Partial wave analysis of NN scattering data in chiral EFT

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- New local regulator for long-range interaction
- Employs πN LECs from recent Roy-Steiner eq. analysis [Hoferichter et al. ‘15]
- NN contact LECs fitted to 2013 Granada database of NN scattering data

See talk by Evgeny Epelbaum

Introduction additional leading F-wave contact interactions (N^4LO^+ ) allowed precise description of data (\( \chi^2/\text{datum} \sim 1 \))

Inclusion of isospin breaking effects limited to Pion-Mass splitting in OPE and charge-dependent short-range interactions in \(^1S_0\)

Framework and precision allow to study full inclusion of IB effects
CIB interactions

Pion-mass-splitting + πγ-exchange = CIB contact

Parameter-free for nuclear forces:

1. \( \delta M_\pi = M_{\pi^\pm} - M_{\pi^0} \)
2. Use physical pion masses: \( M_{\pi^\pm} = 139.57 \text{ MeV}, \ M_{\pi^0} = 134.98 \text{ MeV} \)

References:
- Friar, van Kolck ‘99
- Epelbaum ‘05
- van Kolck ‘98
• Use physical nucleon mass shift $\delta m_N = -1.2933\,\text{MeV}$ where strong and e.m. mass shift contributions can be added together.

• If not, use strong mass shift $\delta m_N^{\text{str.}} = -2.05\,\text{MeV}$ determined via Cottingham sum rule [Gasser, Leutwyler ‘82] See talk by Andre Walker-Loud

Have to determine:

• $2d_{17} - d_{18} - 2d_{19}$ (contributes to CSB OPE and subleading CSB TPE)
• CSB contacts entering S- & P-Waves
Fitting of contacts & charge-dependent OPE

General OPE without Isospin limit:

\[ V_{1\pi}(pp) = f_p^2 \ V(M_{\pi^0}) \]
\[ V_{1\pi}(np) = -f_0^2 \ V(M_{\pi^0}) + (-1)^{t+1} 2f_c^2 \ V(M_{\pi^\pm}) \]
\[ V_{1\pi}(nn) = f_n^2 \ V(M_{\pi^0}) \]

With \( V(M_i) = -\frac{4\pi}{M_{\pi^\pm}^2} \ \vec{\sigma}_1 \cdot \vec{q} \ \vec{\sigma}_2 \cdot \vec{q} \), \( f_0^2 = f_p f_n \)

- Fit 24 + 6 IB contact LECs + \( f_p, f_n, f_c \) = 33 parameters
- Additional input: \( B_d = 2.224575(9) \) MeV and \( b_{np} = -3.7405(9) \) fm
- Increase energy range of data to \( E_{\text{lab}} = 0 - 280 \) MeV

\[ \chi^2/\text{datum} \]
<table>
<thead>
<tr>
<th>( E_{\text{lab}} = ) 0-280 MeV:</th>
<th>Without additional IB effects</th>
<th>With additional IB effects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.023</td>
<td>1.016 ( (\Lambda = 450 ) MeV)</td>
</tr>
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</table>
# Charge-dependence of OPE coupling

- Overall small improvement
- Largest effects for np data at low energies

<table>
<thead>
<tr>
<th>$E_{\text{lab}}$ bin</th>
<th>SMS N$^4$LO$^+$</th>
<th>SMS N$^4$LO$^+$ add. IB</th>
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<tbody>
<tr>
<td><strong>neutron-proton scattering data</strong></td>
<td></td>
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<tr>
<td>0–100</td>
<td>1.077</td>
<td>1.058 (+0.002)</td>
</tr>
<tr>
<td>0–200</td>
<td>1.070</td>
<td>1.055 (+0.002)</td>
</tr>
<tr>
<td>0–300</td>
<td>1.061</td>
<td>1.053 (+0.001)</td>
</tr>
</tbody>
</table>

| **proton-proton scattering data** |                  |                           |
| 0–100                | 0.862            | 0.864                     |
| 0–200                | 0.954            | 0.951                     |
| 0–300                | 0.989            | 0.986                     |

($\Lambda = 450$ MeV)

<table>
<thead>
<tr>
<th>Nijmegen ’93</th>
<th>Granada ’17</th>
<th>N$^4$LO$^+$ $\Lambda = 450$ MeV</th>
<th>N$^4$LO$^+$ $\Lambda = 500$ MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_p^2$</td>
<td>0.0761(4)</td>
<td>0.0769(4)</td>
<td>0.0761(4)</td>
</tr>
<tr>
<td>$f_0^2$</td>
<td>0.075</td>
<td>0.0790(9)</td>
<td>0.0778(9)</td>
</tr>
<tr>
<td>$f_c^2$</td>
<td>0.0772(5)</td>
<td>0.0765(4)</td>
<td>0.0756(4)</td>
</tr>
</tbody>
</table>

$\chi^2$/datum: 1.016 1.022
np-pp phaseshift difference contrib. (preliminary)
np-pp phaseshift difference (lower partial waves)

\[ \delta_{np} - \delta_{pp} \, [\text{deg}] \]

\[ E_{lab} \, [\text{MeV}] \]

preliminary
np-pp phaseshift difference (higher partial waves)
Pinning down (short-range) CSB effects is difficult due to our poor knowledge of the nn-interaction.

Only used $^1S_0$ scattering length $a_{nn} = -18.90$ fm up to now (fix $\tilde{C}_{1S0}^{nn}$).

What about IB $C_{i}'$s?

Large contributions from IB contact interactions in P-Waves:

**Problem**: How much of it is CIB / CSB?

3N bound state energies ($\Lambda=450$ MeV, based on 2NF only):

$^3$He: -7.368 MeV

$^3$H: -8.130 MeV (IB $C_{i}'$s full CSB)

-8.090 MeV (IB $C_{i}'$s full CIB)

40 keV variation in binding energy difference
Summary

1. Extension of SMS potential with complete treatment of isospin breaking effects

2. Small changes/improvements in np force & corresponding 2N observables at low energies

3. Results for charge dependence of OPE coupling constant are consistent with zero, but no definite statement yet

4. Lack of data to determine short-range IB details of nn force

- Includes scattering data from 50ies up to 2013
- uses "3σ-criterion" to reject non-normal-distributed data
- rejection rate 0-300 MeV: np: 31%, pp: 11%

Comparison between theory and experiment via standard $\chi^2$ approach:

$$\chi^2_j = \sum_{i=1}^{n_j} \left( \frac{O_{ij}^{exp} - ZO_{ij}^{theo}}{\delta O_i} \right)^2 + \left( \frac{Z - 1}{\delta_{sys}} \right)^2$$

- Z (inverse relative norm) is chosen to minimize $\chi^2_j$