

Evgeny Epelbaum, RUB

Quark Confinement and the Hadron Spectrum XI,
St. Petersburg, September 8-12, 2014

Chiral effective field theory for light nuclei

Introduction

New chiral NN potentials

Chiral 3NF: Ongoing developments

Summary



Nuclear chiral effective field theory

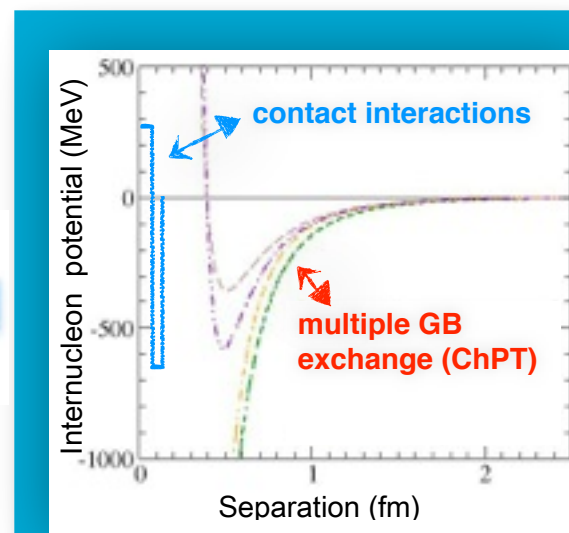
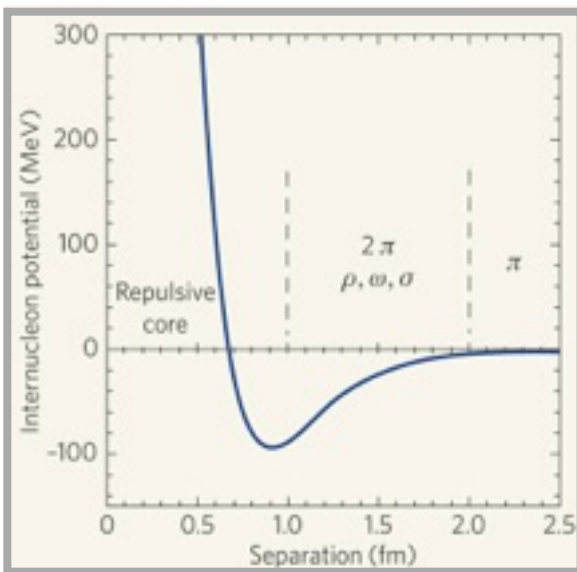
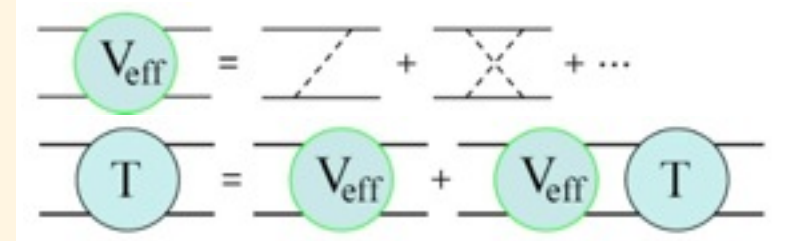
Nuclear chiral EFT

Weinberg, van Kolck, EE, Glöckle, Meißner, Machleidt, Entem...

- Schrödinger equation for nucleons interacting via contact forces and **long-range potentials (pion exchanges)**




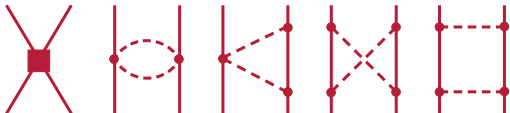






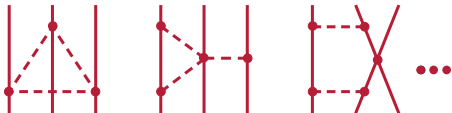
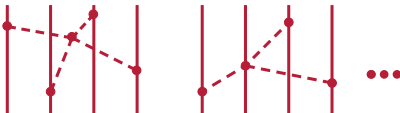
$$\left[\left(\sum_{i=1}^A \frac{-\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3}) \right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \dots}_{\text{derived in ChPT}} \right] |\Psi\rangle = E|\Psi\rangle$$

- access to heavier nuclei (ab initio few-/many-body methods)



- unified description of $\pi\pi$, πN and NN
- consistent many-body forces and currents
- systematically improvable
- bridging different reactions (electroweak, π -prod., ...)
- precision physics with/from light nuclei

Chiral expansion of nuclear forces

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO (Q^0)			
NLO (Q^2)			
N ² LO (Q^3)			
N ³ LO (Q^4)			

2N force: accurate N³LO potentials are available [Entem-Machleidt '03](#); [EE-Glöckle-Meißner '04](#)

3N force: N²LO 3NF included in most calculations

N³LO 3NF worked out [Bernard, EE, Krebs Meißner '08,'11](#); (probably) not yet converged → higher orders
numerical PWD developed [Golak, Skibinski, Krebs, Hebeler, ...](#), first results available [Witala et al.'13](#)

4N force: leading (i.e. N³LO) terms worked out [EE '06](#); contrib. to ⁴He BE ~ few 100 keV [Rospežnik et al. '06](#)

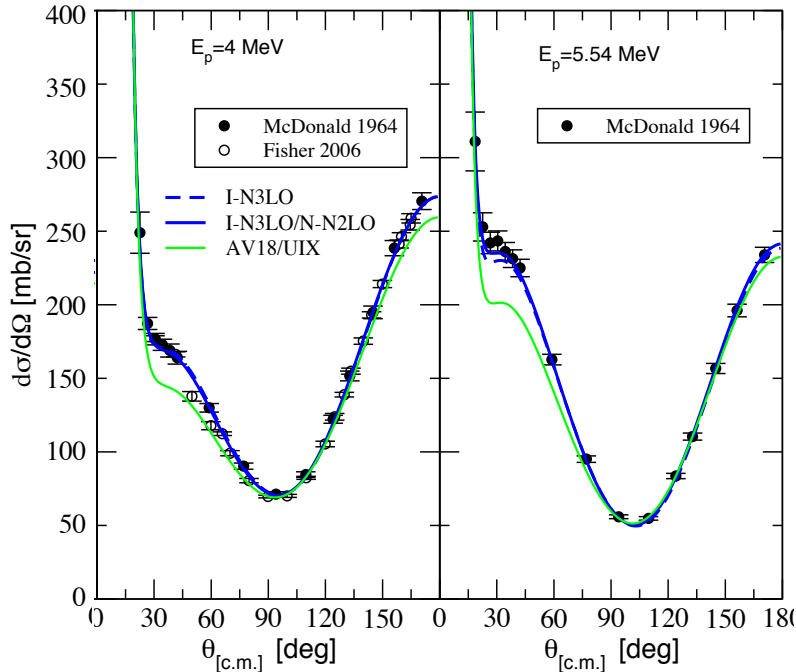
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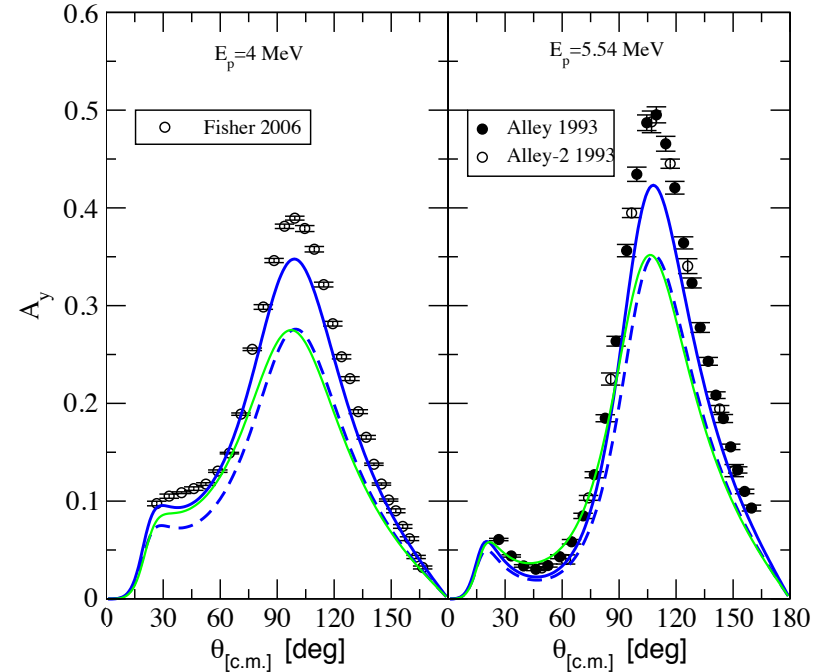
The „standard“ nuclear chiral Hamiltonian has been extensively tested in few- and many-body systems

Chiral Hamiltonian & the 3N/4N continuum

p - ^3He differential cross section



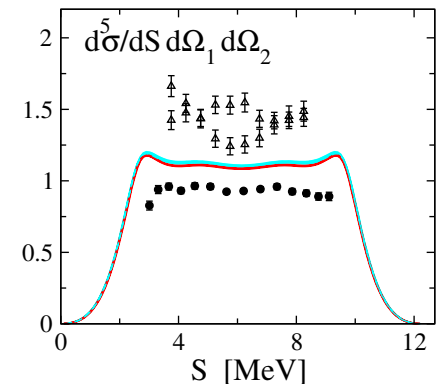
A_y -puzzle in p - ^3He elastic scattering



LECs D,E tuned to the ^3H and ^4He binding energies, figure from Viviani et al., arXiv:1004.1306

To summarize:

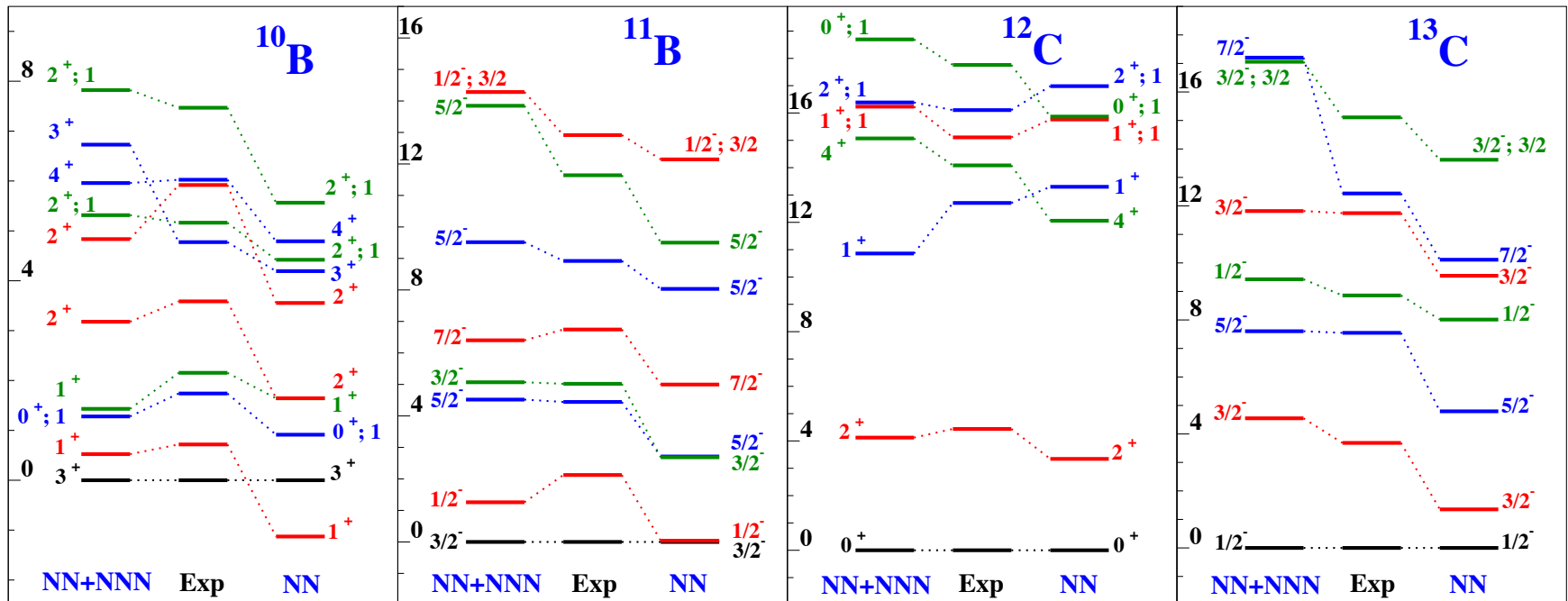
- Nd scattering: accurate description at low energy except for A_y (fine tuned) and **Space Star breakup configuration**
- Uncertainty increases with energy (**higher-order 3NF?**)
- **4N continuum**: an emerging field...



Chiral Hamiltonian & nuclear structure

Ab initio methods (NCSM, GFMC, CCM, Lattice, ...) + renormalization ideas (SRG, $V_{\text{low-k}}$, UCOM)
 + computational resources \longrightarrow precision ab initio nuclear structure calculations

NCSM calculation of p-shell nuclei with chiral 2NF+3NF Navratil et al. '07



- sensitive to details of the 3NF
- promising results (neutron-rich nuclei, long lifetime of ^{14}C , neutron star radii, ...)
- still room for improvement and some open questions

The quest for high-precision chiral Hamiltonian

Corrections to the 3NF beyond N^2LO

Ishikawa, Robilotta '06; Bernard, EE, Krebs, Meißner '07,'11;
Krebs, Gasparyan, EE'12,'13; EE, Gasparyan, Krebs, Schat, to appear

Convergence, effects of the Δ , partial wave decomposition, determination of the LECs, impact on 3N/4N scattering observables and nuclear structure...

→ **Low Energy Nuclear Physics International Collaboration (LENPIC)**
Bochum-Bonn-Cracow-Darmstadt-Iowa-Jülich-Kyushu-Ohio-Orsay

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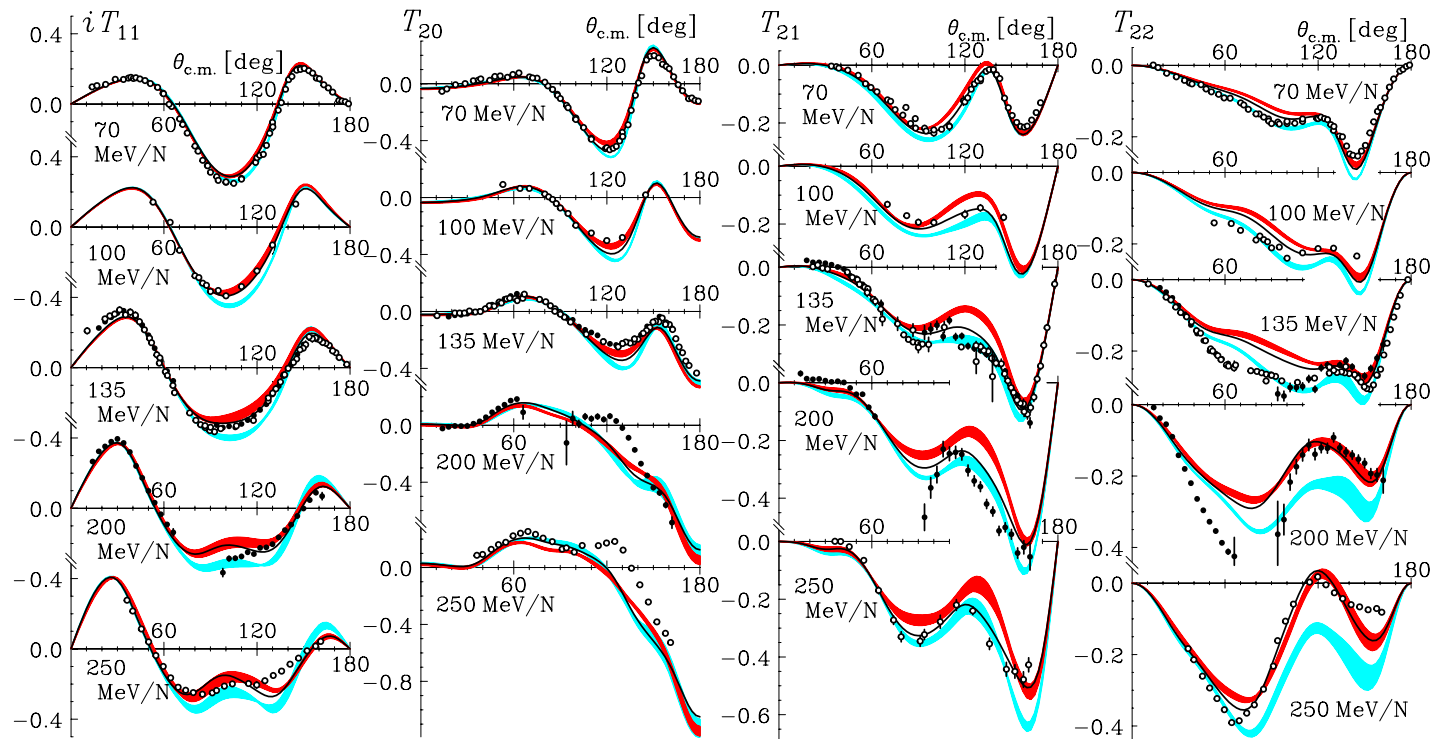
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Actually, the spin structure of the 3NF is still poorly understood in spite of decades of effort! (one of the biggest challenges in nuclear physics)



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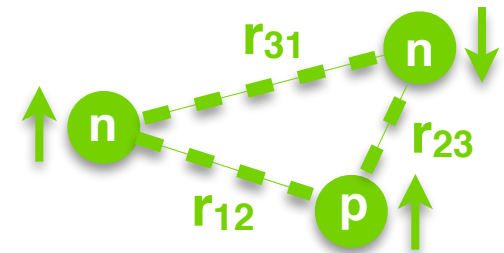
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Most general structure of a local, isospin-symmetric 3NF

Krebs, Gasparyan, EE '13; Phillips, Schat '13; EE, Gasparyan, Krebs, Schat, in preparation

Generators \mathcal{G} in momentum space	Generators $\tilde{\mathcal{G}}$ in coordinate space
$\mathcal{G}_1 = 1$	$\tilde{\mathcal{G}}_1 = 1$
$\mathcal{G}_2 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3$	$\tilde{\mathcal{G}}_2 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3$
$\mathcal{G}_3 = \vec{\sigma}_1 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_3 = \vec{\sigma}_1 \cdot \vec{\sigma}_3$
$\mathcal{G}_4 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_4 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_3$
$\mathcal{G}_5 = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_5 = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_2$
$\mathcal{G}_6 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3)$	$\tilde{\mathcal{G}}_6 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3)$
$\mathcal{G}_7 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	$\tilde{\mathcal{G}}_7 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})$
$\mathcal{G}_8 = \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_8 = \hat{r}_{23} \cdot \vec{\sigma}_1 \hat{r}_{23} \cdot \vec{\sigma}_3$
$\mathcal{G}_9 = \vec{q}_1 \cdot \vec{\sigma}_3 \vec{q}_3 \cdot \vec{\sigma}_1$	$\tilde{\mathcal{G}}_9 = \hat{r}_{23} \cdot \vec{\sigma}_3 \hat{r}_{12} \cdot \vec{\sigma}_1$
$\mathcal{G}_{10} = \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{10} = \hat{r}_{23} \cdot \vec{\sigma}_1 \hat{r}_{12} \cdot \vec{\sigma}_3$
$\mathcal{G}_{11} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{11} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \hat{r}_{23} \cdot \vec{\sigma}_1 \hat{r}_{23} \cdot \vec{\sigma}_2$
$\mathcal{G}_{12} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{12} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \hat{r}_{23} \cdot \vec{\sigma}_1 \hat{r}_{12} \cdot \vec{\sigma}_2$
$\mathcal{G}_{13} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{13} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \hat{r}_{12} \cdot \vec{\sigma}_1 \hat{r}_{23} \cdot \vec{\sigma}_2$
$\mathcal{G}_{14} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{14} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \hat{r}_{12} \cdot \vec{\sigma}_1 \hat{r}_{12} \cdot \vec{\sigma}_2$
$\mathcal{G}_{15} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{q}_2 \cdot \vec{\sigma}_1 \vec{q}_2 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{15} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \hat{r}_{13} \cdot \vec{\sigma}_1 \hat{r}_{13} \cdot \vec{\sigma}_3$
$\mathcal{G}_{16} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_2 \vec{q}_3 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{16} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \hat{r}_{12} \cdot \vec{\sigma}_2 \hat{r}_{12} \cdot \vec{\sigma}_3$
$\mathcal{G}_{17} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{17} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \hat{r}_{23} \cdot \vec{\sigma}_1 \hat{r}_{12} \cdot \vec{\sigma}_3$
$\mathcal{G}_{18} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{\sigma}_3 \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	$\tilde{\mathcal{G}}_{18} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{\sigma}_3 \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})$
$\mathcal{G}_{19} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_3 \cdot \vec{q}_1 \vec{q}_1 \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)$	$\tilde{\mathcal{G}}_{19} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_3 \cdot \hat{r}_{23} \hat{r}_{23} \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)$
$\mathcal{G}_{20} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot \vec{q}_3 \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	$\tilde{\mathcal{G}}_{20} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \hat{r}_{23} \vec{\sigma}_3 \cdot \hat{r}_{12} \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})$



Assuming hermiticity, time reversal & parity invariance, **20 structure functions** are needed:

$$V(r_{12}, r_{23}, r_{31}) = \sum_{i=1}^{20} \tilde{\mathcal{G}}_i F_i(r_{12}, r_{23}, r_{31})$$

+ permutations

$$V(q_1, q_2, q_3) = \sum_{i=1}^{20} \mathcal{G}_i F_i(q_1, q_2, q_3)$$

+ permutations

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New generation of chiral NN potentials

Reduce finite- Λ artifacts by employing regularization which maintains the analytic structure of the amplitude → **better performance at higher energies.**

l_{ocal}-chiral potentials @ LO, NLO, N^2LO

Gezerlis et al, PRL 111 (13) 032501; arXiv:1406.0454; Lynn et al. arXiv:1406.2787

i_{mproved}-chiral potentials up to N^3LO

EE, Krebs, Meißner, Golak, Skibinski, Witala; work in progress

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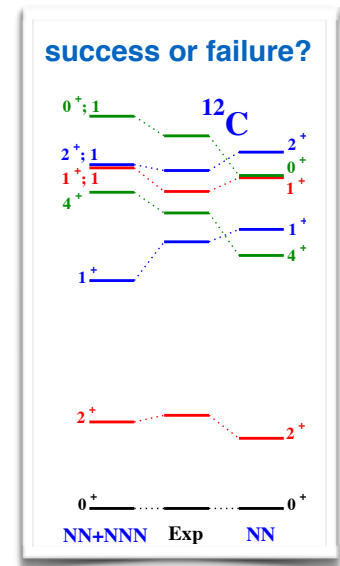
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Reliable estimation of theoretical uncertainties

is needed in order to test chiral dynamics in nuclear systems, identify/resolve possible puzzles (is A_Y -puzzle a real puzzle?), make reliable predictions and guide new experiments



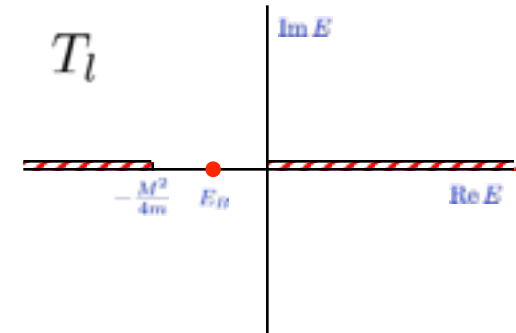
New chiral NN potential (preliminary)

- Regularization which maintains the analytic structure

Old EM/EGM potentials

$$V_{\text{long-range}}^{\text{reg}}(\vec{p}', \vec{p}) = V_{\text{long-range}}(\vec{p}', \vec{p}) \exp\left[-\frac{p^n + p'^n}{\Lambda^n}\right]$$

affects the discontinuity across the left-hand cut



New potentials

$$V_{\text{long-range}}^{\text{reg}}(\vec{r}) = V_{\text{long-range}}(\vec{r}) f_{\text{reg}}\left(\frac{r}{R}\right) \longrightarrow V_{\text{long-range}}^{\text{reg}}(\vec{q}) = V_{\text{long-range}}(\vec{q}) - \underbrace{\int d^3l V_{\text{long-range}}(\vec{q} - \vec{l}) f_{\text{reg}}(\vec{l})}_{\text{manifestly short-range}}$$

Fourier Transform[$f(r/R) - 1$]

no distortion of the long-range potential!

- No need for an additional spectral function regularization in the TPEP

- Use LECs c_i, d_i from πN scattering without any fine tuning

	Entem-Machleidt potential	Epelbaum-Glöckle-Meißner potential	πN scattering to leading one loop (Q^3)
c_1	-0.81	-0.81	-0.81 ± 0.15^b
c_2	2.80	3.28	3.28 ± 0.23^c
c_3	-3.20	-3.40	-4.69 ± 1.34^b
c_4	5.40	3.40	3.40 ± 0.04^b
$\bar{d}_1 + \bar{d}_2$	3.06	3.06	3.06 ± 0.21^c
\bar{d}_3	-3.27	-3.27	-3.27 ± 0.73^c
\bar{d}_5	0.45	0.45	0.45 ± 0.42^c
$\bar{d}_{14} - \bar{d}_{15}$	-5.65	-5.65	-5.65 ± 0.41^c

were tuned to improve the fit

taken on the lower side to avoid deeply-bound states

Convergence of the χ -expansion (preliminary)

- Total cross section for neutron-proton scattering

$E_{\text{lab}} = 25 \text{ MeV}$ [$p = 108 \text{ MeV}$]

$$R = 0.9 \text{ fm} [\Lambda = 440 \text{ MeV}]: \quad \sigma_{\text{tot}} = \overbrace{396}^{Q^0} - \overbrace{15}^{Q^2} - \overbrace{1}^{Q^3} + \overbrace{0}^{Q^4} = 380 \text{ mbarn}$$

$Q \sim 0.3 \rightarrow \text{expect: } \sim 40 \quad \sim 12 \quad \sim 4$

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$E_{\text{lab}} = 200 \text{ MeV}$ [$p = 306 \text{ MeV}$]

$$R = 0.9 \text{ fm } [\Lambda = 440 \text{ MeV}]: \quad \sigma_{\text{tot}} = 35 + \overbrace{15} - \overbrace{7} + \overbrace{0} = 43 \text{ mbarn}$$

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$$R = 1.2 \text{ fm } [\Lambda = 330 \text{ MeV}]: \quad \sigma_{\text{tot}} = 17 + 6 + 1 + 14 = 38 \text{ mbarn}$$

$Q \sim 1 \rightarrow \text{expect no convergence!}$

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$Q \sim 0.7 \rightarrow \text{expect: } \sim 17 \quad \sim 12 \quad \sim 8$

$$R = 1.2 \text{ fm } [\Lambda = 330 \text{ MeV}]: \quad \sigma_{\text{tot}} = 17 + 6 + 1 + 14 = 38 \text{ mbarn}$$

$Q \sim 1 \rightarrow \text{expect no convergence!}$

● Total cross section for neutron-deuteron scattering

$E_{\text{lab}} = 10 \text{ MeV}$ [$p = 69 \text{ MeV}$]

$$R = 0.9 \text{ fm } [\Lambda = 440 \text{ MeV}]: \quad \sigma_{\text{tot}} = 918 + \overbrace{85} + \overbrace{2} + \overbrace{11} = 1046 \text{ mbarn}$$

$Q \sim 0.3 \rightarrow \text{expect: } \sim 80 \quad \sim 24 \quad \sim 7$

$E_{\text{lab}} = 200 \text{ MeV}$ [$p = 306 \text{ MeV}$]

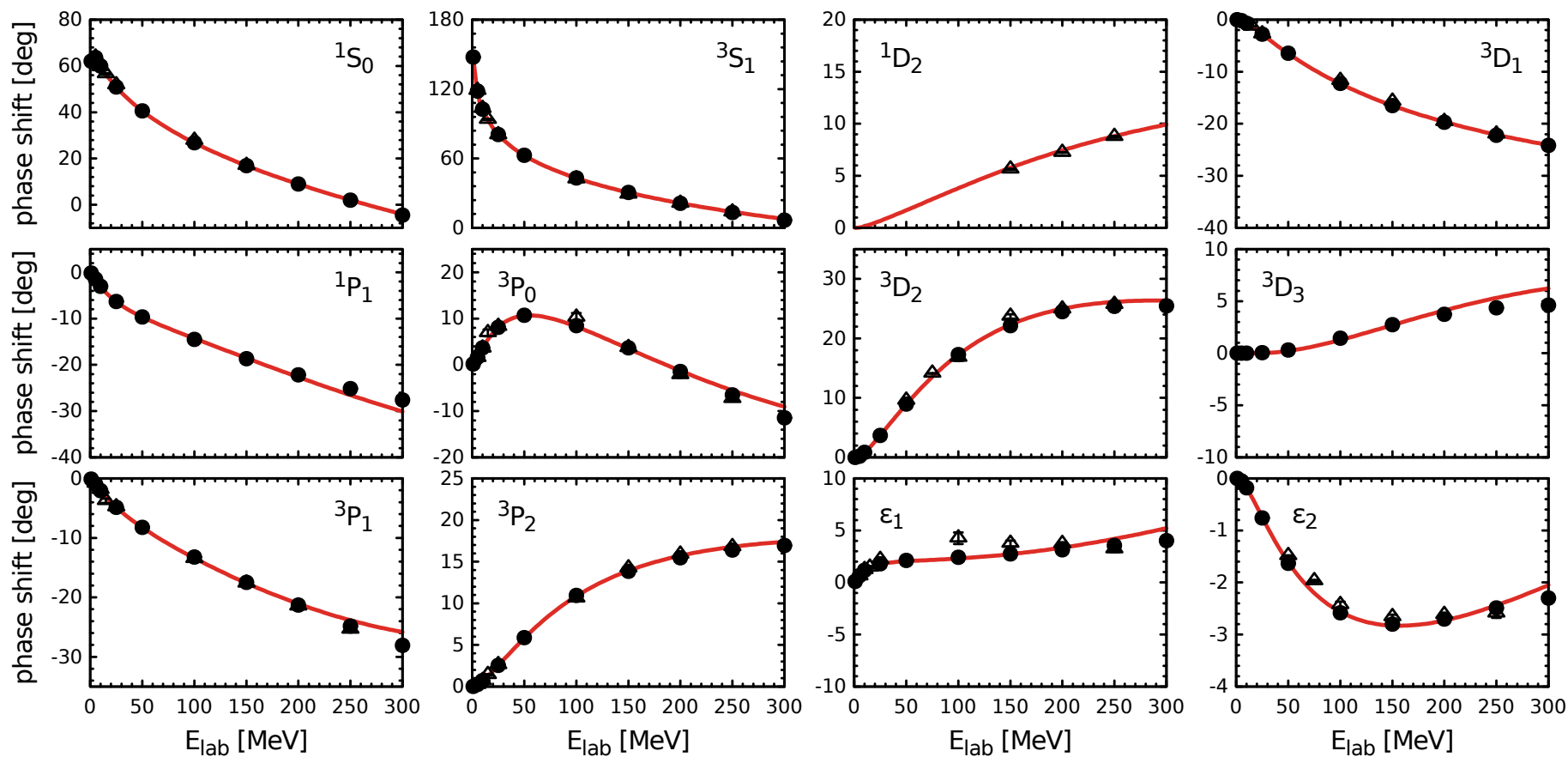
$$R = 0.9 \text{ fm } [\Lambda = 440 \text{ MeV}]: \quad \sigma_{\text{tot}} = 43 + \overbrace{11} + \overbrace{8} - \overbrace{1} = 61 \text{ mbarn}$$

$Q \sim 0.7 \rightarrow \text{expect: } \sim 21 \quad \sim 14 \quad \sim 10$

$$R = 1.2 \text{ fm } [\Lambda = 330 \text{ MeV}]: \quad \sigma_{\text{tot}} = 20 + 10 + 4 + 20 = 54 \text{ mbarn}$$

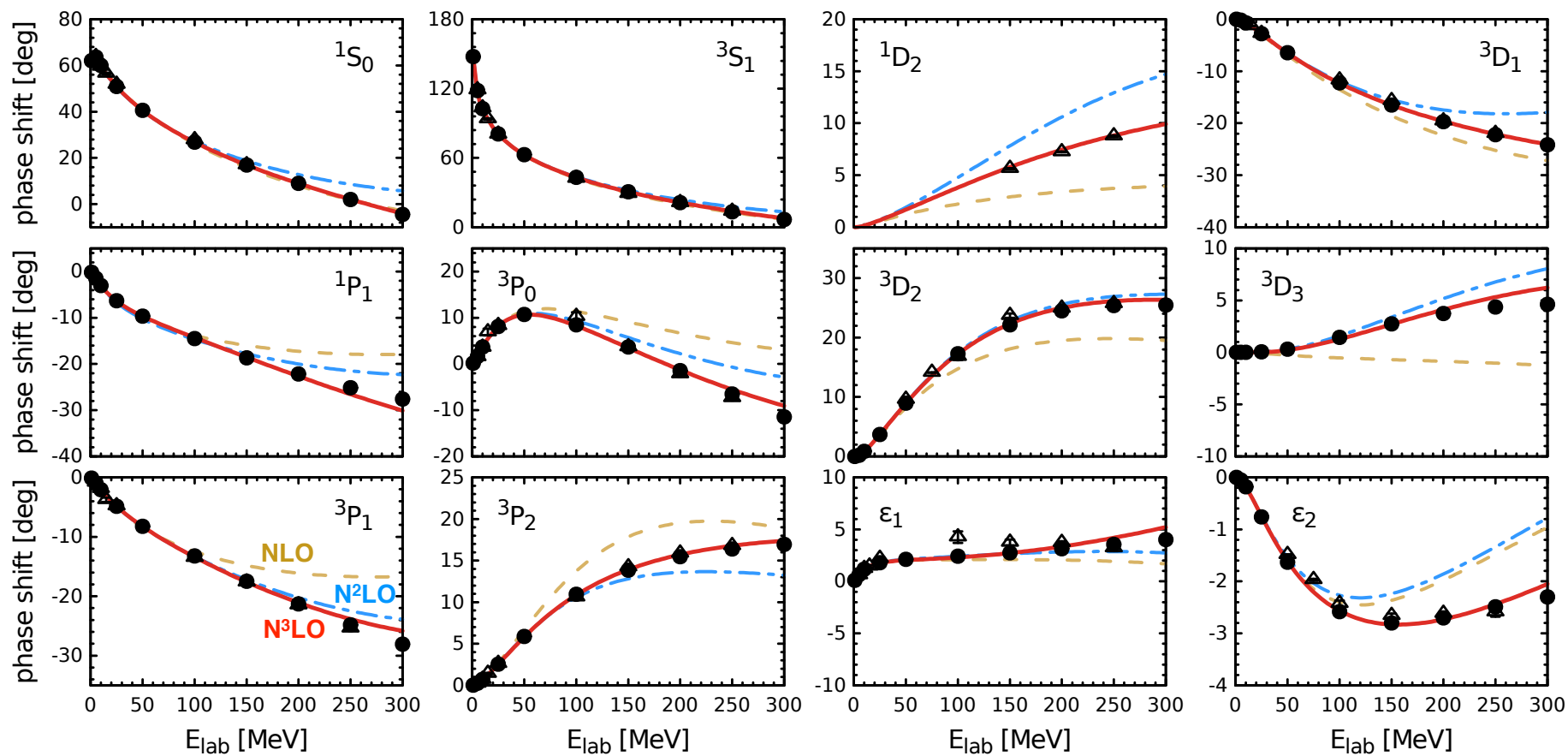
$Q \sim 1 \rightarrow \text{expect no convergence!}$

Uncertainty in phase shifts (preliminary)

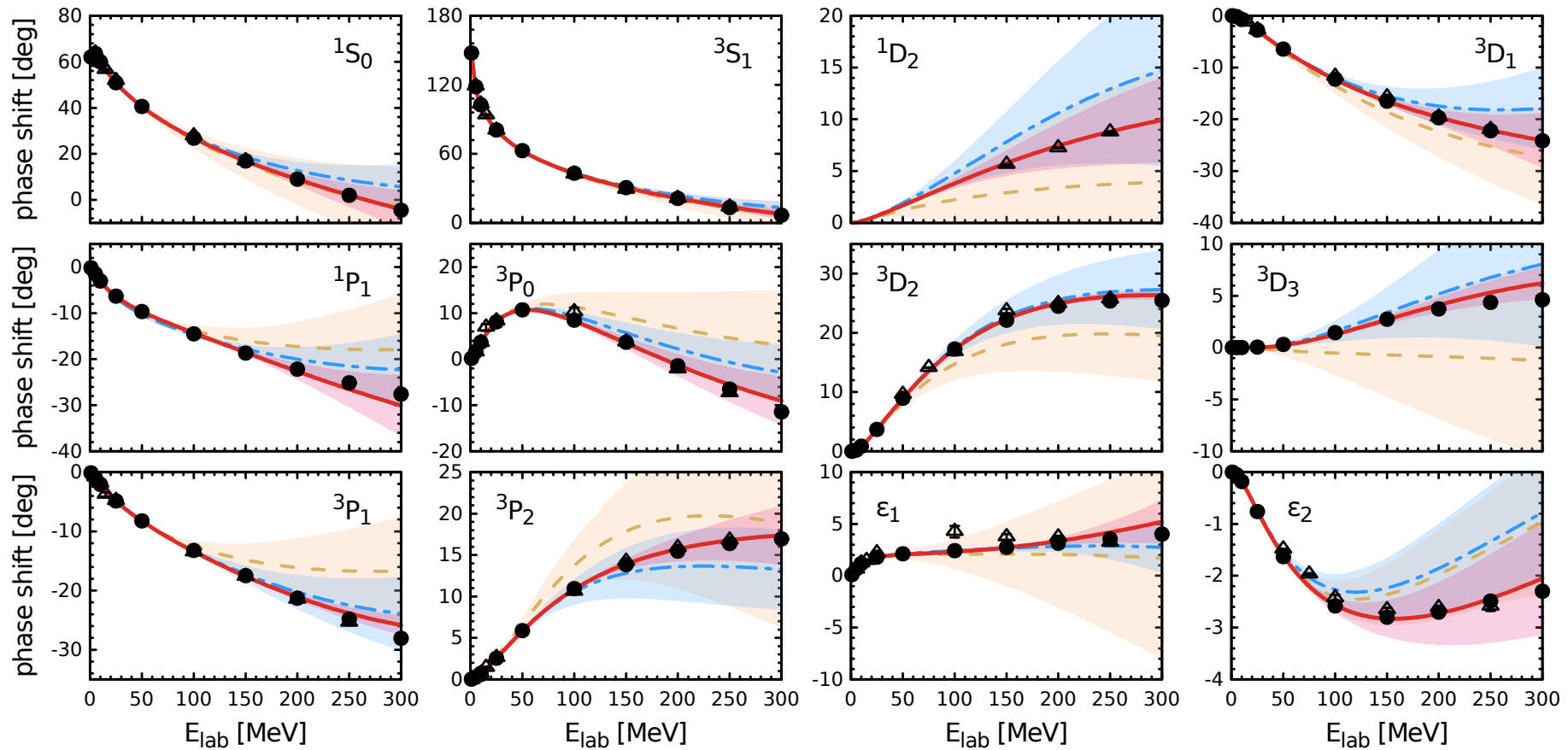


The description of phase shifts at N³LO is nearly perfect. **But what is the expected theoretical uncertainty?**

Uncertainty in phase shifts (preliminary)



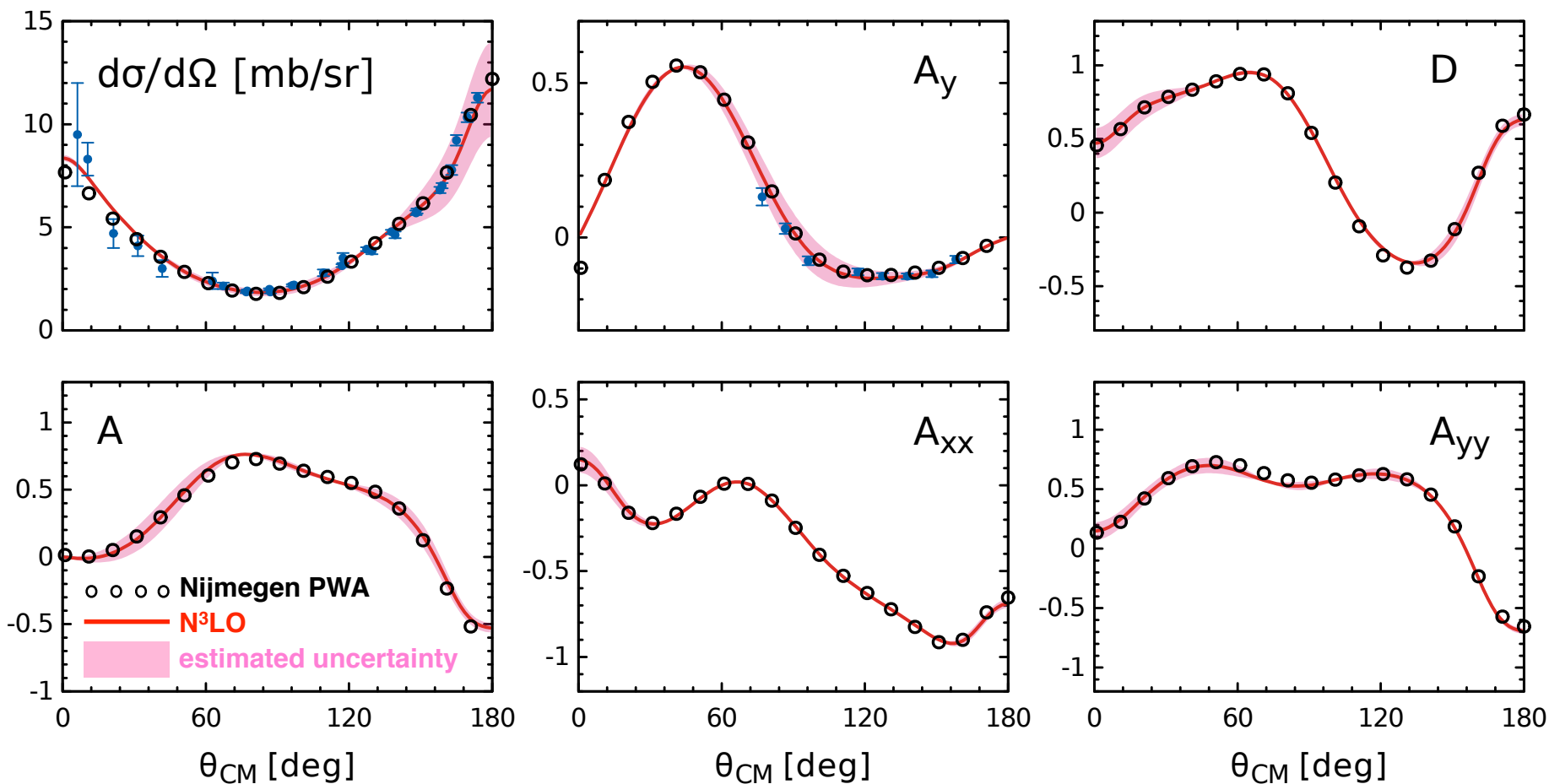
Uncertainty in phase shifts (preliminary)



For an observable $X(p)$, we assign: $\Delta X^{(n)}(p) = \text{Abs}(X^{(n)}(p) - X^{(n-1)}(p)) \times \text{Max}\left(\frac{p}{\Lambda}, \frac{M_\pi}{\Lambda}\right)$
to estimate the size of corrections beyond the order Q^n .
(if higher-order corrections are available, we use them as a lower bound)

Nucleon-nucleon scattering (preliminary)

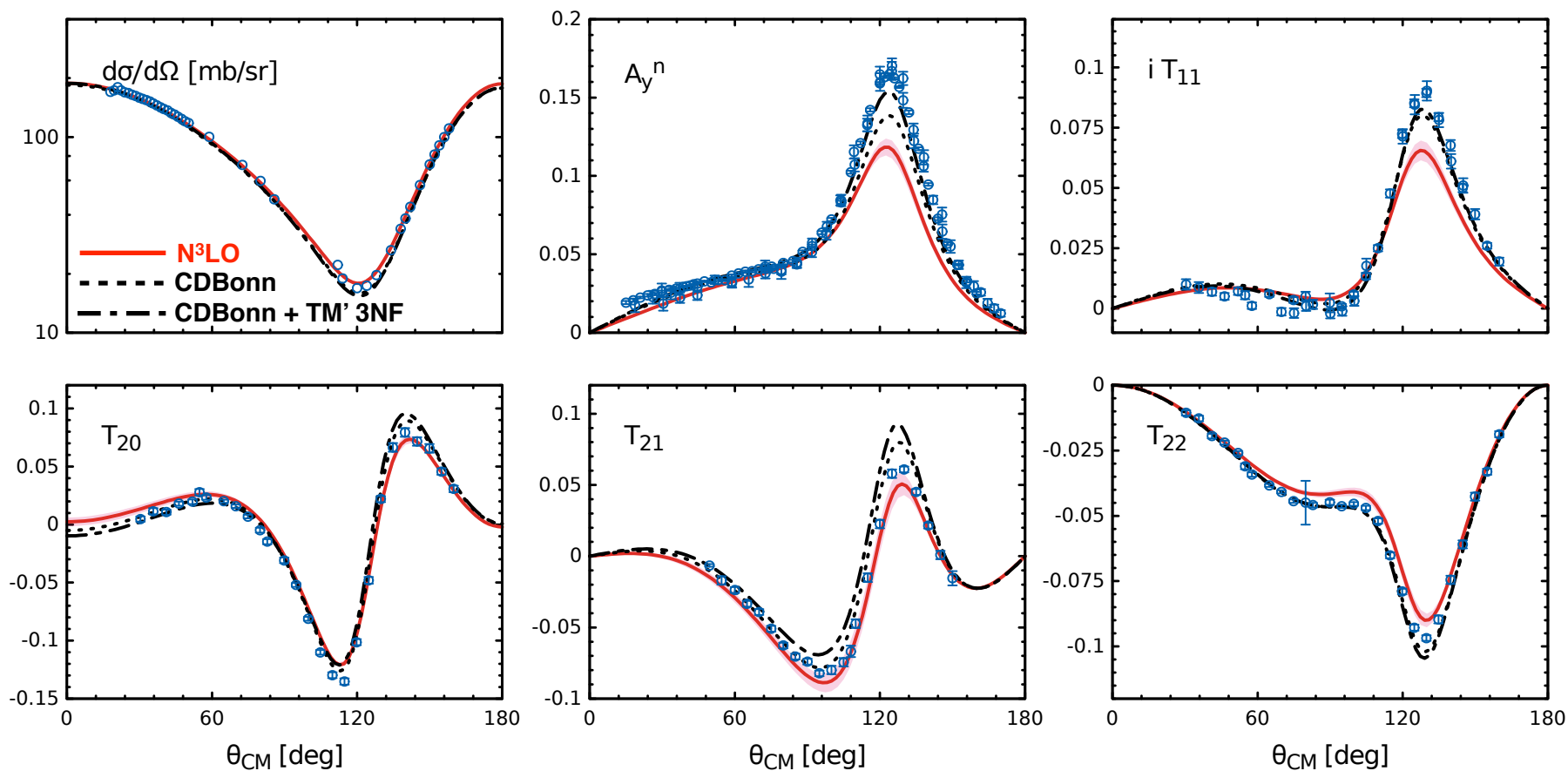
Selected neutron-proton scattering observables at $E_{\text{lab}} = 200$ MeV
(preliminary results with i_{improved} -chiral $N^3\text{LO}$ potential)



At $N^3\text{LO}$, 2N observables are accurately described up to at least $E_{\text{lab}} \sim 200$ MeV

Elastic Nd scattering with N³LO 2NF

Selected neutron-deuteron scattering observables at $E_{\text{lab}} = 10$ MeV
(preliminary results with i_{improved} -chiral N³LO potential)

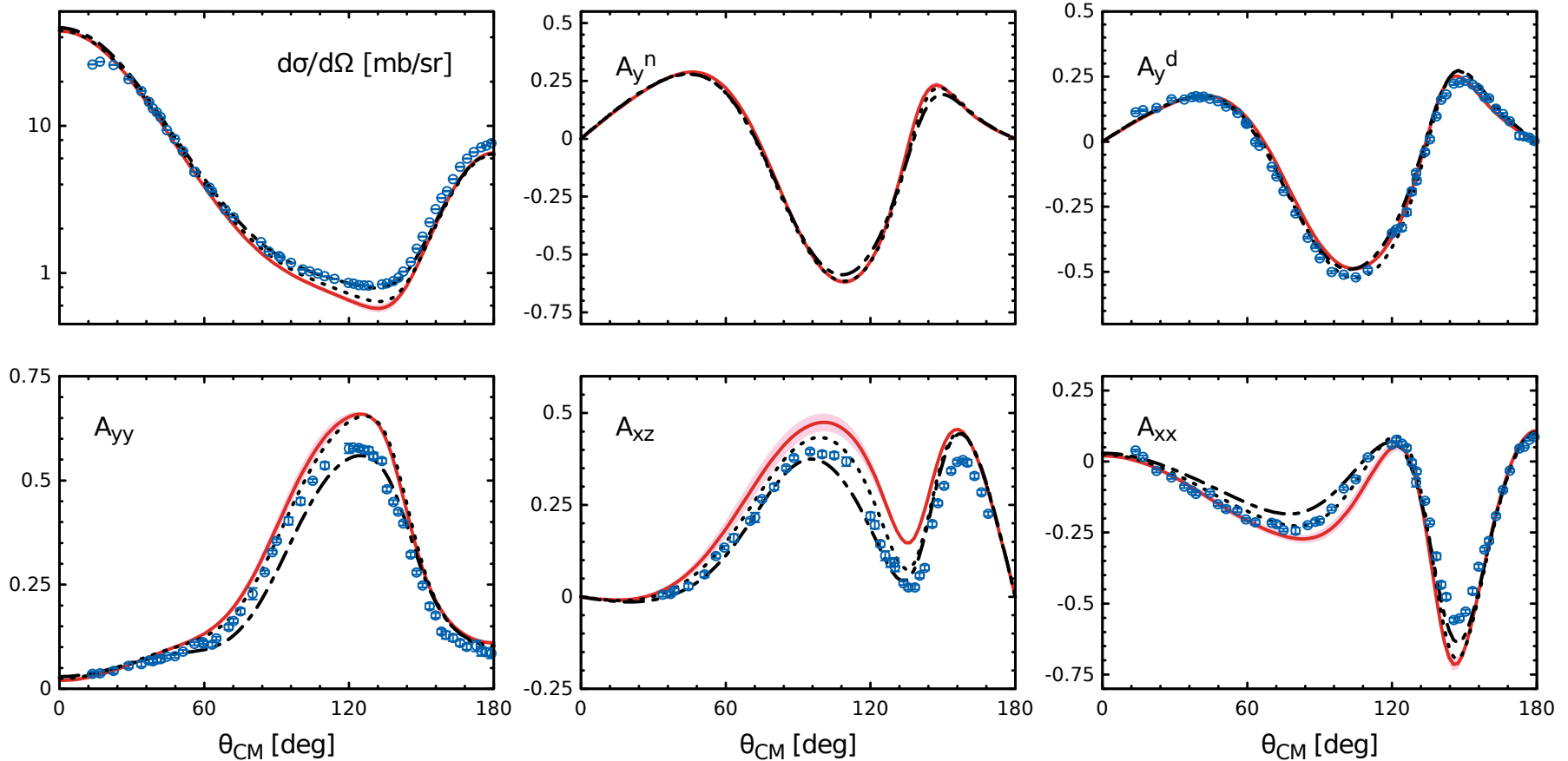


Clear room for 3NF effects in A_y and iT_{11}

Most of the data are Coulomb-corrected pd data

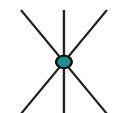
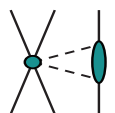
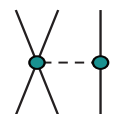
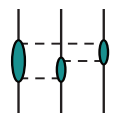
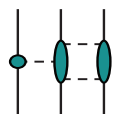
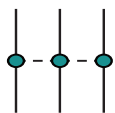
Elastic Nd scattering with N³LO 2NF

Selected neutron-deuteron scattering observables at $E_{\text{lab}} = 70$ MeV
(preliminary results with i_{improved} -chiral N³LO potential)



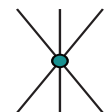
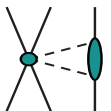
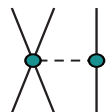
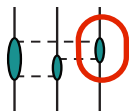
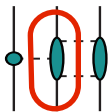
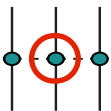
Clear room for 3NF effects in the cross section and A_{ij}

Chiral expansion of the 3NF (Δ -less EFT)

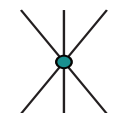
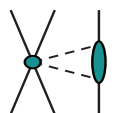
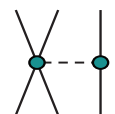
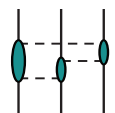
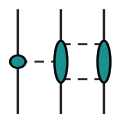
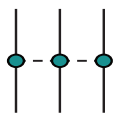


Chiral expansion of the 3NF (Δ -less EFT)

3NF structure functions at large distance are model-independent and parameter-free predictions based on χ symmetry of QCD + exp. information on πN system

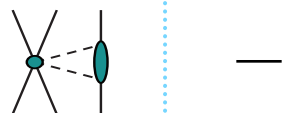
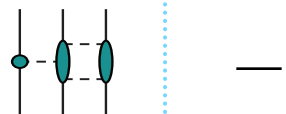
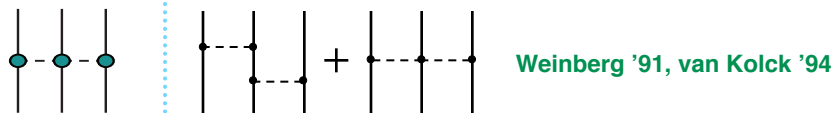


Chiral expansion of the 3NF (Δ -less EFT)



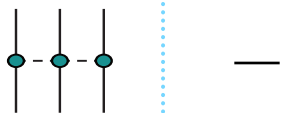
Chiral expansion of the 3NF (Δ -less EFT)

NLO (Q^2)

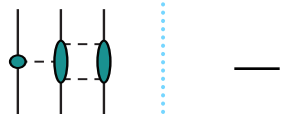


Chiral expansion of the 3NF (Δ -less EFT)

NLO (Q^2)



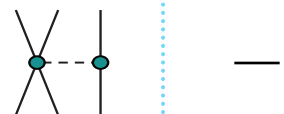
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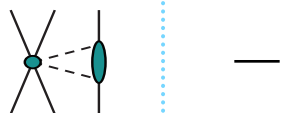
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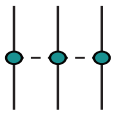
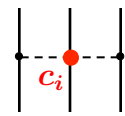
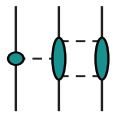
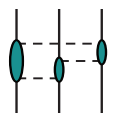
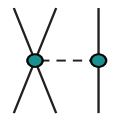
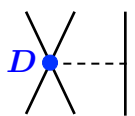
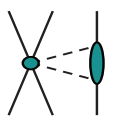
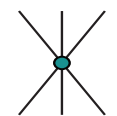
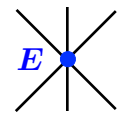


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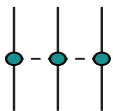
Chiral expansion of the 3NF (Δ -less EFT)

	NLO (Q^2)	N ² LO (Q^3)
	—	
	—	—
	—	—
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	—	—
	—	

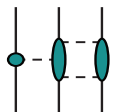
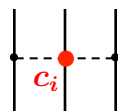
Chiral expansion of the 3NF (Δ -less EFT)

NLO (Q^2)

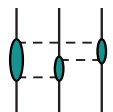
N²LO (Q^3)



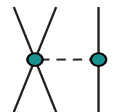
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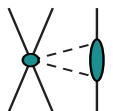
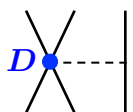
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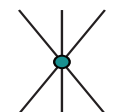
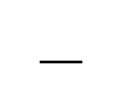
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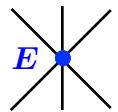
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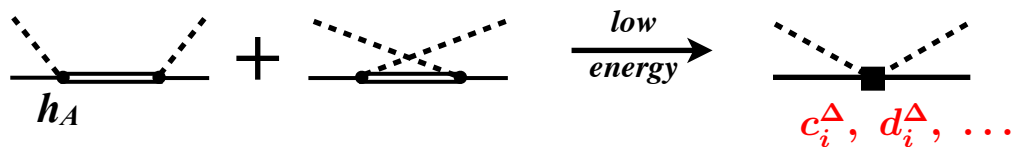


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Notice: c_i receive large $\Delta(1232)$ contributions

Bernard, Kaiser, Meißner '97

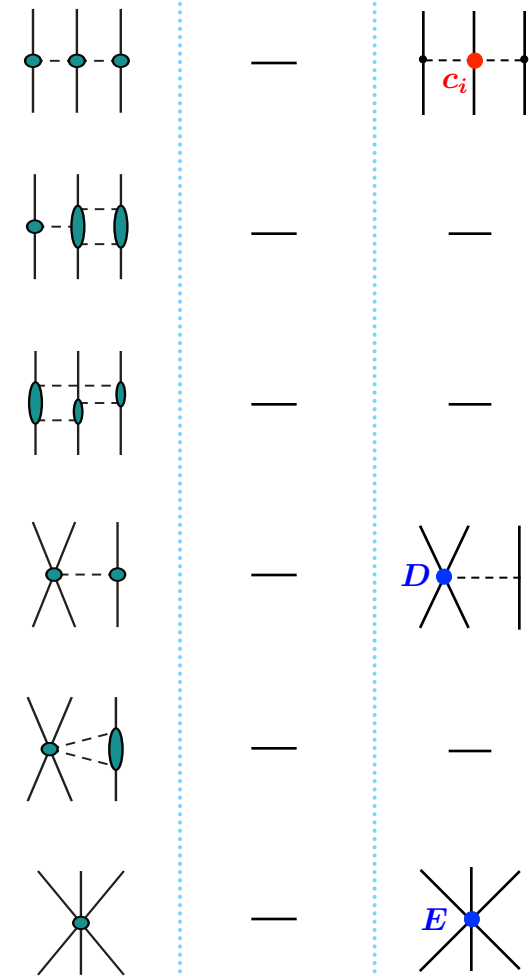


$$c_2^\Delta = -c_3^\Delta = 2c_4^\Delta = \frac{4h_A^2}{9(m_\Delta - m_N)} \simeq 2.8 \text{ GeV}^{-1}$$

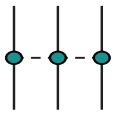
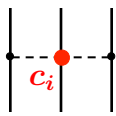
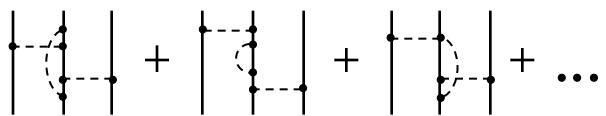
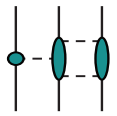
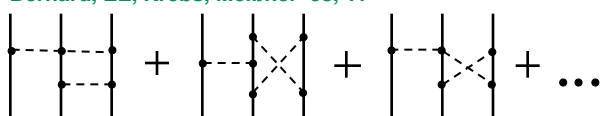
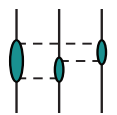
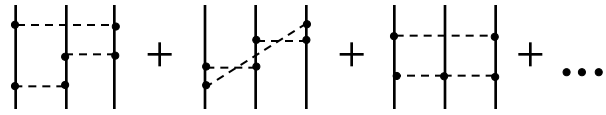
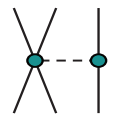
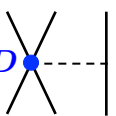
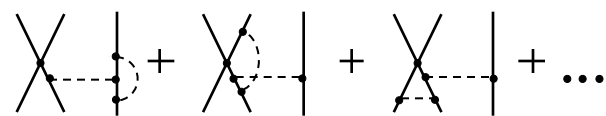
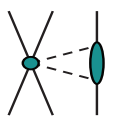
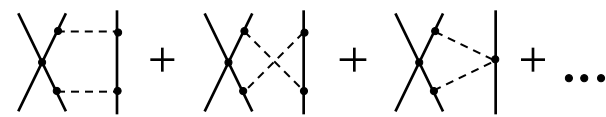
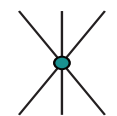
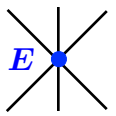
Chiral expansion of the 3NF (Δ -less EFT)

NLO (Q^2)

N²LO (Q^3)



Chiral expansion of the 3NF (Δ -less EFT)

	NLO (Q^2)	N ² LO (Q^3)	N ³ LO (Q^4)
	—		
	—	—	
	—	—	
	—		
	—	—	
	—		—

Ishikawa, Robilotta '08
Bernard, EE, Krebs, Meißner '08,'11

Chiral expansion of the 3NF (Δ -less EFT)

	NLO (Q^2)	N ² LO (Q^3)	N ³ LO (Q^4)
	—		
	—	—	
	—	—	
	—		
	—	—	
	—		—

Ishikawa, Robilotta '08
Bernard, EE, Krebs, Meißner '08,'11

- parameter-free!
- Δ -effects are missing (except for the 2π 3NF) \rightarrow expect large N⁴LO corrections

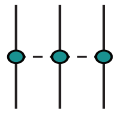
Chiral expansion of the 3NF (Δ -less EFT)

NLO (Q^2)

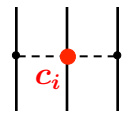
N²LO (Q^3)

N³LO (Q^4)

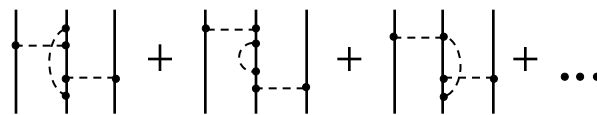
N⁴LO (Q^5)



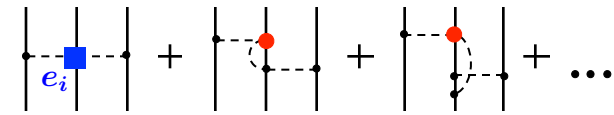
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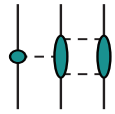
c_i



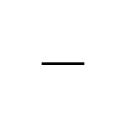
Ishikawa, Robilotta '08
Bernard, EE, Krebs, Meißner '08,'11



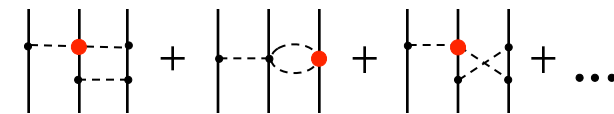
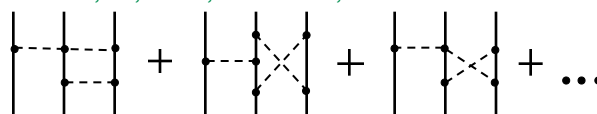
e_i
Krebs, Gasparyan, EE '12



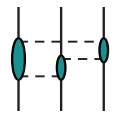
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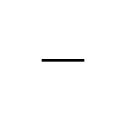
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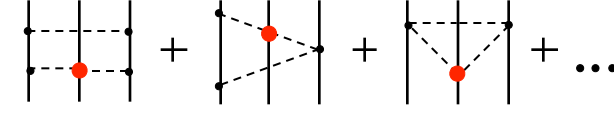
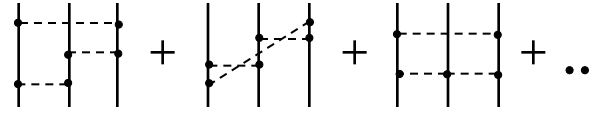
Krebs, Gasparyan, EE '13



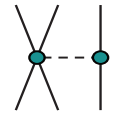
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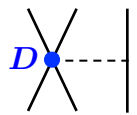
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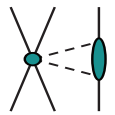
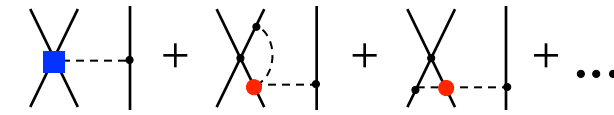
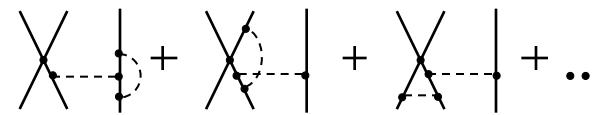
Krebs, Gasparyan, EE '13



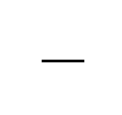
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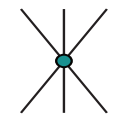
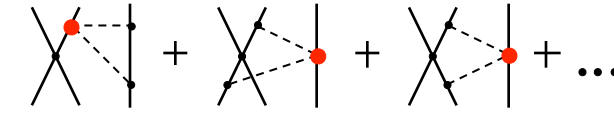
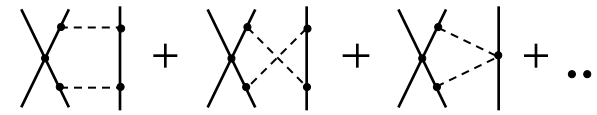
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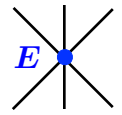
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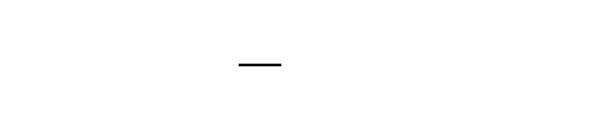
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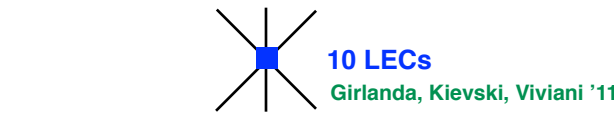
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E



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10 LECs
Girlanda, Kievski, Viviani '11

- parameter-free!
- Δ -effects are missing (except for the 2π 3NF) \rightarrow expect large N⁴LO corrections

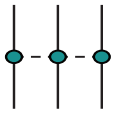
Chiral expansion of the 3NF (Δ -less EFT)

NLO (Q^2)

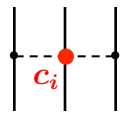
N²LO (Q^3)

N³LO (Q^4)

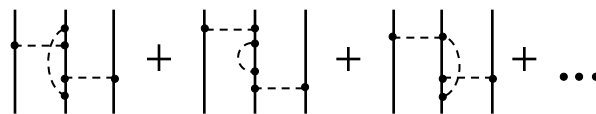
N⁴LO (Q^5)



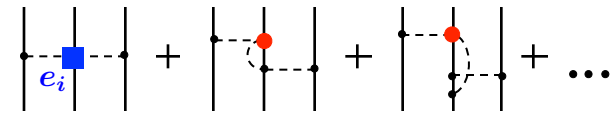
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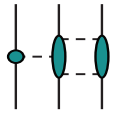
c_i



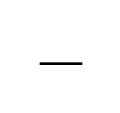
Ishikawa, Robilotta '08
Bernard, EE, Krebs, Meißner '08,'11



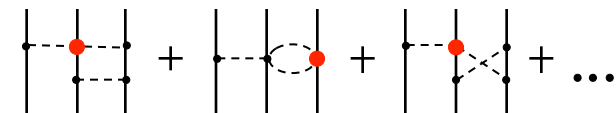
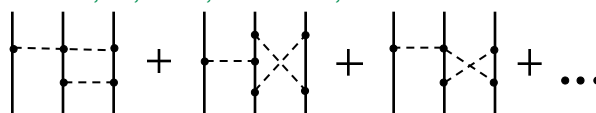
Krebs, Gasparyan, EE '12



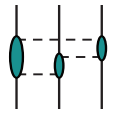
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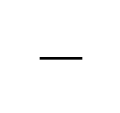
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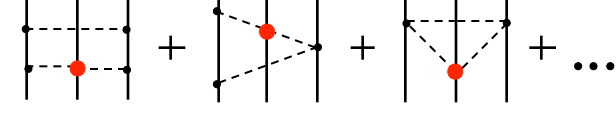
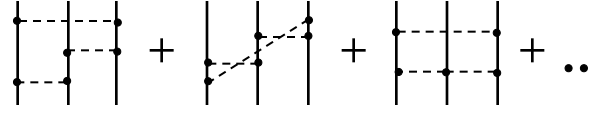
Krebs, Gasparyan, EE '13



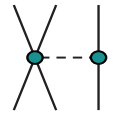
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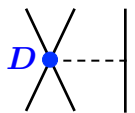
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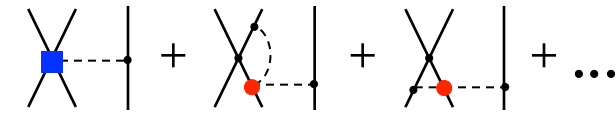
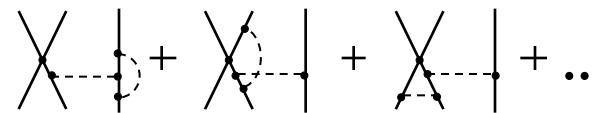
Krebs, Gasparyan, EE '13



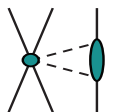
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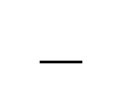
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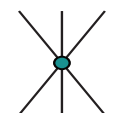
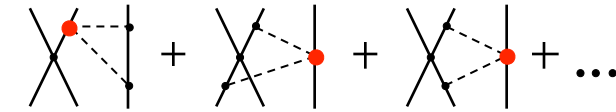
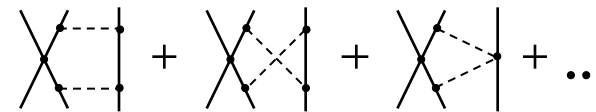
e_i



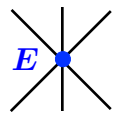
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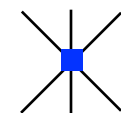


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E

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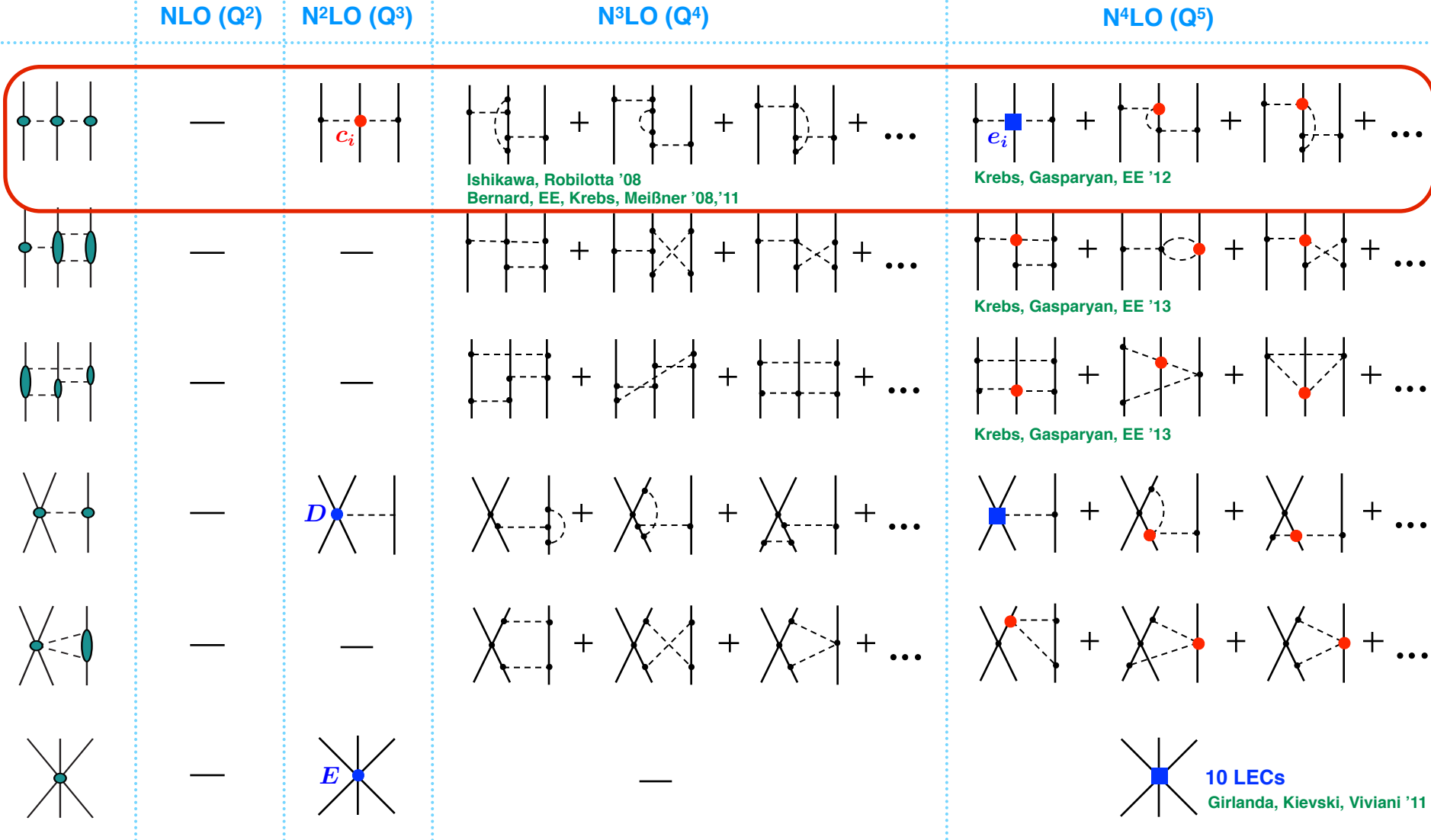
10 LECs

Girlanda, Kievski, Viviani '11

- parameter-free!
- Δ -effects are missing (except for the 2π 3NF) \rightarrow expect large N⁴LO corrections

- long range parameter-free (after determination of LECs in π N)
- converged?? (graphs $\sim c_i^2, c_i^3 \dots$)

Chiral expansion of the 3NF (Δ -less EFT)



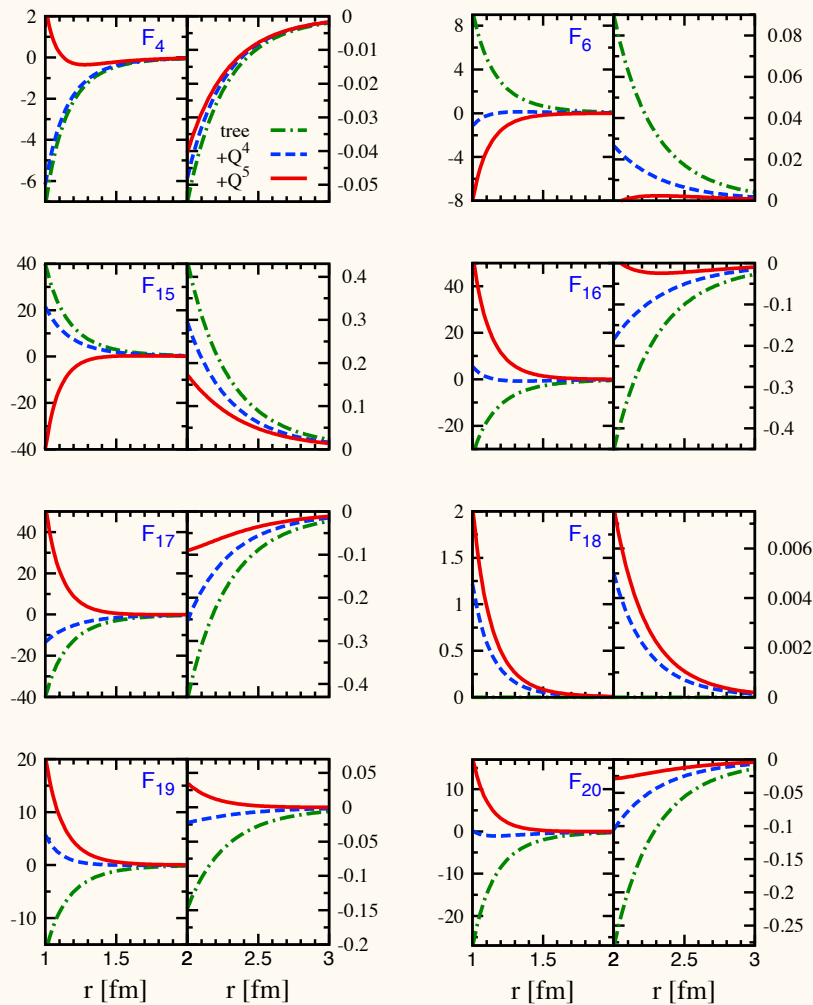
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2π 3NF: Δ -full vs Δ -less (preliminary)

Krebs, Gasparyan, EE, to appear

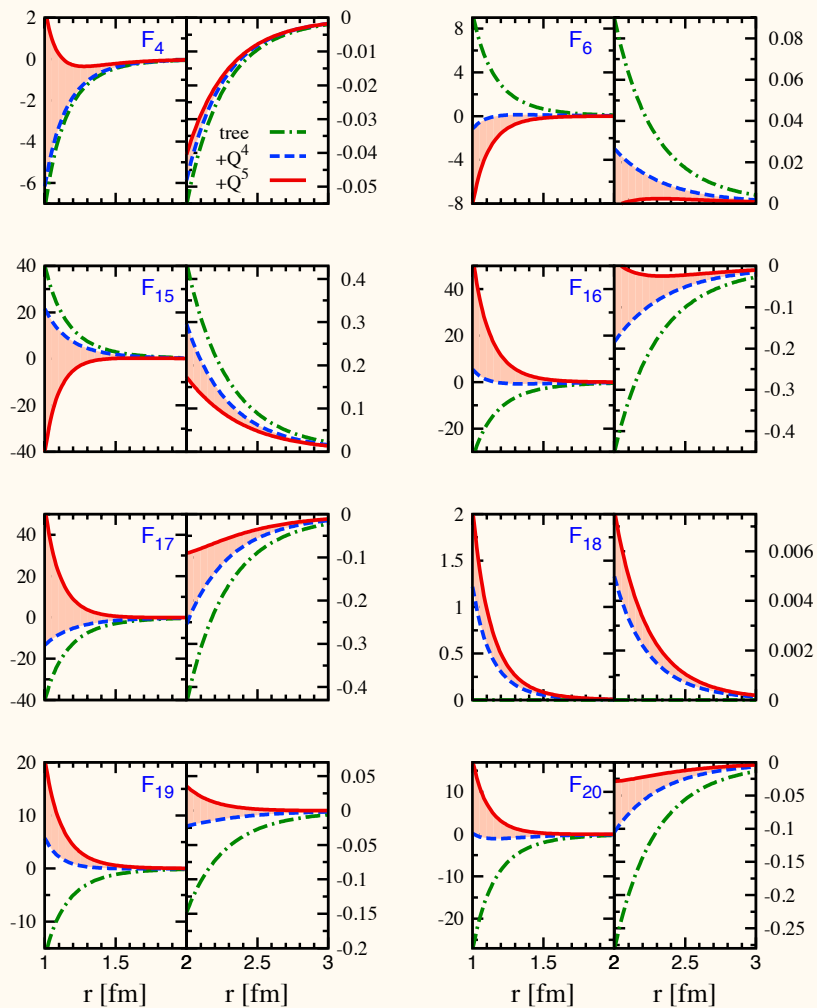
$F_i(r_{12}=r_{23}=r_{31})$ in units of MeV: Δ -less EFT



2π 3NF: Δ -full vs Δ -less (preliminary)

Krebs, Gasparyan, EE, to appear

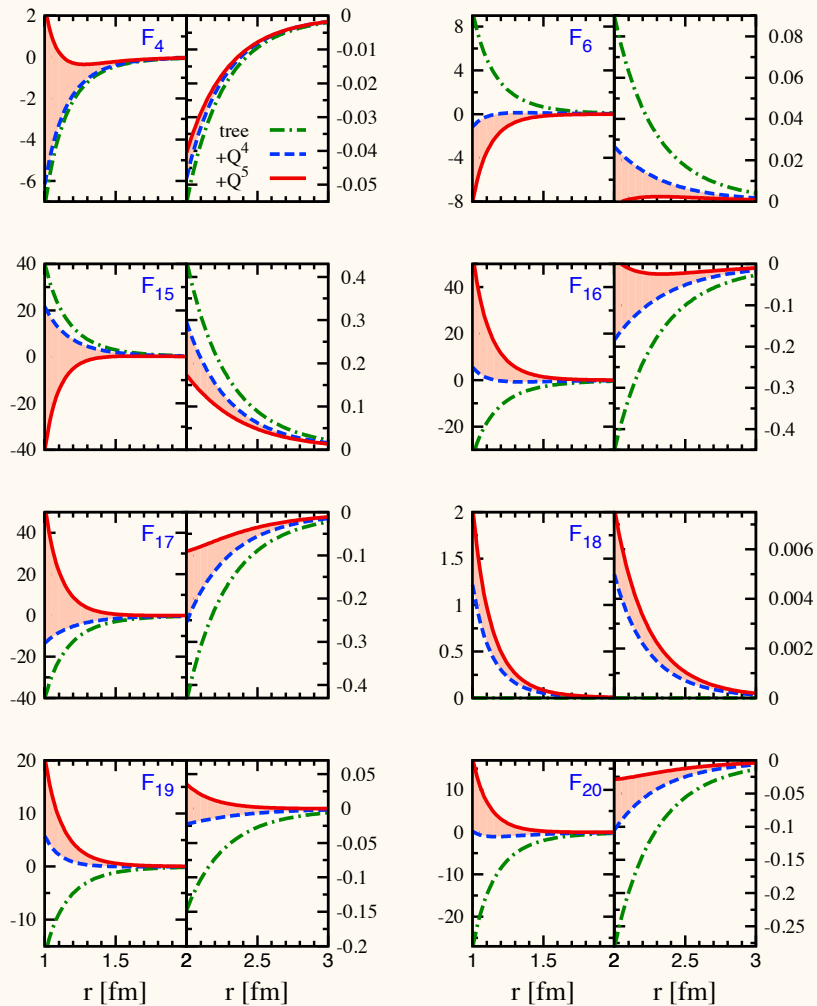
$F_i(r_{12}=r_{23}=r_{31})$ in units of MeV: Δ -less EFT



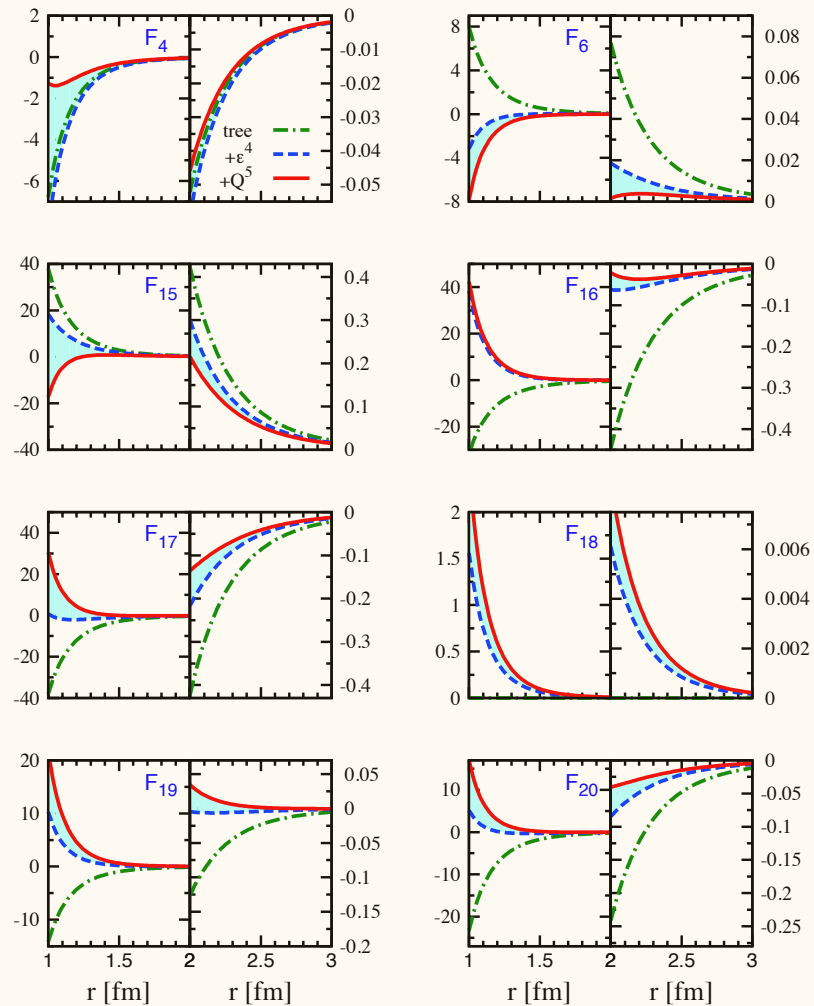
2π 3NF: Δ -full vs Δ -less (preliminary)

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$F_i(r_{12}=r_{23}=r_{31})$ in units of MeV: Δ -less EFT



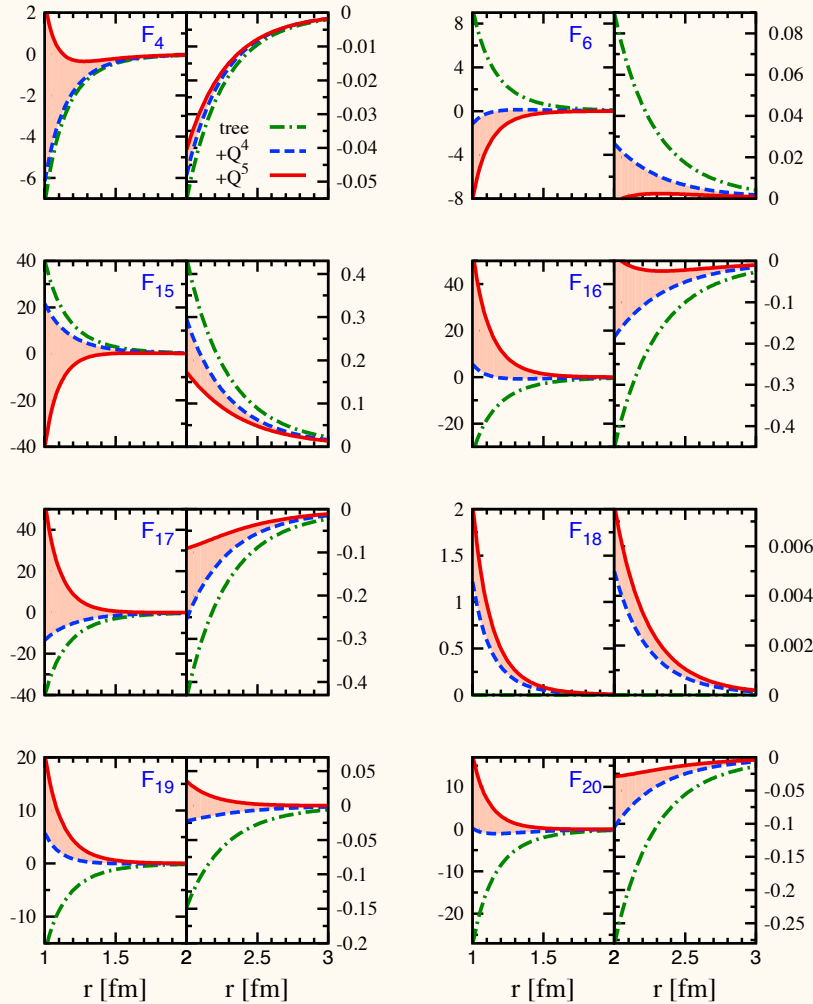
$F_i(r_{12}=r_{23}=r_{31})$ in units of MeV: Δ -full EFT



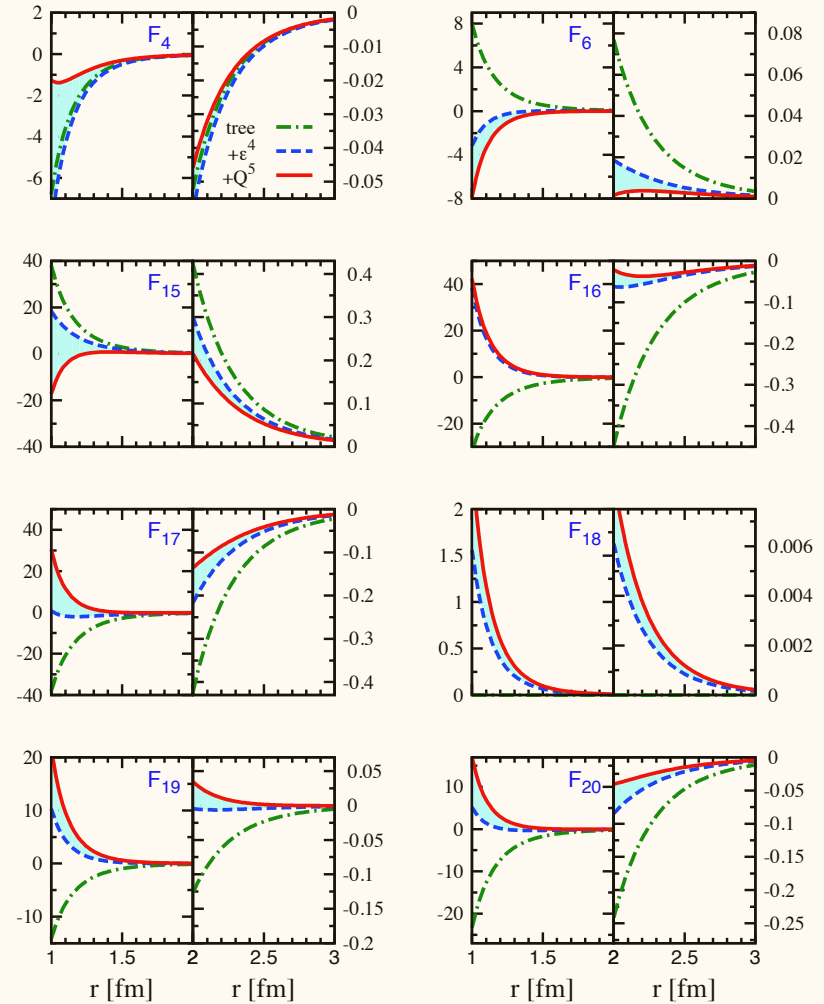
2π 3NF: Δ -full vs Δ -less (preliminary)

Krebs, Gasparyan, EE, to appear

$F_i(r_{12}=r_{23}=r_{31})$ in units of MeV: Δ -less EFT



$F_i(r_{12}=r_{23}=r_{31})$ in units of MeV: Δ -full EFT



- Δ -full and Δ -less EFT predictions agree well with each other
- Δ -full approach shows clearly a superior convergence
- remarkably, the final 2π 3NF turns out to be rather weak at large distances...

Summary

Chiral two-nucleon force

- A new generation of chiral NN potentials is underway
 - regulator maintains analytic structure of the ampl. → better performance at high E
 - no need for additional spectral function regularization
 - LECs from πN without fine tuning

Error estimations

- Instead of a cutoff variation, we suggest to use a standard ChPT procedure by estimating the size of the (missing) higher-order corrections.

Three-nucleon force

- Clear evidence for missing 3NF effects in elastic Nd scattering at $E \sim 70-150$ MeV
- 3NF beyond N²LO:
 - Worked out completely at N³LO and, to a large extent, at N⁴LO. The N⁴LO contributions are driven by the Δ and appear to be large (as expected).
 - Alternatively, EFT with explicit Δ DOF. For 2π 3NF both approaches yield similar results (with the Δ -full approach converging faster). Other topologies in progress.
- Good progress on the PWD of the 3NF.