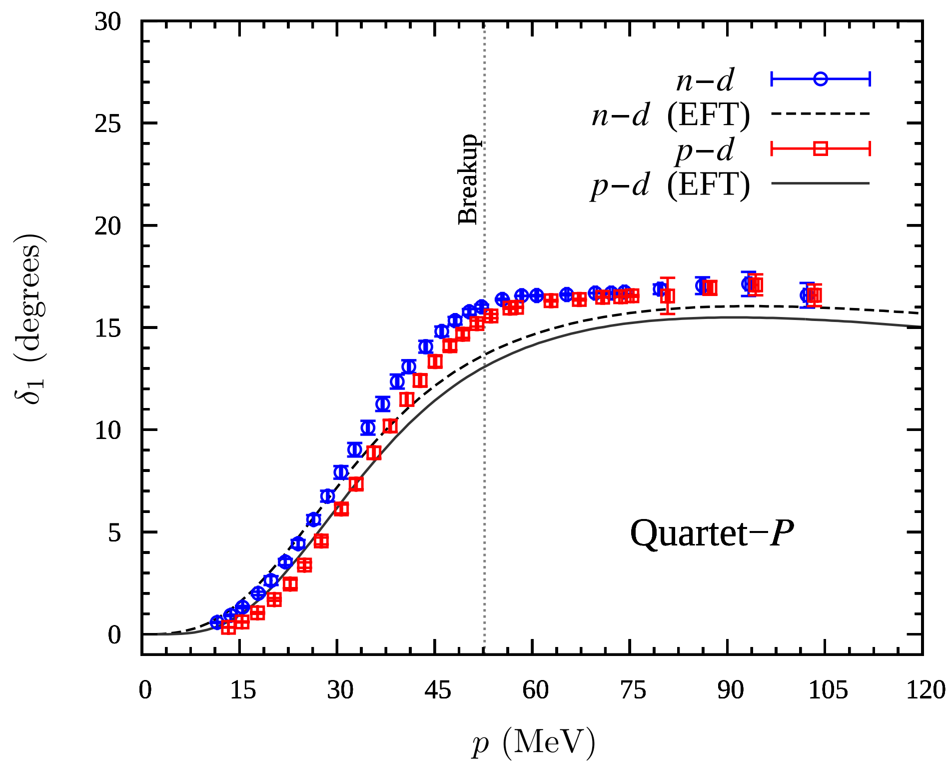
ANOTHER TEST: NUCLEON–DEUTERON SCATTERING

Elhatisari, Lee, UGM, Rupak, Eur. Phys. J. A **52** (2016) 174

- Use improved methods (cluster states projected on sph. harmonics, etc.)
  & algorithmic improvements
- Precision calculation of proton-deuteron and neutron-deuteron scattering

Pionless EFT: König, Hammer, Gabbiani, Bedaque, Rupak, Griesshammer, van Kolck, 1998-2011





Nuclear binding near a quantum phase transition

Elhatisari, Li, Rokash, Alarcon, Du, Klein, Lu, UGM, Epelbaum, Krebs, Lähde, Lee, Rupak,
Phys. Rev. Lett. **117** (2016) 132501 [arXiv:1602.04539]

Editors' suggestion, featured in Physics viewpoint: D.J. Dean, Physics 9 (2016) 106

GENERAL CONSIDERATIONS

- *Ab initio* chiral EFT is an excellent theoretical framework

- not guaranteed to work well with increasing A

→ possible sources of problems:

higher-body forces, higher orders, cutoff dependence, ...

- very many ways of formulating chiral EFT at any given order (smearing etc.)

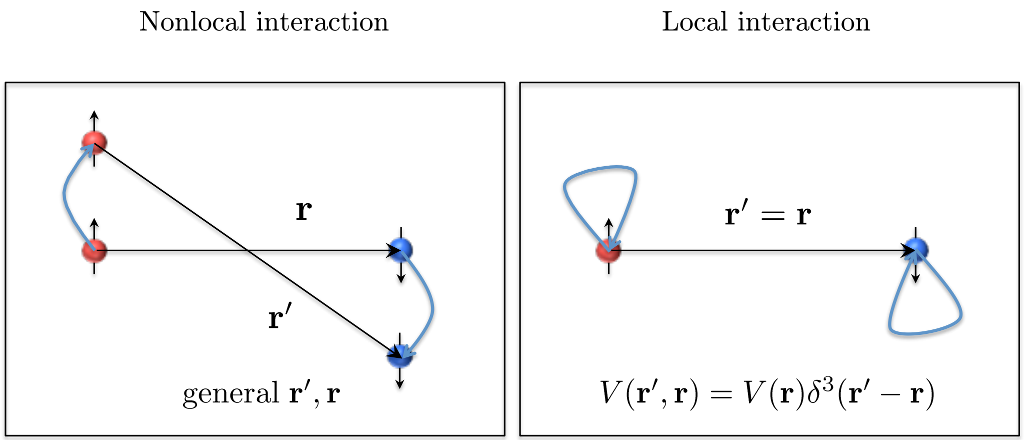
→ use not only NN scattering and light nuclei BEs
but also light nucleus-nucleus scattering data
to pin down the pertinent interactions

→ troublesome corrections might be small

→ investigate these issues using two seemingly equivalent interactions
[not a precision study!]

LOCAL and NON–LOCAL INTERACTIONS

- General potential: $V(\vec{r}, \vec{r}\,')$



- Two types of interactions:

  local: $\vec{r} = \vec{r}\,'$

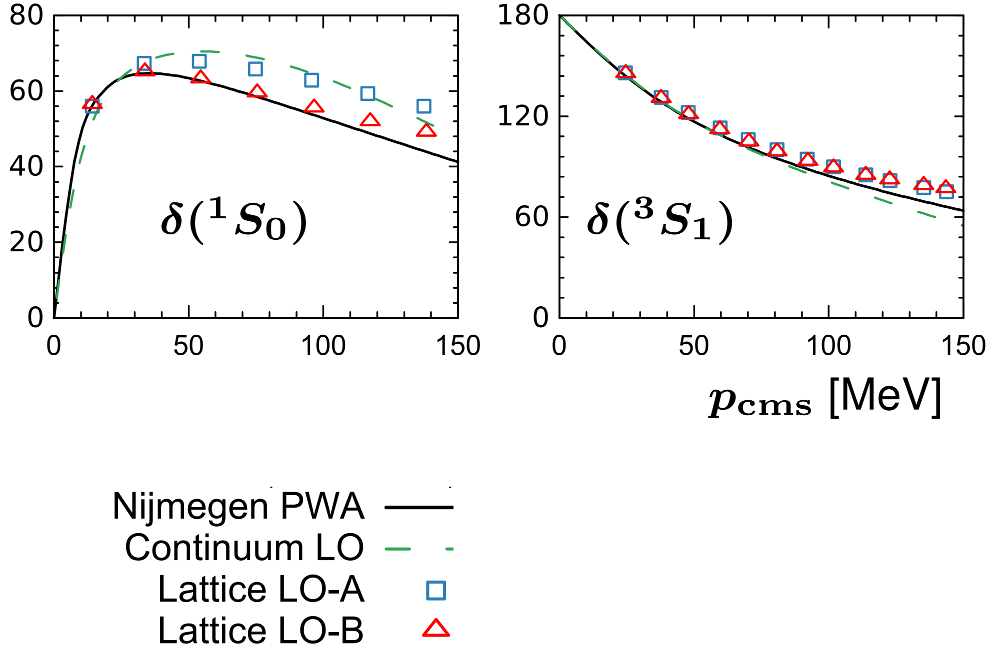
  non-local: $\vec{r} \neq \vec{r}\,'$

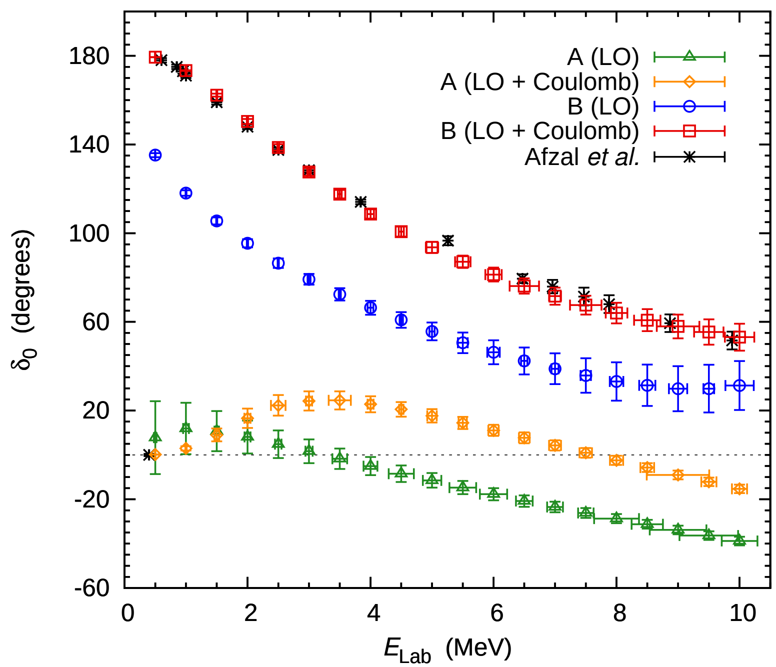
- Taylor two very different interactions:

Interaction A at LO (+ Coulomb)	Interaction B at LO (+ Coulomb)
Non-local short-range interactions	Non-local short-range interactions
+ One-pion exchange interaction	+ Local short-range interactions
(+ Coulomb interaction)	+ One-pion exchange interaction
	(+ Coulomb interaction)
$\rightarrow$ tuned to NN phase shifts	$\rightarrow$ tuned to NN + α-α phase shifts

NN and ALPHA–ALPHA PHASE SHIFTS

- Both interactions very similar for NN but **not** for α-α phase shifts:



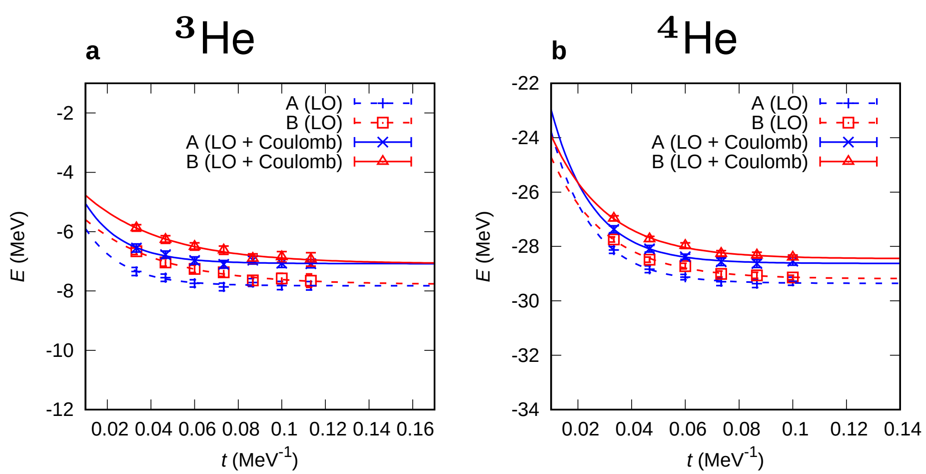


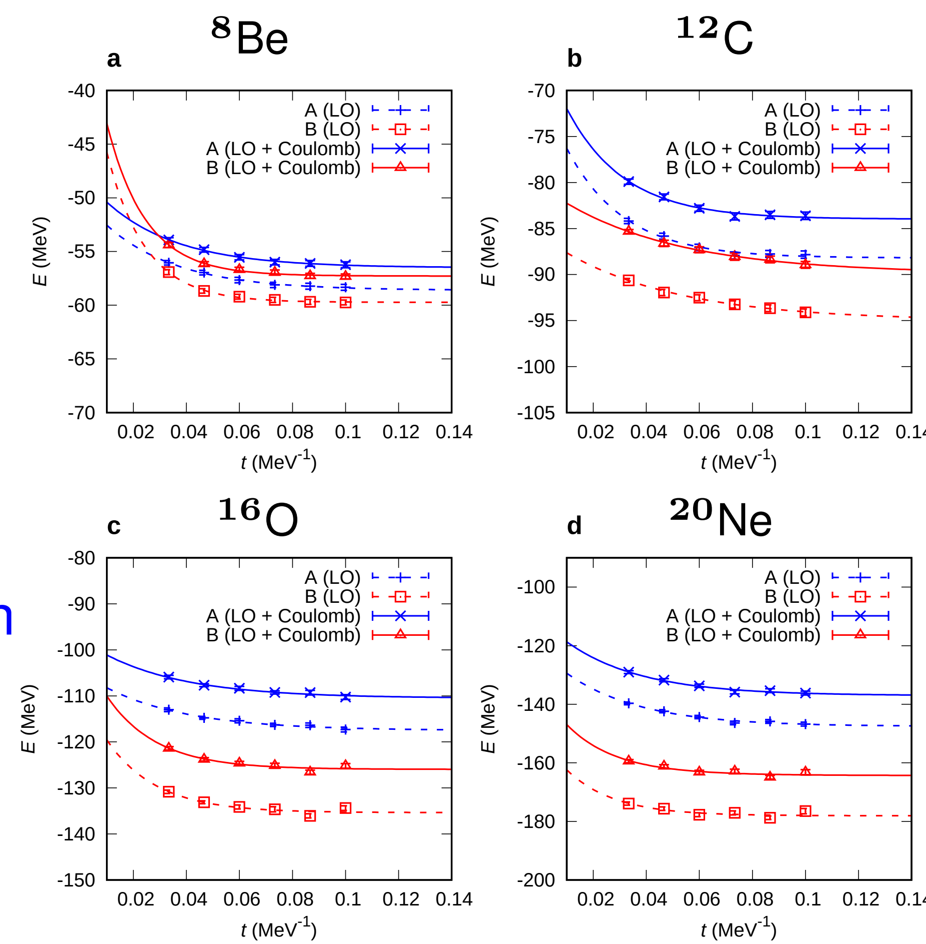
$\longrightarrow$ Interaction A fails, interaction B fitted

$\hookrightarrow$ consequences for nuclei?

GROUND STATE ENERGIES I

- Ground state energies for alpha-type nuclei plus ^{3}He:





- small errors, excellent thermalization

$\Rightarrow$ Both interactions significantly reduce the Monte Carlo sign oscillations (original motivation)

GROUND STATE ENERGIES I

- Ground state energies for alpha-type nuclei (in MeV):

	A (LO)	A (LO+C.)	B (LO)	B (LO+C.)	Exp.
^{4}He	$-29.4(4)$	$-28.6(4)$	$-29.2(1)$	$-28.5(1)$	-28.3
^{8}Be	$-58.6(1)$	$-56.5(1)$	$-59.7(6)$	$-57.3(7)$	-56.6
^{12}C	$-88.2(3)$	$-84.0(3)$	$-95.0(5)$	$-89.9(5)$	-92.2
^{16}O	$-117.5(6)$	$-110.5(6)$	$-135.4(7)$	$-126.0(7)$	-127.6
^{20}Ne	$-148(1)$	$-137(1)$	$-178(1)$	$-164(1)$	-160.6

- B (LO+Coulomb) quite close to experiment (within 2% or better)

- A (LO) describes a Bose condensate of particles:

$$E(^8\mathrm{Be})/E(^4\mathrm{He}) = 1.997(6) \quad E(^{12}\mathrm{C})/E(^4\mathrm{He}) = 3.00(1)$$

$$E(^{16}\mathrm{O})/E(^4\mathrm{He}) = 4.00(2) \quad E(^{20}\mathrm{Ne})/E(^4\mathrm{He}) = 5.03(3)$$

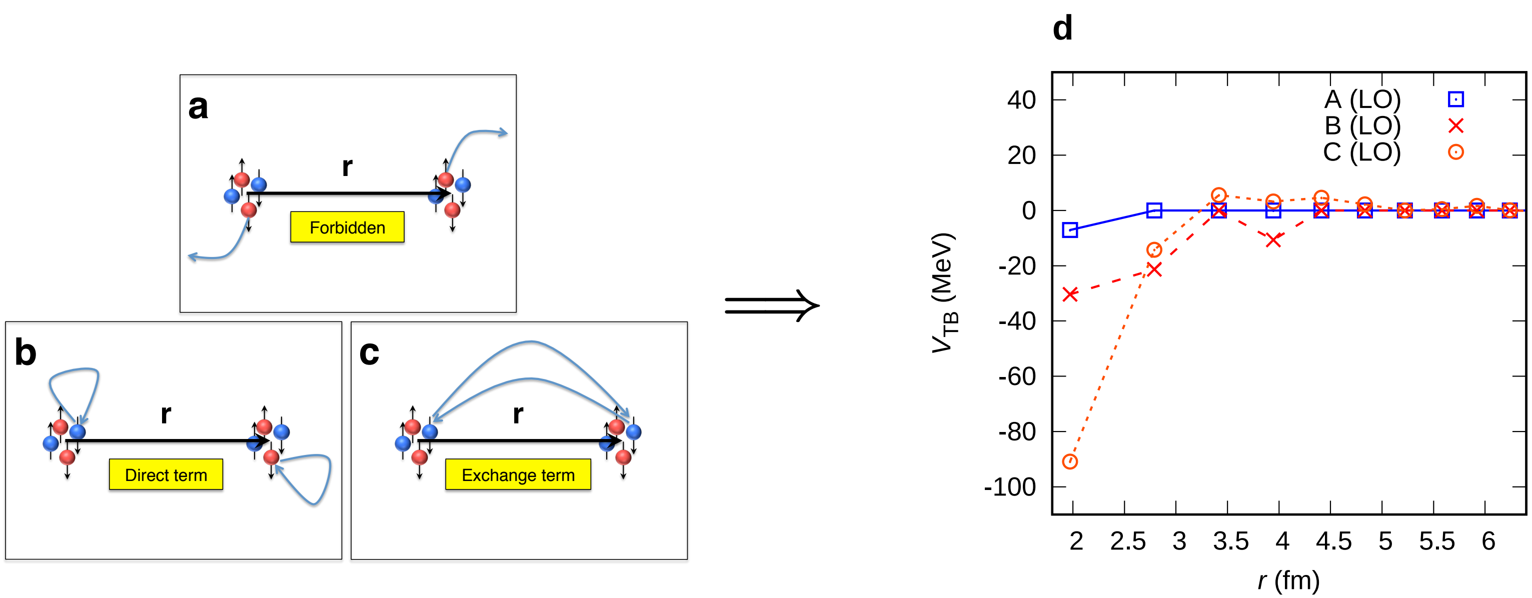
FIRST INSIGHT

- Interaction B was tuned to the nucleon-nucleon phase shifts, the deuteron binding energy, and the S-wave α-α phase shift

- Interaction A starts from interaction B, but *all* local short-distance interactions are switched off, then the LECs of the non-local terms are refitted to describe the nucleon-nucleon phase shifts and the deuteron binding energy

$\rightarrow$ The alpha-alpha interaction is sensitive to the degree of locality of the NN int.

$\rightarrow$ Qualitative understanding: tight-binding approximation (eff. α-α int.)



CONSEQUENCES for NUCLEI and NUCLEAR MATTER

- Define a one-parameter family of interactions that interpolates between the interactions A and B:

$$\boxed{V_\lambda = (1-\lambda)\, V_A + \lambda\, V_B}$$

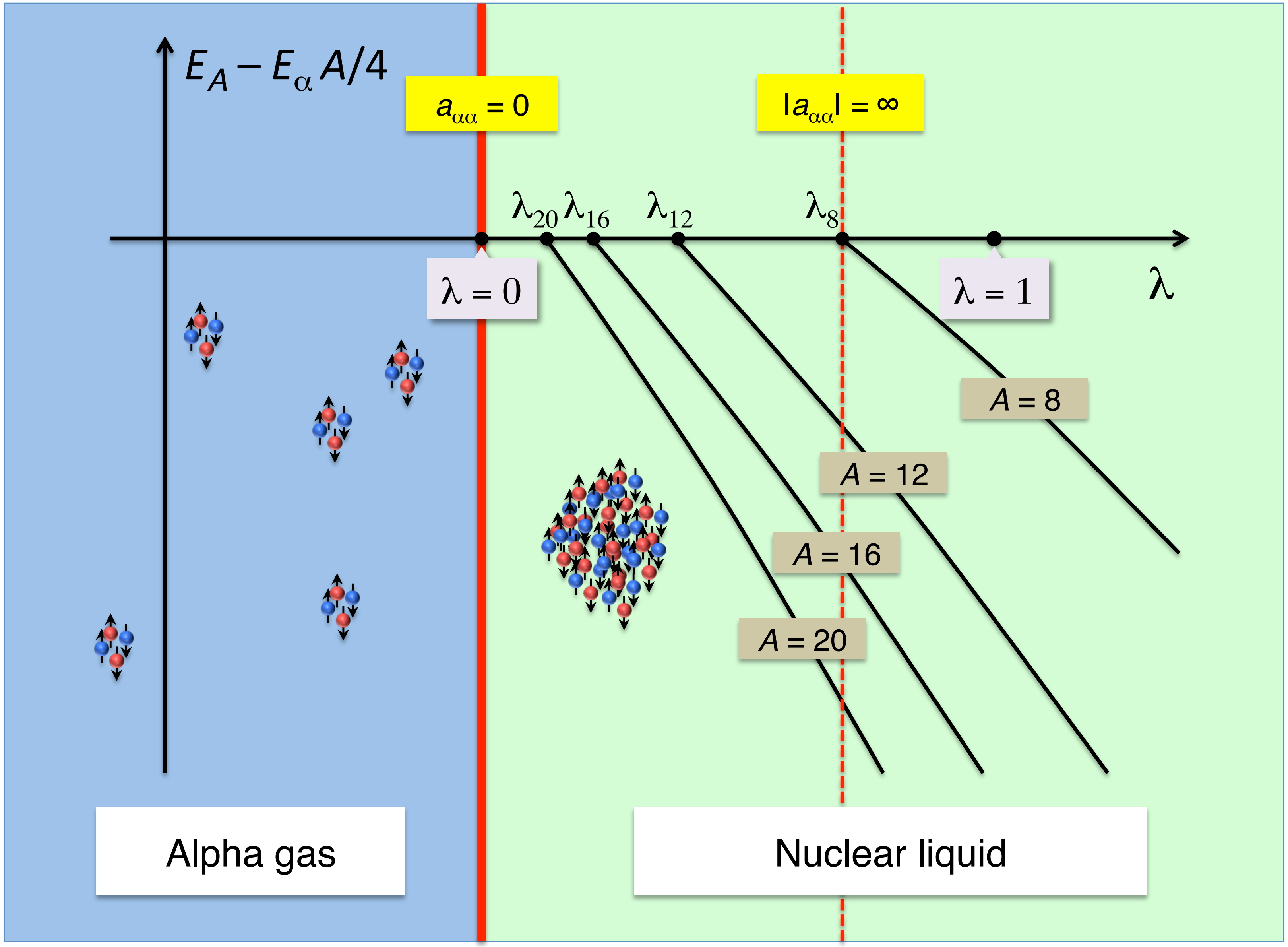
- To discuss the many-body limit, we turn off the Coulomb interaction and explore the zero-temperature phase diagram

- As a function of λ, there is a quantum phase transition at the point where the alpha-alpha scattering length vanishes

Stoff, Phys. Rev. A **49** (1994) 3824

- The transition is a first-order transition from a Bose-condensed gas of alpha particles to a nuclear liquid

ZERO–TEMPERATURE PHASE DIAGRAM



$$\lambda_8 = 0.7(1)$$
$$\lambda_{12} = 0.3(1)$$
$$\lambda_{16} = 0.2(1)$$
$$\lambda_{20} = 0.2(1)$$
$$\lambda_\infty = 0.0(1)$$

FURTHER CONSEQUENCES

- By adjusting the parameter λ in *ab initio* calculations, one can move the of any α-cluster state up and down to alpha separation thresholds.

$\rightarrow$ This can be used as a new window to view the structure of these exotic nuclear states

- In particular, one can tune the α-α scattering length to infinity!

$\rightarrow$ In the absence of Coulomb interactions, one can thus make contact to **universal Efimov physics**:

for a review, see Braaten, Hammer, Phys. Rept. **428** (2006) 259

Hoyle state of ^{12}C	$\xrightarrow{\lambda}$	Universal Efimov trimer
Second 0^+ of ^{16}O	$\xrightarrow{\lambda}$	Universal Efimov tetramer

LOCAL/NON–LOCAL INTERACTIONS on the LATTICE

- Local operators/densities:

$$a(\mathbf{n}), a^\dagger(\mathbf{n}) \quad [\mathbf{n} \text{ denotes a lattice point}]$$

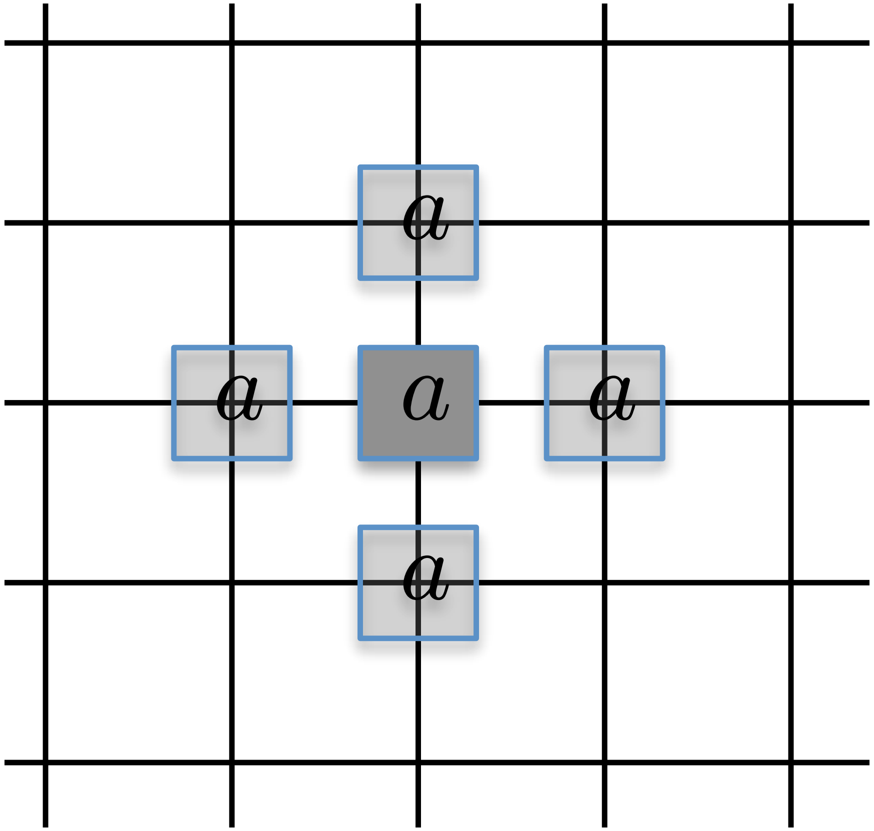
$$\rho_L(\mathbf{n}) = a^\dagger(\mathbf{n})a(\mathbf{n})$$

- Non-local operators/densities:

$$a_{NL}(\mathbf{n}) = a(\mathbf{n}) + s_{NL} \sum_{\langle \mathbf{n}' \, \mathbf{n} \rangle} a(\mathbf{n}')$$

$$a^\dagger_{NL}(\mathbf{n}) = a^\dagger(\mathbf{n}) + s_{NL} \sum_{\langle \mathbf{n}' \, \mathbf{n} \rangle} a^\dagger(\mathbf{n}')$$

$$\rho_{NL}(\mathbf{n}) = a^\dagger_{NL}(\mathbf{n})a_{NL}(\mathbf{n})$$

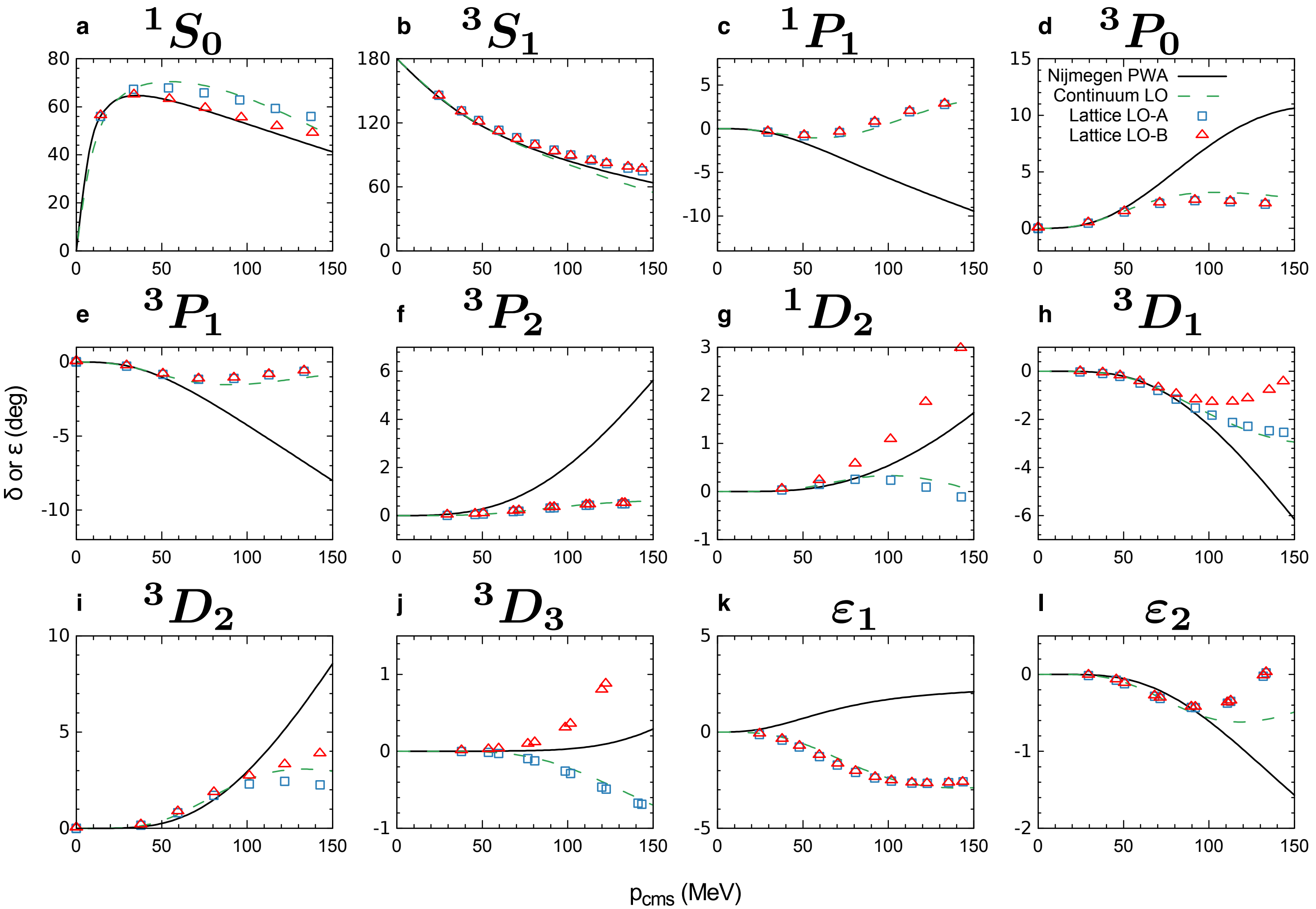


→ where $\sum_{\langle \mathbf{n}' \, \mathbf{n} \rangle}$ denotes the sum over nearest-neighbor lattice sites of $\mathbf{n}$

→ the smearing parameter s_{NL} is determined when fitting to the phase shifts

NUCLEON–NUCLEON PHASE SHIFTS

- Show results for NN [and α-α] phase shifts for both interactions:



$\rightarrow$ both interactions very similar

Neutron-proton scattering at NNLO for varying lattice spacings

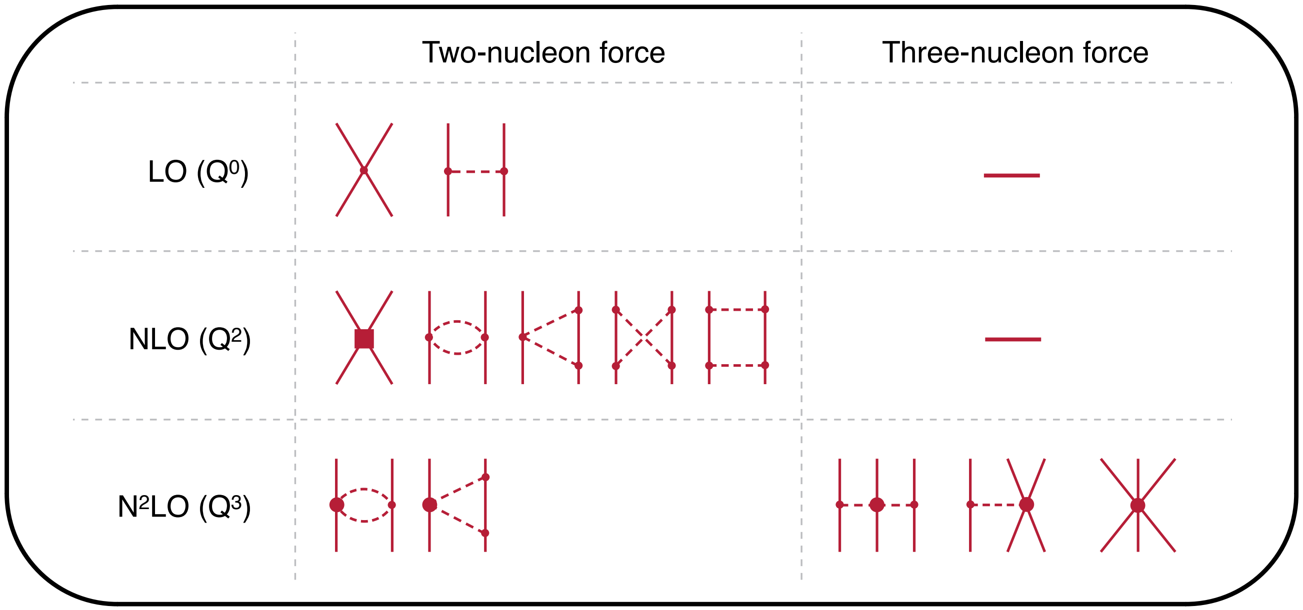
Alarcón, Du, Klein, Lähde, Lee, Li, Luu, UGM

Eur. Phys. J. **A** (2017) in print [arXiv:1702.05319]

NUCLEAR FORCES at NNLO

for details, see: Epelbaum, Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773

- Potential at next-to-next-to-leading order $[Q = \{p/\Lambda, M_\pi/\Lambda\}]$:



- NN potential to NNLO [all πN and $\pi\pi N$ LECs fixed from πN scattering]:

$$V_{\rm NN} = V^{(0)}_{\rm LO} + V^{(2)}_{\rm NLO} + V^{(3)}_{\rm NNLO}$$

$$= V^{\rm cont}_{\rm LO} + V^{\rm OPE}_{\rm LO} + V^{\rm cont}_{\rm NLO} + V^{\rm TPE}_{\rm NLO} + V^{\rm TPE}_{\rm NNLO}$$

NUCLEAR FORCES at NNLO continued

- Analytic expressions [2+7 LECs]:

$$V_{\rm LO}^{\rm cont} = C_S + C_T\,(\vec\sigma_1\cdot\vec\sigma_2)$$

$$V_{\rm LO}^{\rm OPE} = -\frac{g_A^2}{4F_\pi^2}\,\tau_1\cdot\tau_2\,\frac{(\vec\sigma_1\cdot\vec q)\,(\vec\sigma_2\cdot\vec q)}{q^{\,2}+M_\pi^2}$$

$\vec q$ = t-channel mom. transfer

$$V_{\rm NLO}^{\rm cont} = C_1 q^2 + C_2 k^2 + \left(C_3 q^2 + C_4 k^2\right)(\vec\sigma_1\cdot\vec\sigma_2) + iC_5\tfrac{1}{2}\,(\vec\sigma_1+\vec\sigma_2)\cdot(\vec q\times\vec k)$$
$$+\, C_6\,(\vec\sigma_1\cdot\vec q)\,(\vec\sigma_2\cdot\vec q) + C_7(\vec\sigma_1\cdot\vec k)(\vec\sigma_2\cdot\vec k)$$

$\vec k$ = u-channel mom. transfer

$$V_{\rm NLO}^{\rm TPE} = -\frac{\tau_1\cdot\tau_2}{384\pi^2F_\pi^4}L(q)\Big[4M_\pi^2\left(5g_A^4-4g_A^2-1\right)+q^2\left(23g_A^4-10g_A^2-1\right)$$
$$+\,\frac{48g_A^4M_\pi^4}{4M_\pi^2+q^2}\Big] - \frac{3g_A^4}{64\pi^2F_\pi^4}L(q)\left[(\vec q\cdot\vec\sigma_1)\,(\vec q\cdot\vec\sigma_2)-q^2\,(\vec\sigma_1\cdot\vec\sigma_2)\right]$$

- Loop function: $L(q) = \frac{1}{2q}\sqrt{4M_\pi^2+q^2}\,\ln\frac{\sqrt{4M_\pi^2+q^2}+q}{\sqrt{4M_\pi^2+q^2}-q}$

$$\to 1 + \frac{1}{3}\frac{q^2}{4M_\pi^2} + \cdots \text{ for } q \ll \Lambda$$

$\rightarrow$ for coarse lattices $a \simeq 2$ fm, the TPE at N(N)LO can be absorbed in the LECs C_i

$\rightarrow$ no longer true as a decreases, need to account for the TPE explicitely

A FEW DETAILS ON THE FITS

- Fits in large & fixed volumes, vary a from 1 to 2 fm:

a^{-1} [MeV]	a [fm]	L	La [fm]
100	1.97	32	63.14
120	1.64	38	62.48
150	1.32	48	63.14
200	0.98	64	63.14

- OPE and TPE LECs completely fixed ($g_A \sim g_{\pi NN}$ and $c_{1,2,3,4}$ from RS analysis)

Hoferichter, Ruiz de Elvira, Kubis, UGM, Phys. Rev. Lett. **115** (2015) 092301

- Smeared LO S-wave contact interactions: $f(\vec{q}) \equiv f_0^{-1} \exp\left(-b_s \frac{\vec{q}^4}{4}\right)$

- Partial-wave projection of the contact interactions

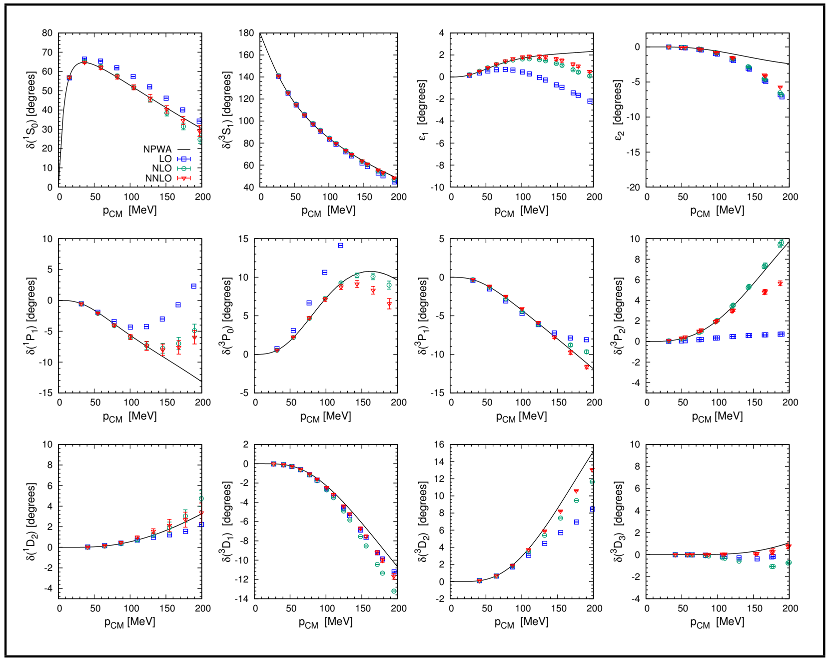
$\rightarrow$ fit b_s and two S-wave LECs C_i at LO up to $p_{\rm cm} = 100$ MeV

$\rightarrow$ w/ b_s fixed, fit two/seven S/P-wave LECs C_i at NLO/NNLO up to $p_{\rm cm} = 150$ MeV

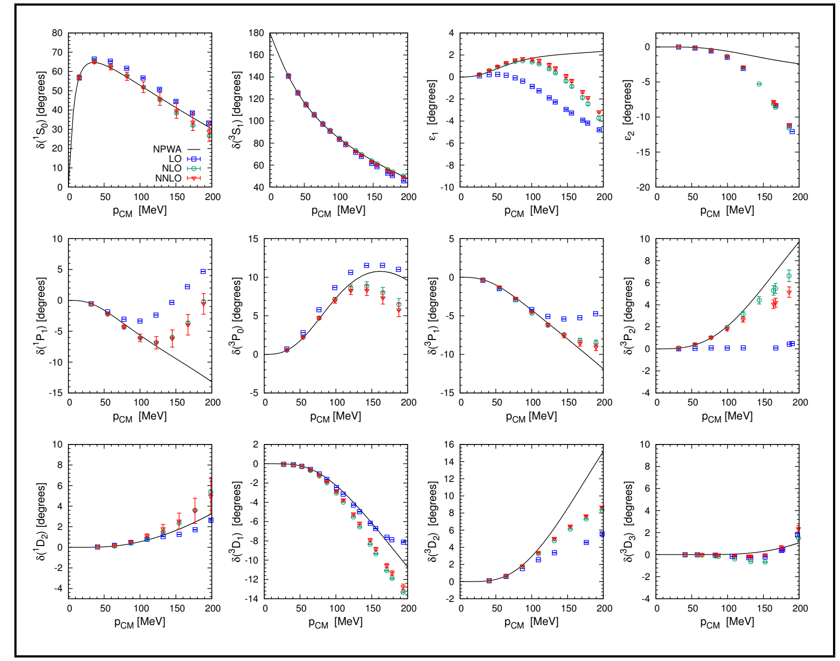
$\rightarrow$ treat NLO and NNLO corrections perturbatively and non-perturbatively

RESULTS for VARIOUS LATTICE SPACINGS - nonpert.

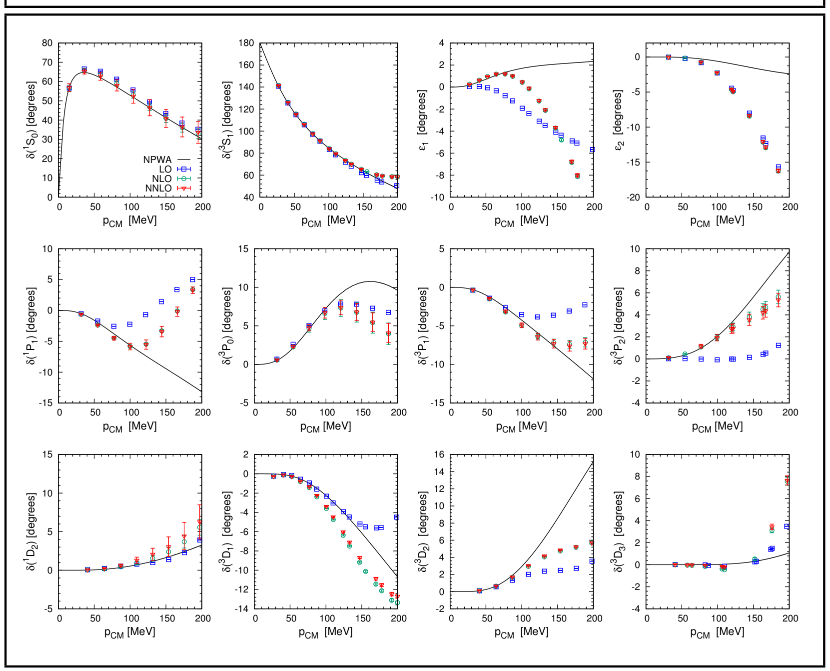
$a = 0.98$ fm



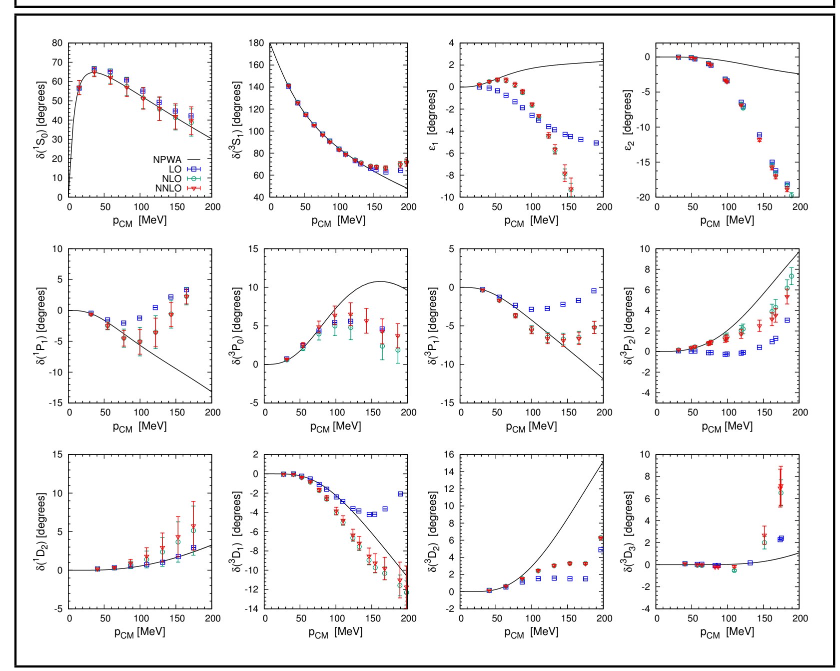
$a = 1.32$ fm



$a = 1.64$ fm



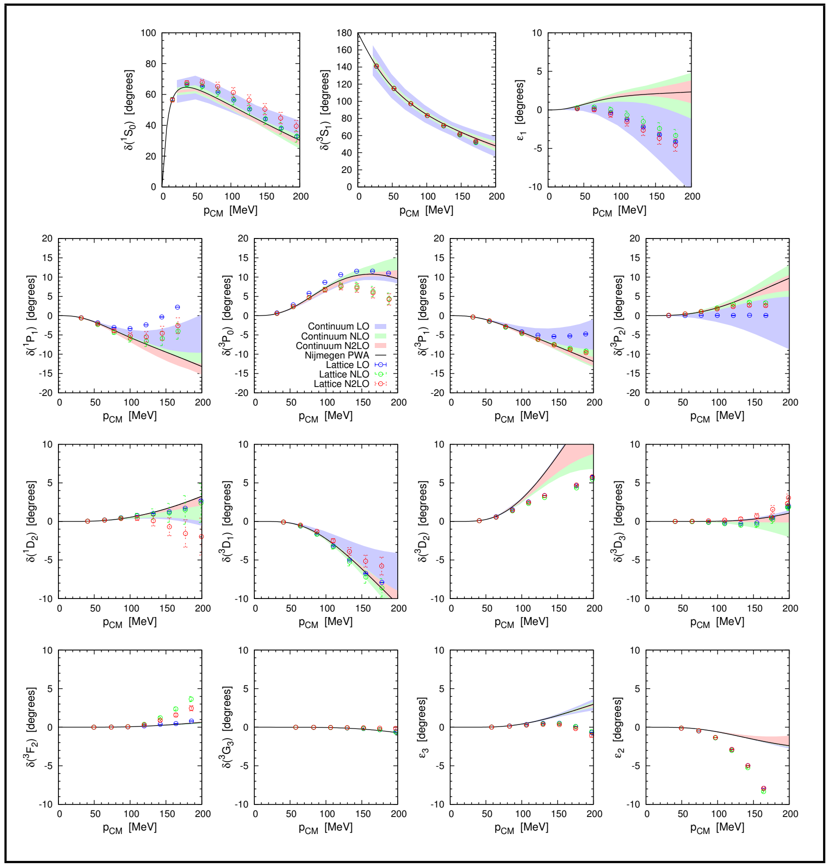
$a = 1.97$ fm



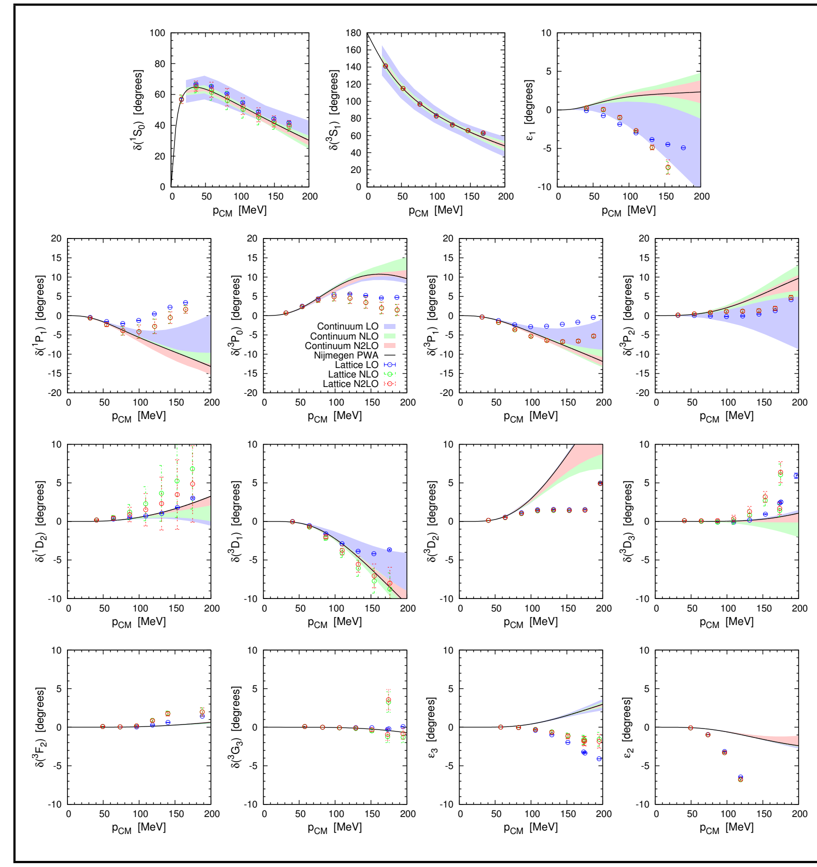
RESULTS for VARIOUS LATTICE SPACINGS - pert.

- perturbative treatment of NLO and NNLO corrections

$a = 1.32$ fm



$a = 1.97$ fm



$\rightarrow$ up to $p_{\rm cm} \simeq 150$ MeV, physics is indendependent of a $\surd$

$\rightarrow$ description consistent with the continuum within error bands $\surd$

$\rightarrow$ explore this for nuclei —- work in progress / stay tuned