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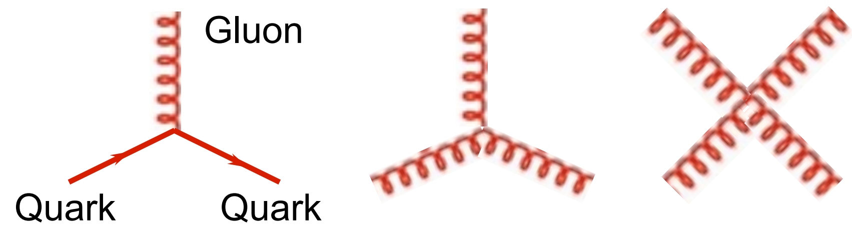
A new generation of chiral nuclear forces

In collaboration with: H.Krebs (Bochum), U.-G.Meißner (Bonn/Jülich), A.Nogga (Jülich), J.Golak, R.Skibinsky, H.Witala (Cracow), H.Kamada (Kyushu)

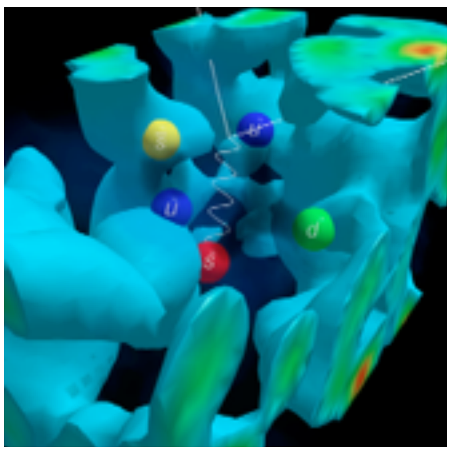
RUB

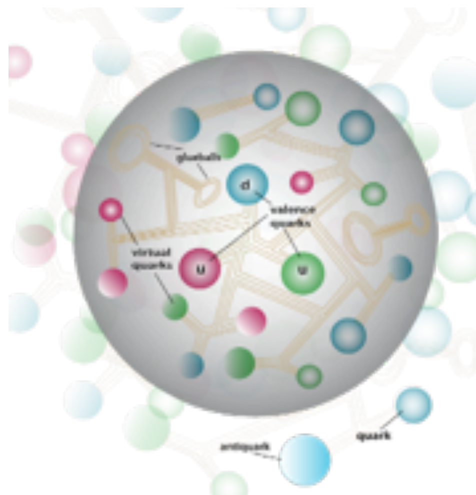
Facets of strong interactions

$$\mathcal{L}_{\rm QCD} = \bar{q}(i\not{D} - m)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu}$$



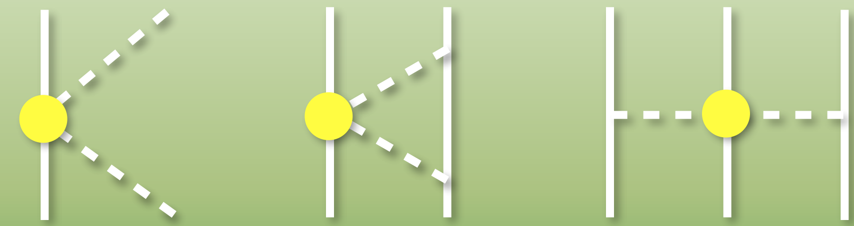
Seemingly very simple formulation is responsible for extremely complex phenomena!





From QCD to nuclear physics

→ **effective chiral Lagrangian** ⟶ **(low-energy) nuclear physics**



Chiral perturbation theory

- **Ideal world [$m_u = m_d = 0$], zero-energy limit:** non-interacting massless GBs
  (+ strongly interacting massive hadrons)

- **Real world [$m_u,\ m_d \ll \Lambda_{QCD}$], low energy:** weakly interacting light GBs
  (+ strongly interacting massive hadrons)

$\longrightarrow$ expand about the ideal world (ChPT)

Chiral Perturbation Theory

Chiral Perturbation Theory: expansion of the scattering amplitude in powers of

Weinberg, Gasser, Leutwyler, Meißner, ...

$$Q = \frac{\text{momenta of pions and nucleons or } M_\pi \sim 140\text{ MeV}}{\text{hard scales [at best } \Lambda_\chi = 4\pi F_\pi \sim 1\text{ GeV]}}$$

Manohar, Georgi '84

Tool: Feynman calculus using the effective chiral Lagrangian

low-energy constants

$$\mathcal{L}_\pi = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \ldots$$

$$\mathcal{L}_{\pi N} = \underbrace{\bar{N}\left(i\gamma^\mu D_\mu[\pi] - m + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu[\pi]\right)N}_{\mathcal{L}_{\pi N}^{(1)}} + \underbrace{\sum_i \boldsymbol{c}_i\, \bar{N}\hat{O}_i^{(2)}[\pi]N}_{\mathcal{L}_{\pi N}^{(2)}} + \underbrace{\sum_i \boldsymbol{d}_i\, \bar{N}\hat{O}_i^{(3)}[\pi]N}_{\mathcal{L}_{\pi N}^{(3)}} + \ldots$$

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Pion-nucleon scattering up to Q^4 in heavy-baryon ChPT

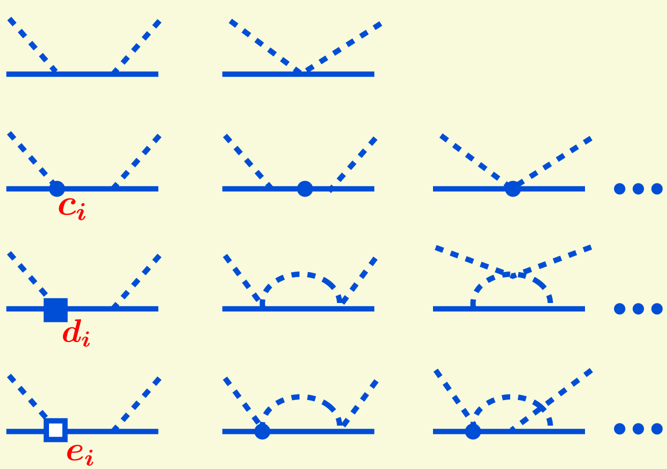
Fettes, Meißner '00; Krebs, Gasparyan, EE '12

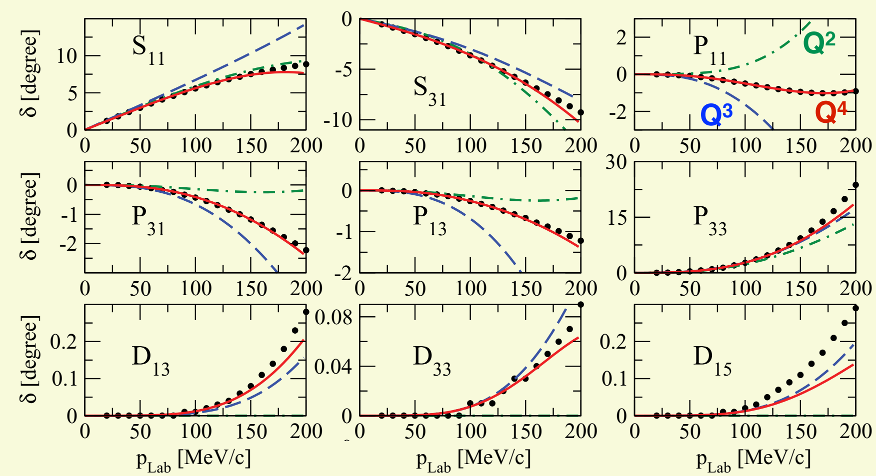
Order Q:

Order Q^2:

Order Q^3:

Order Q^4:



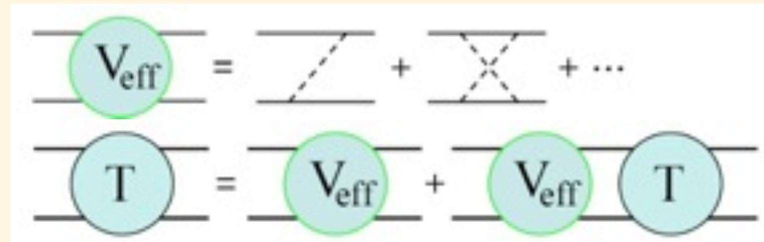


Nuclear chiral effective field theory

Nuclear chiral EFT

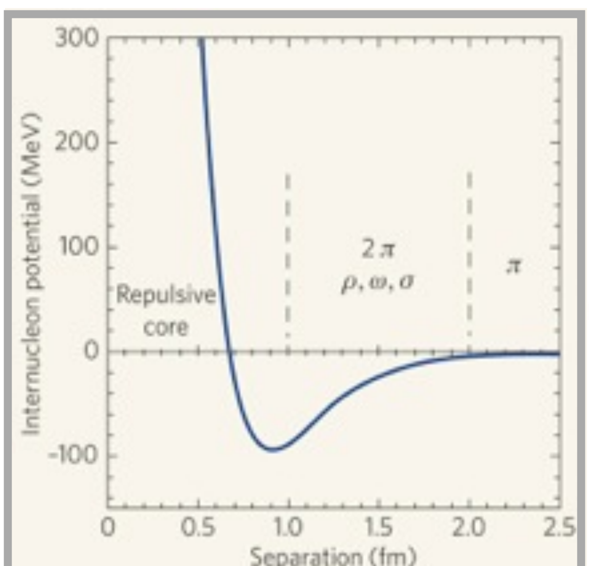
Weinberg, van Kolck, EE, Glöckle, Meißner, Machleidt, Entem...

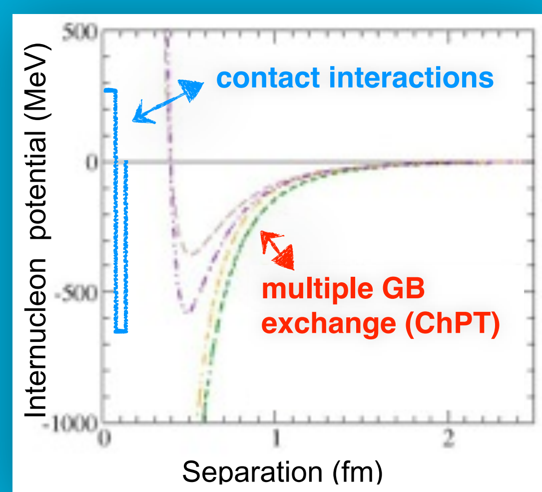
- Schrödinger equation for nucleons interacting via contact forces and long-range potentials (pion exchanges)



$$\left[\left(\sum_{i=1}^{A}\frac{-\vec{\nabla}_i^2}{2m_N}+\mathcal{O}(m_N^{-3})\right)+\underbrace{V_{2N}+V_{3N}+V_{4N}+\ldots}_{\text{derived in ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$

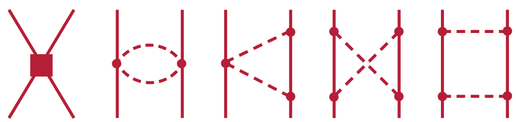
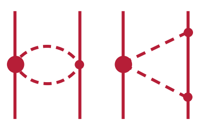
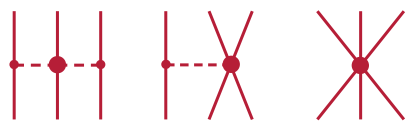
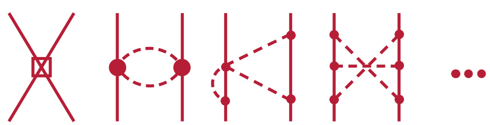
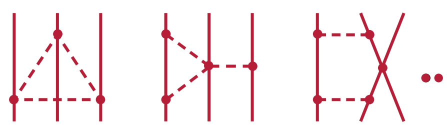
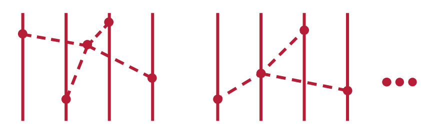
- access to heavier nuclei (ab initio few-/many-body methods)





- unified description of $\pi\pi$, πN and NN
- consistent many-body forces and currents
- systematically improvable
- bridging different reactions (electroweak, π-prod., ...)
- precision physics with/from light nuclei

Chiral expansion of nuclear forces

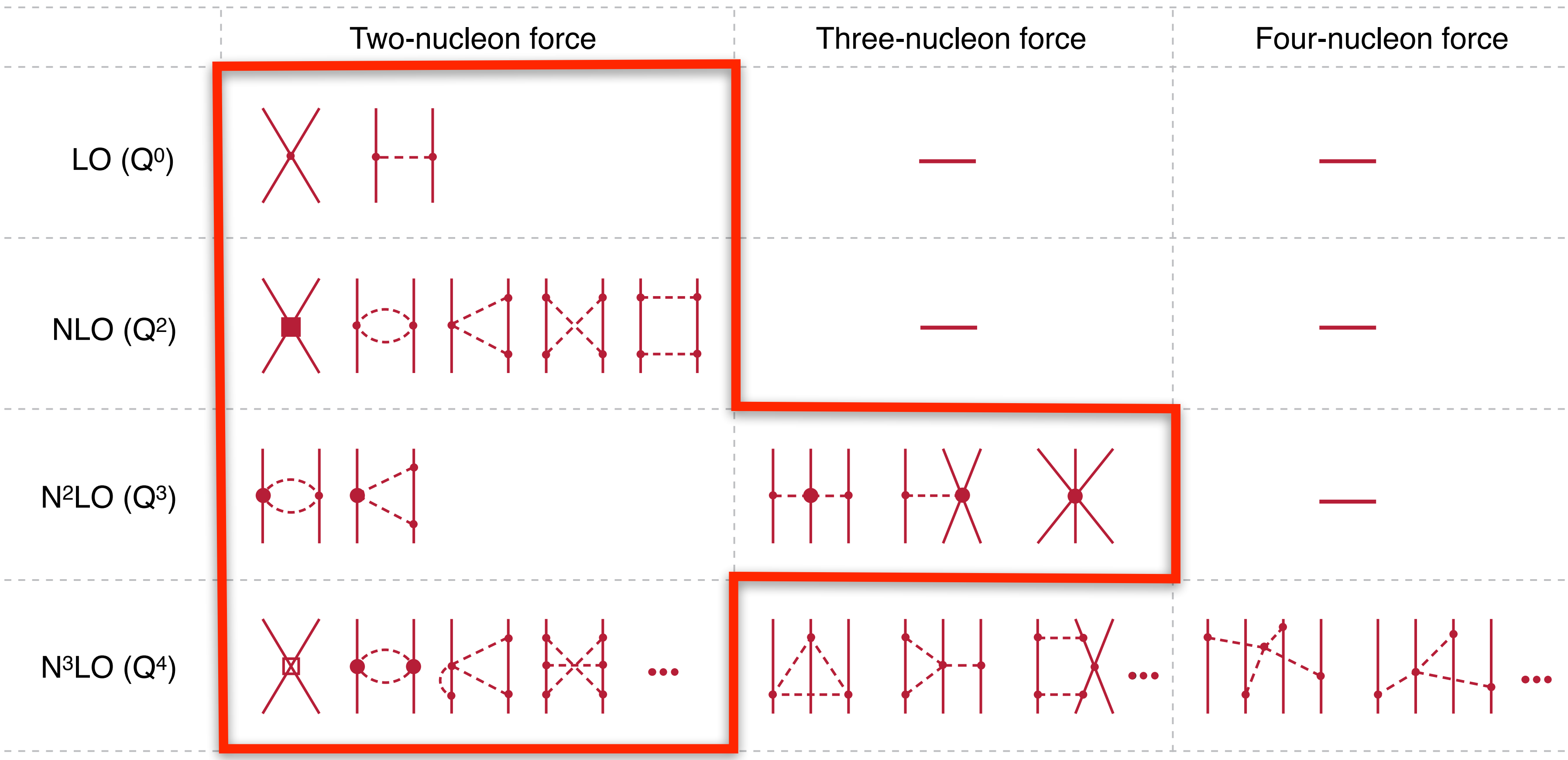
	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO (Q^0)		—	—
NLO (Q^2)		—	—
N^{2}LO (Q^3)			—
N^{3}LO (Q^4)			

2N force: accurate N^{3}LO potentials are available Entem-Machleidt ’03; EE-Glöckle-Meißner ’04

3N force: N^{2}LO 3NF included in most calculations
N^{3}LO 3NF worked out Bernard, EE, Krebs Meißner ’08,’11; (probably) not yet converged → higher orders
numerical PWD developed Golak, Skibinski, Krebs, Hebeler, …, first results available Witala et al.’13

4N force: leading (i.e. N^{3}LO) terms worked out EE ’06; contrib. to ^{4}He BE ~ few 100 keV Rospedzik et al. ’06

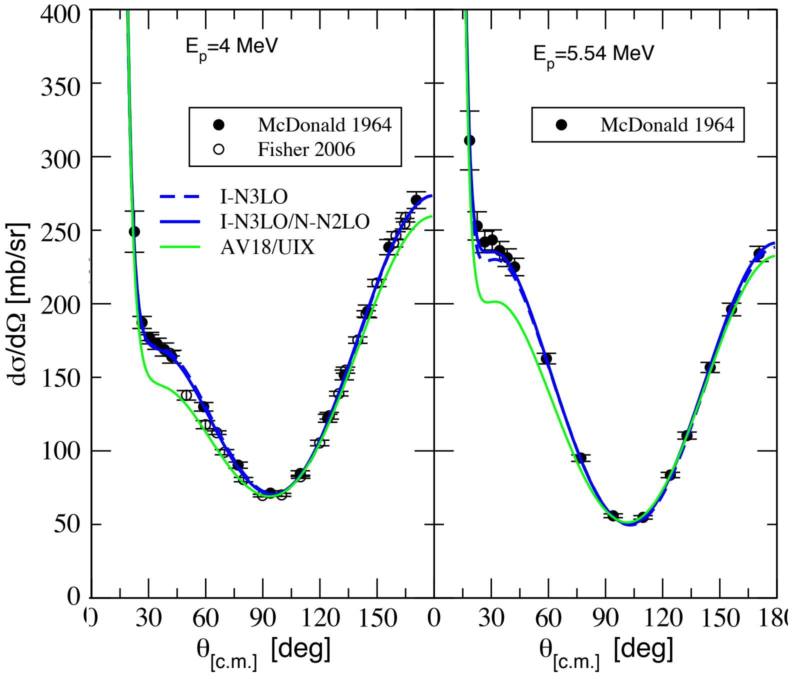
Chiral expansion of nuclear forces



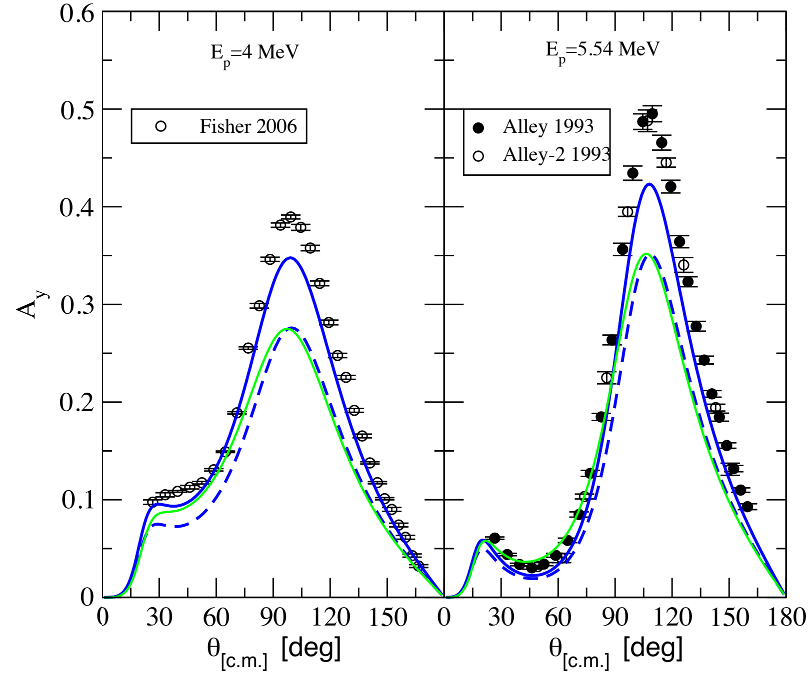
The „standard“ nuclear chiral Hamiltonian has been extensively tested in few- and many-body systems

Chiral Hamiltonian & the 3N/4N continuum

p-^{3}He differential cross section



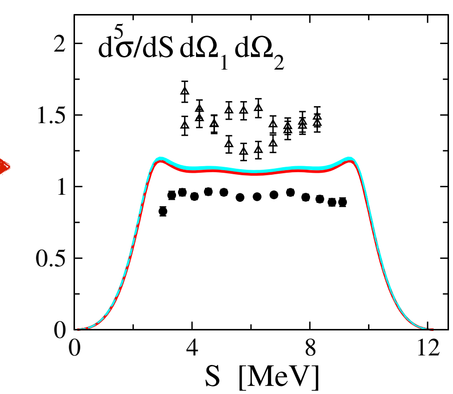
A_y-puzzle in p-^{3}He elastic scattering



LECs D,E tuned to the ^{3}H and ^{4}He binding energies, figure from Viviani et al., arXiv:1004.1306

To summarize:

- Nd scattering: accurate description at low energy except for A_y (fine tuned) and Space Star breakup configuration
- Uncertainty increases with energy (higher-order 3NF?)
- 4N continuum: an emerging field...

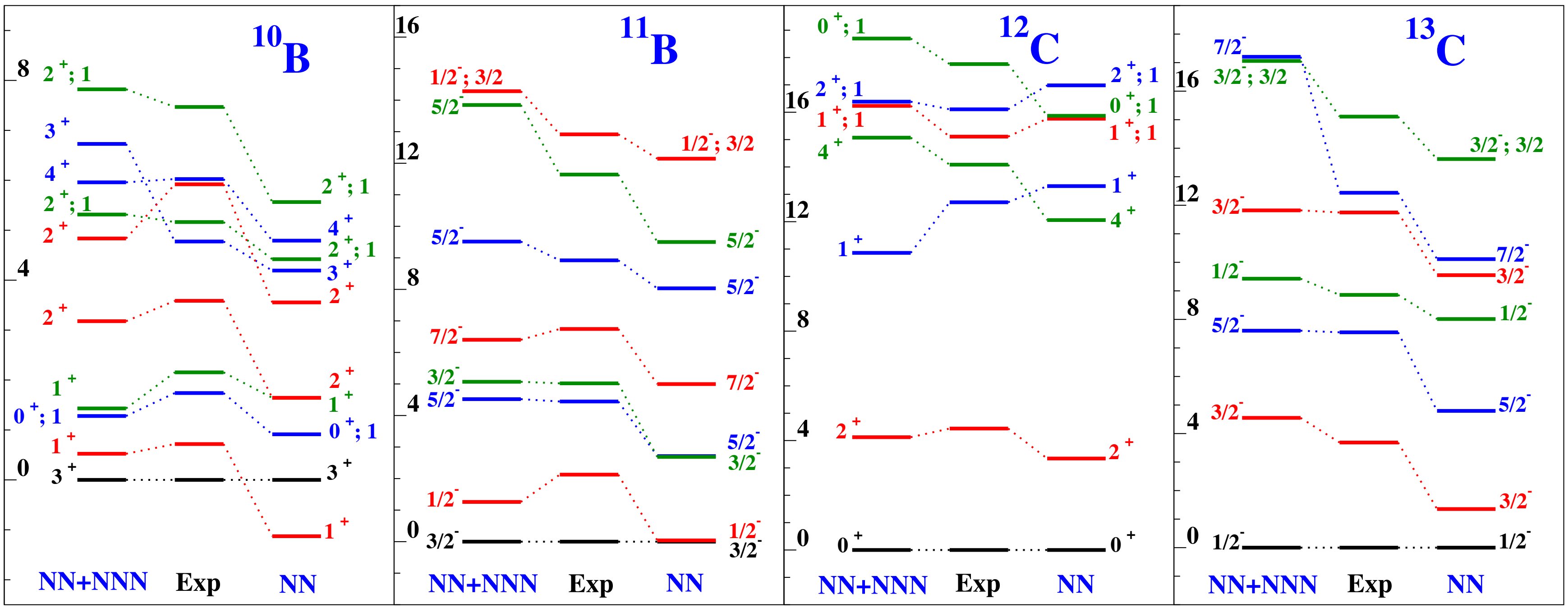


Chiral Hamiltonian & nuclear structure

Ab initio methods (NCSM, GFMC, CCM, Lattice, ...) + renormalization ideas (SRG, $V_{low\text{-}k}$, UCOM)
+ computational resources → precision ab initio nuclear structure calculations

NCSM calculation of p-shell nuclei with chiral 2NF+3NF

Navratil et al. '07



- sensitive to details of the 3NF
- promising results (neutron-rich nuclei, long lifetime of ^{14}C, neutron star radii, ...)
- still room for improvement and some open questions

The quest for high-precision chiral Hamiltonian

Corrections to the 3NF beyond N^2LO

Convergence, effects of the Δ, partial wave decomposition, determination of the LECs, impact on 3N/4N scattering observables and nuclear structure…

→ **Low Energy Nuclear Physics International Collaboration (LENPIC)**

Bochum-Bonn-Cracow-Darmstadt-Iowa-Jülich-Kyushu-Ohio-Orsay

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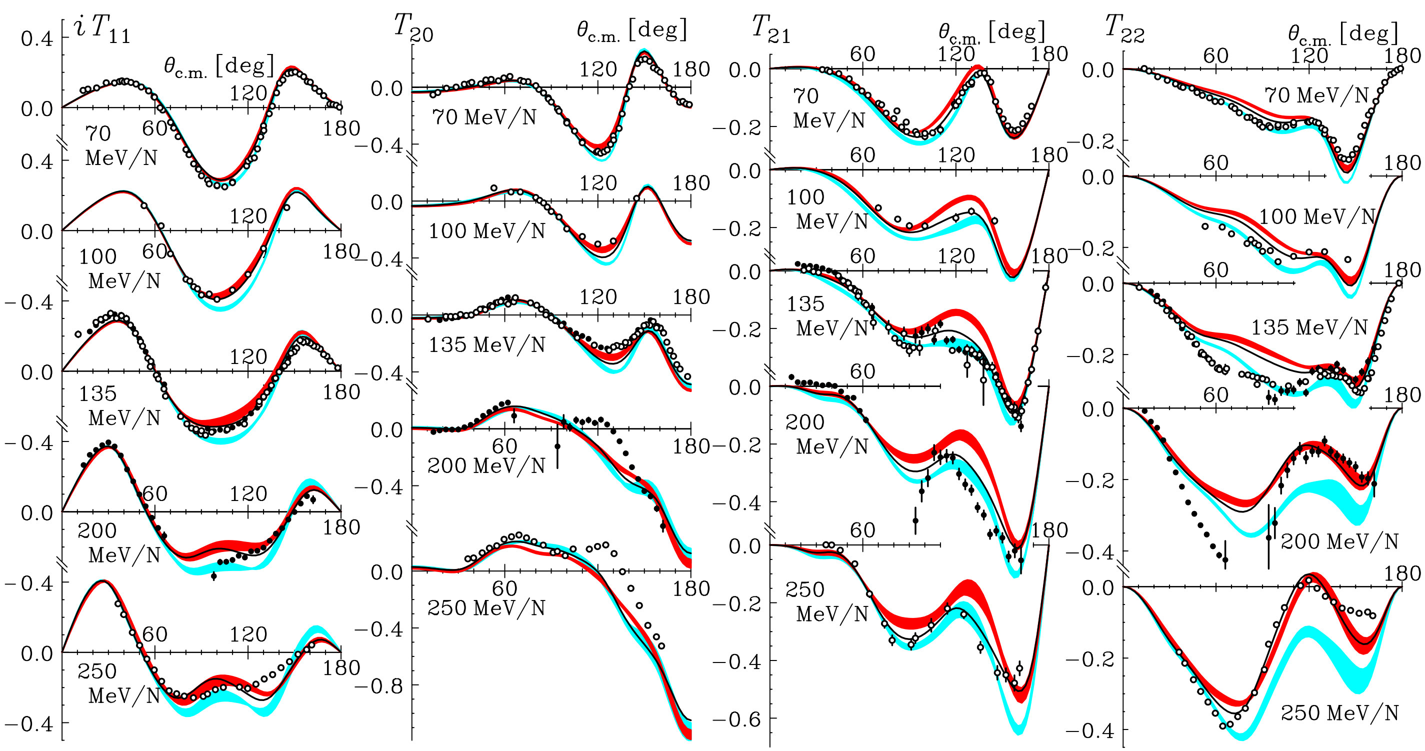
Actually, the spin structure of the 3NF is still poorly understood in spite of decades of effort! (one of the biggest challenges in nuclear physics)



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Improving the accuracy/applicability range of chiral potentials

Chiral nuclear potentials are regularized using momentum-space cutoff Λ ~ 500 MeV. Reducing finite-Λ artifacts is expected to increase the accuracy/applicability range of chiral forces.

→ **New generation of chiral NN potentials using a local regulator**

l$_{\text{ocal}}$-chiral potentials @ LO, NLO, N^{2}LO

Gezerlis et al, PRL 111 (13) 032501; arXiv:1406.0454; Lynn et al. arXiv:1406.2787

i$_{\text{mproved}}$-chiral potentials up to N^{3}LO

EE, Krebs, Meißner, Golak, Skibinski, Witala; work in progress

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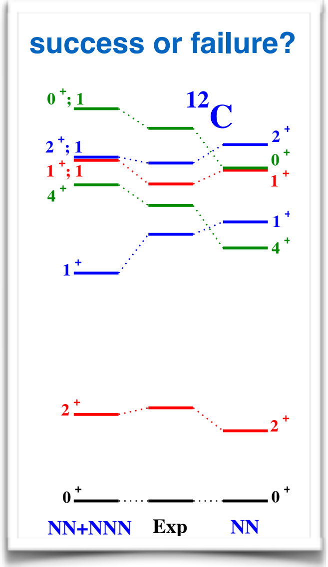

Reliable estimation of theoretical uncertainties

is needed in order to test chiral dynamics in nuclear systems, identify/resolve possible puzzles (is A_y-puzzle a real puzzle?), make reliable predictions and guide new experiments

The quest for high-precision chiral Hamiltonian

Corrections to the 3NF beyond N²LO ⟵ **Seminar at PKU on Aug. 29**

Convergence, effects of the Δ, partial wave decomposition, determination of the LECs, impact on 3N/4N scattering observables and nuclear structure…

⟶ **Low Energy Nuclear Physics International Collaboration (LENPIC)**

Bochum-Bonn-Cracow-Darmstadt-Iowa-Jülich-Kyushu-Ohio-Orsay

today's talk

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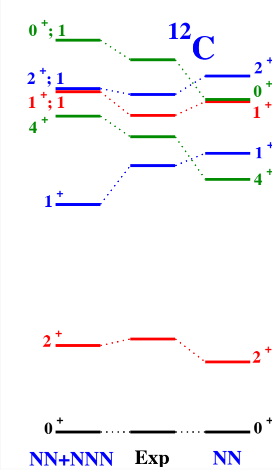
$\mathbf{i}_{\text{mproved}}$**-chiral** potentials up to N³LO

EE, Krebs, Meißner, Golak, Skibinski, Witala; work in progress

success or failure?



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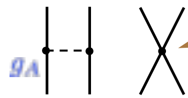
Nucleon-nucleon force up to N^{3}LO

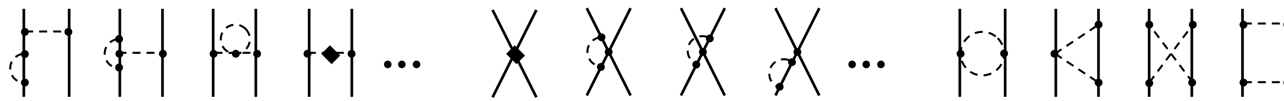
Ordonez et al. ’94; Friar & Coon ’94; Kaiser et al. ’97; E.E. et al. ’98,‘03; Kaiser ’99-’01; Higa, Robilotta ’03; …

- **LO (Q^0):**



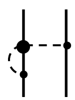
2 LECs

- **NLO (Q^2):**

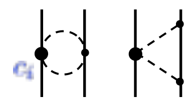


renormalization of 1π-exchange — 7 LECs — renormalization of contact terms — leading 2π-exchange

- **N^{2}LO (Q^3):**

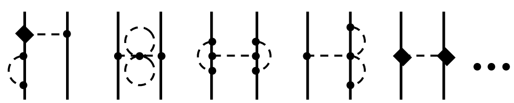


renormalization of 1π-exchange

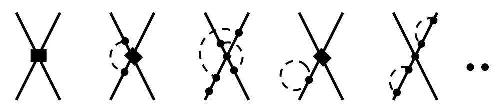


subleading 2π-exchange

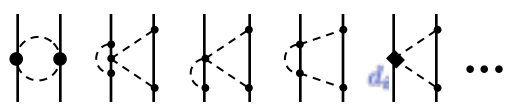
- **N^{3}LO (Q^4):**



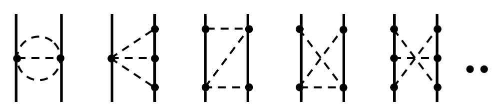
renormalization of 1π-exchange



15 LECs — renormalization of contact terms



sub-subleading 2π-exchange



3π-exchange (small)

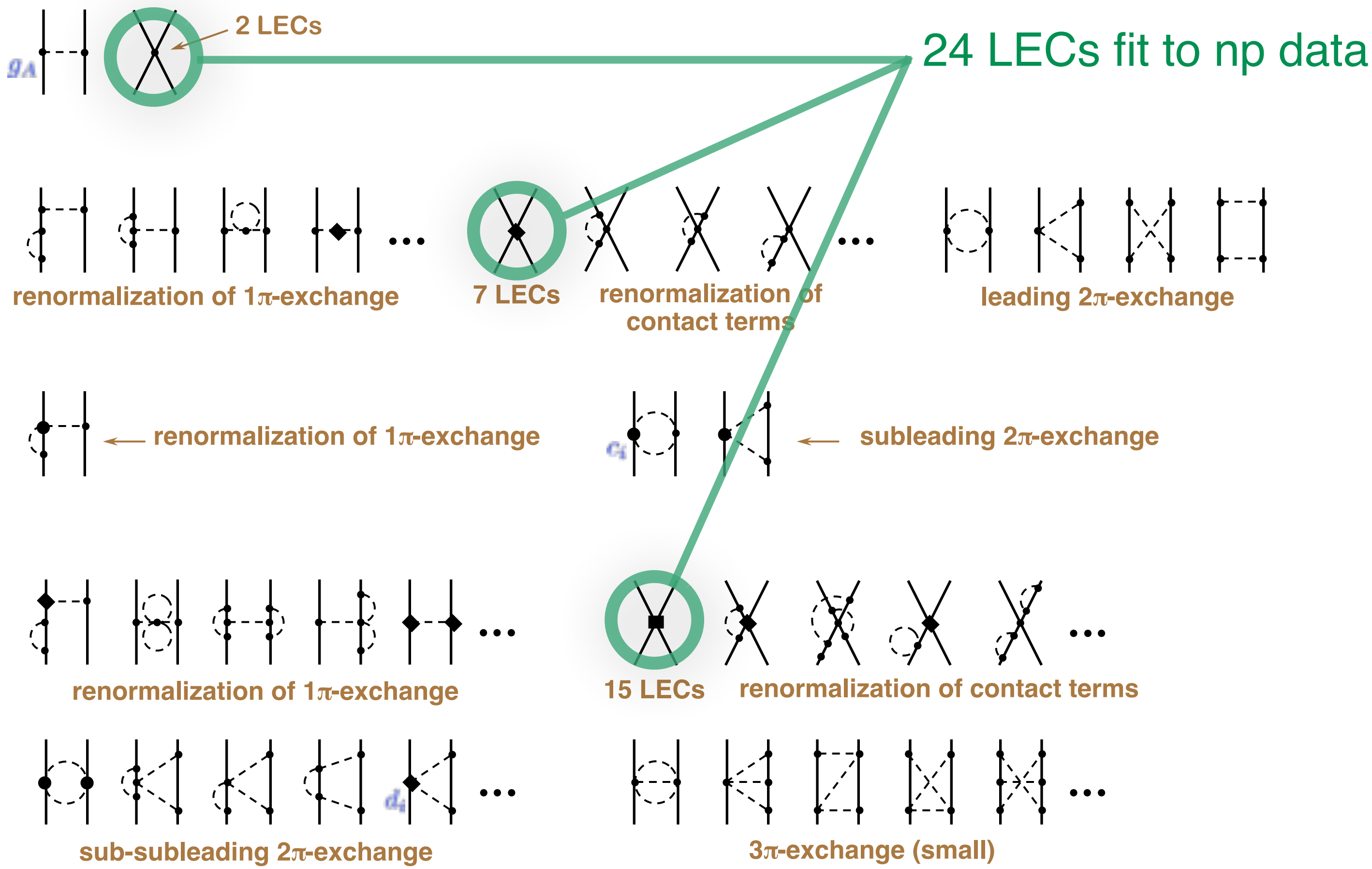
\+ isospin-breaking corrections…

van Kolck et al. ’93, ’96; Friar et al. ’99, ’03, ’04; Niskanen ’02; Kaiser ’06; E.E. et al. ’04,’05,’07; …

Nucleon-nucleon force up to N^{3}LO

Ordonez et al. ’94; Friar & Coon ’94; Kaiser et al. ’97; E.E. et al. ’98,‘03; Kaiser ’99-’01; Higa, Robilotta ’03; …

- LO (Q^0):
- NLO (Q^2):
- N^{2}LO (Q^3):
- N^{3}LO (Q^4):



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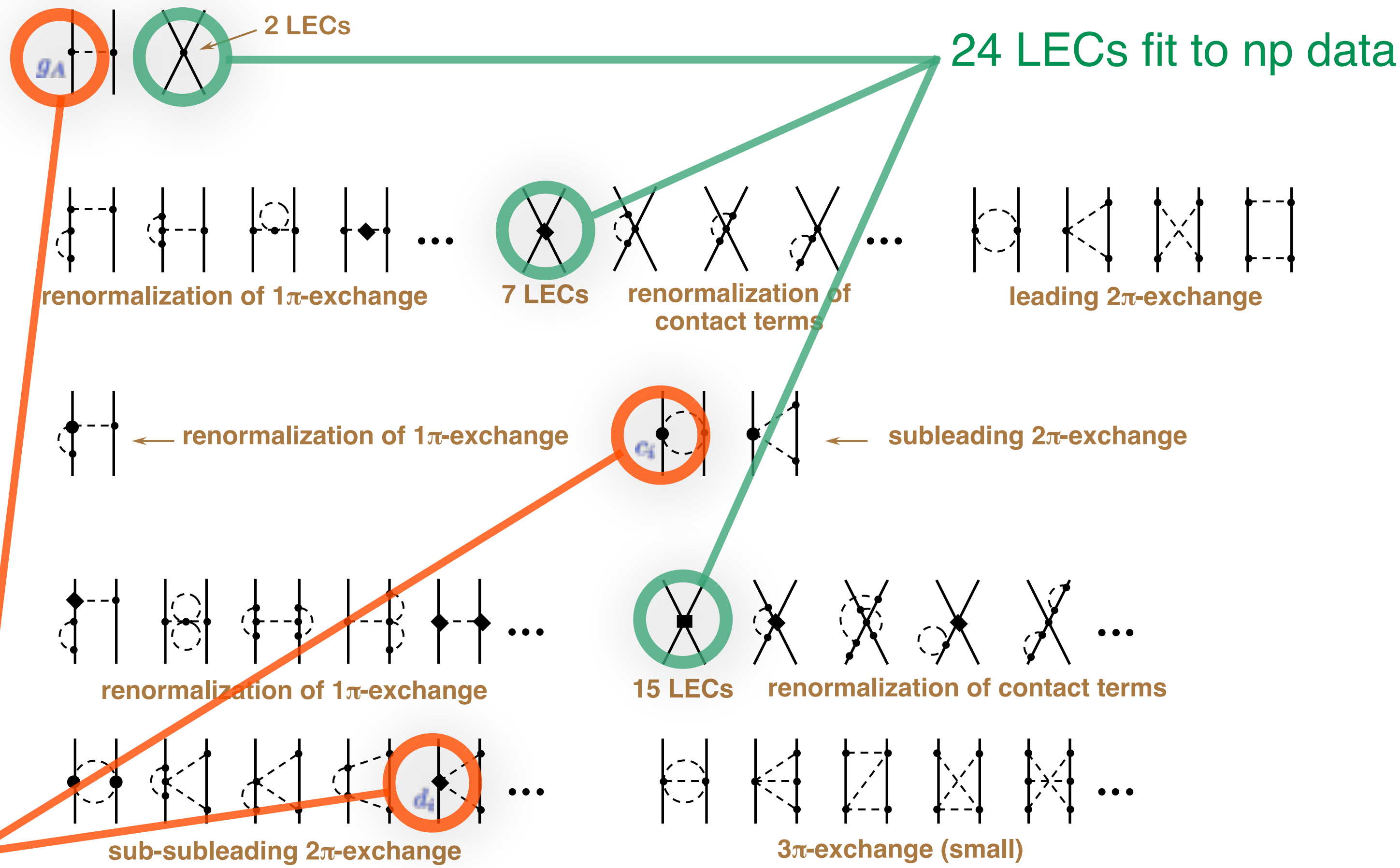


LECs fixed from πN

➞ long-range tail of the nuclear force fixed by chiral symmetry and exp. information on the πN system

+ isospin-breaking corrections…

van Kolck et al. ’93, ’96; Friar et al. ’99, ’03, ’04; Niskanen ’02; Kaiser ’06; E.E. et al. ’04,’05,’07; …

Low-energy constants

- **$_{\text{improved}}$-chiral potential:** adopt LECs from πN scattering without additional fine tuning
(impact of the uncertainty in the LECs needs to be addressed in the future)

low-energy constants

$$\mathcal{L}_\pi = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \ldots$$

$$\mathcal{L}_{\pi N} = \underbrace{\bar{N}\left(i\gamma^\mu D_\mu[\pi] - m + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu[\pi]\right)N}_{\mathcal{L}_{\pi N}^{(1)}} + \underbrace{\sum_i \boldsymbol{c_i}\, \bar{N}\hat{O}_i^{(2)}[\pi]N}_{\mathcal{L}_{\pi N}^{(2)}} + \underbrace{\sum_i \boldsymbol{d_i}\, \bar{N}\hat{O}_i^{(3)}[\pi]N}_{\mathcal{L}_{\pi N}^{(3)}} + \ldots$$

	Entem-Machleidt potential	Epelbaum-Glöckle-Meißner potential	πN scattering to leading one loop (Q^3)
c_1	–0.81	–0.81	-0.81 ± 0.15^b
c_2	2.80	3.28	3.28 ± 0.23^c
c_3	–3.20	–3.40	-4.69 ± 1.34^b
c_4	5.40	3.40	3.40 ± 0.04^b
$\bar{d}_1 + \bar{d}_2$	3.06	3.06	3.06 ± 0.21^c
$\bar{d}_3$	–3.27	–3.27	-3.27 ± 0.73^c
$\bar{d}_5$	0.45	0.45	0.45 ± 0.42^c
$\bar{d}_{14} - \bar{d}_{15}$	–5.65	–5.65	-5.65 ± 0.41^c

were tuned to improve the fit

taken on the lower side to avoid deeply-bound states

- 2 [LO] + 7 [NLO, N²LO] + 15 [N³LO] contact interactions fitted to np phase shifts

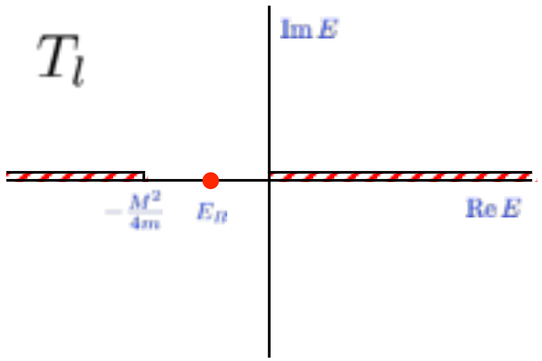
Regularization

Employ regularization which maintains the analytic structure of the amplitude

Old EM/EGM potentials

$$V^{\rm reg}_{\rm long-range}(\vec{p}\,', \vec{p}\,) = V_{\rm long-range}(\vec{p}\,', \vec{p}\,) \;\exp\left[- \frac{p^n + p'^n}{\Lambda^n} \right]$$

affects the discontinuity across the left-hand cut
(i.e. some distortions of the long-range potential)



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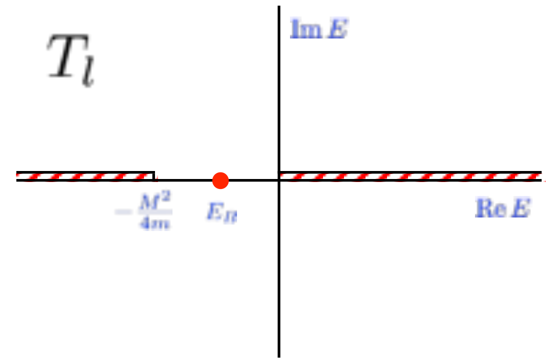


New potentials

$$V^{\rm reg}_{\rm long-range}(\vec r\,) = V_{\rm long-range}(\vec r\,)\ f_{\rm reg}\left(\frac{r}{R}\right)$$

FourierTransform$[f(r/R) - 1]$

$$V^{\rm reg}_{\rm long-range}(\vec q\,) = V_{\rm long-range}(\vec q\,) - \underbrace{\int d^3 l\ V_{\rm long-range}(\vec q - \vec l\,)\ \tilde f_{\rm reg}(\vec l\,)}_{\text{manifestly short-range}}$$

Advantages:

No distortion of the long-range potential ⟶ better performance (at high energy)

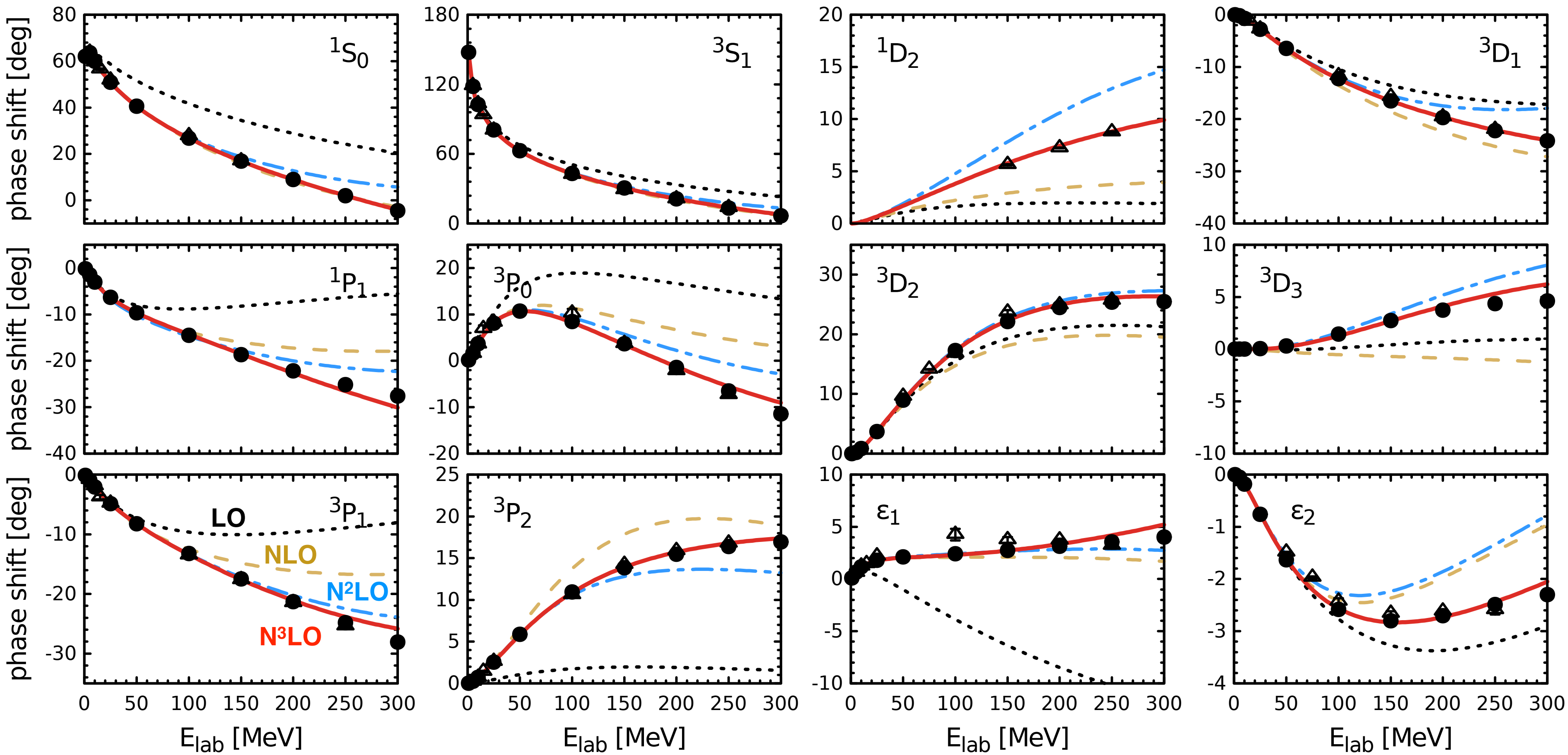
No need for an additional spectral function regularization in the TPEP

Choice of cutoff R: chiral expansion of π-exchanges breaks down for r ~ 0.8 fm Baru et al.'12

⟶ it is natural to choose R ~ 1 fm

(Preliminary) Results

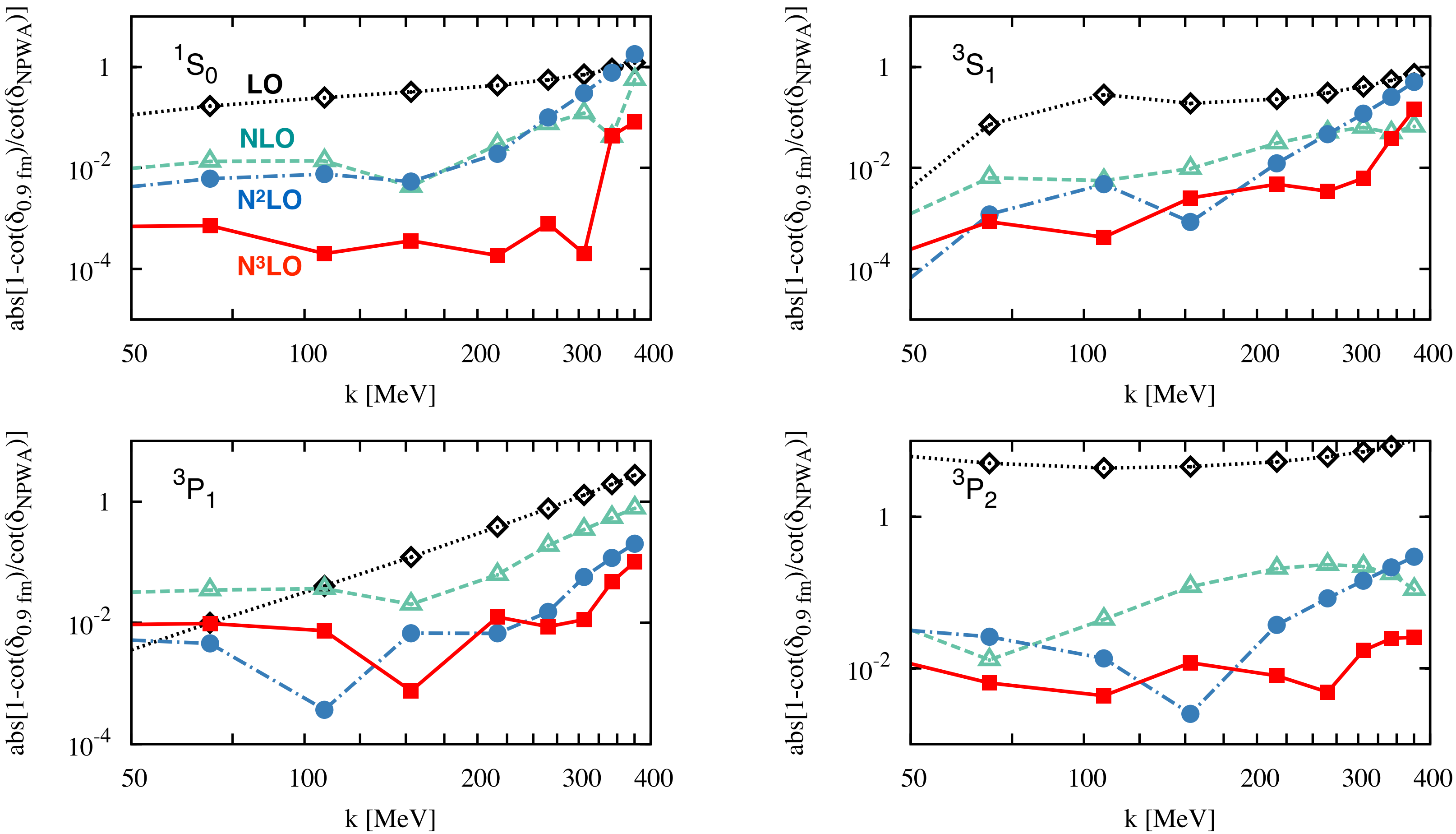
NN phase shifts order by order



results are shown for the hardest cutoff R = 0.9 fm

NN phase shifts order by order

Absolute error in selected phase shifts (Lepage plots)



results are shown for the hardest cutoff R = 0.9 fm

Cutoff dependence of phase shifts

Residual R-dependence probes the size of neglected contact terms:

LO: neglected order-Q^2 contact terms [NLO]
NLO, N^2LO: neglected order-Q^4 contact terms [N^3LO]
N^3LO: neglected order-Q^6 contact terms [N^5LO]

$\Bigg\}$ should decrease from LO to NLO(N^2LO) to N^3LO

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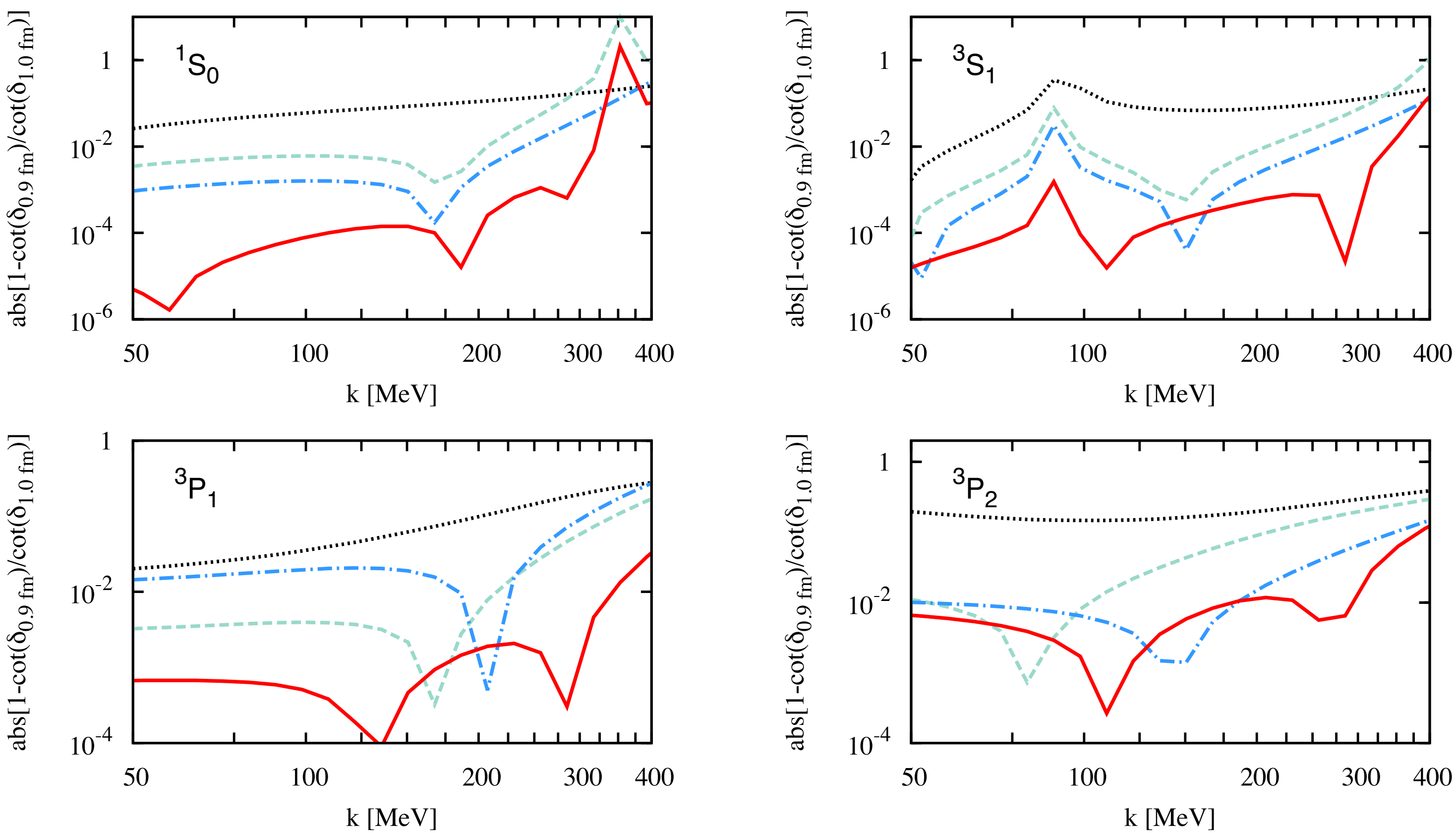
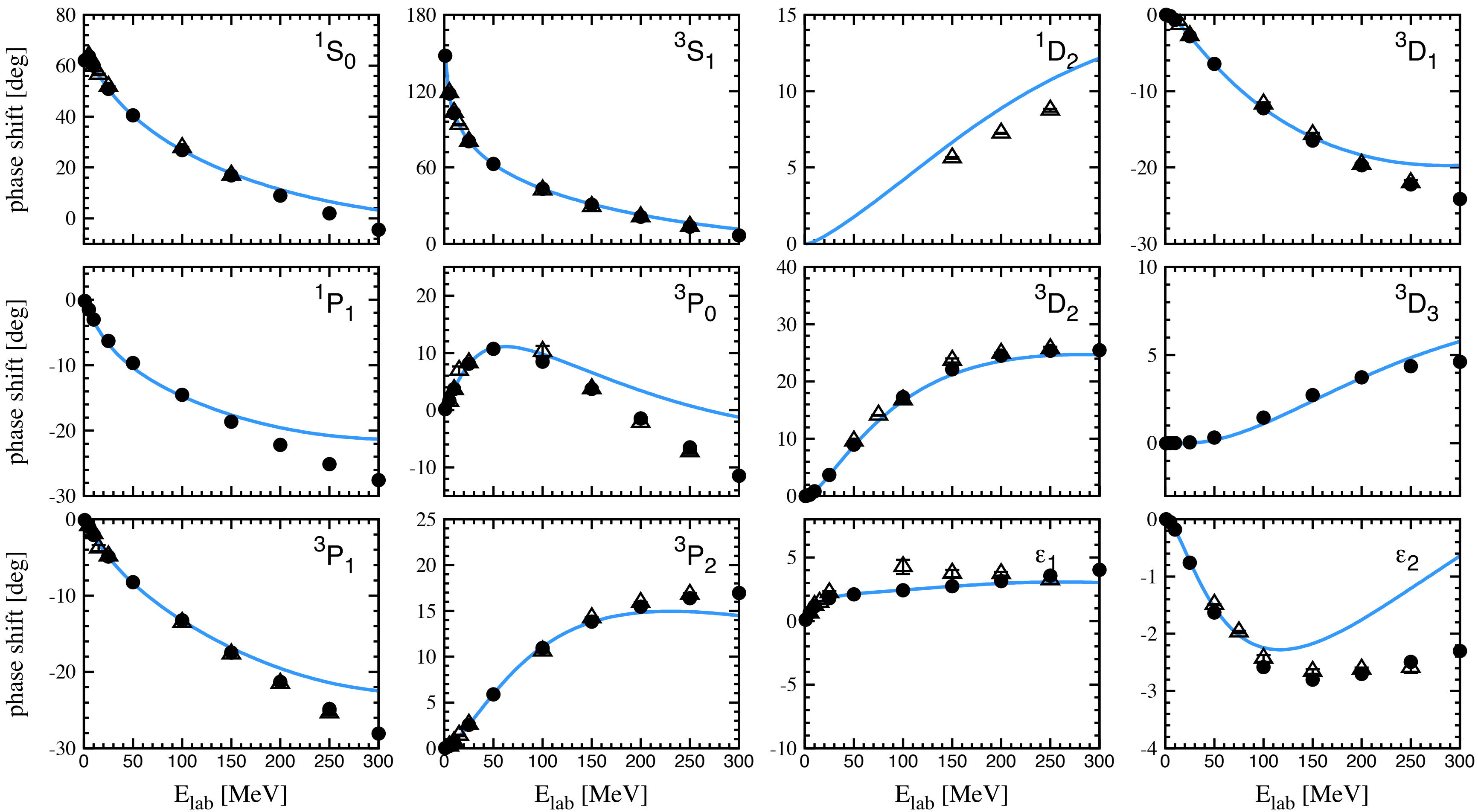


Cutoff dependence of phase shifts

Neutron-proton phase shifts



R ~ 0.9 fm [E_{lab} ~ 410 MeV]

Cutoff dependence of phase shifts

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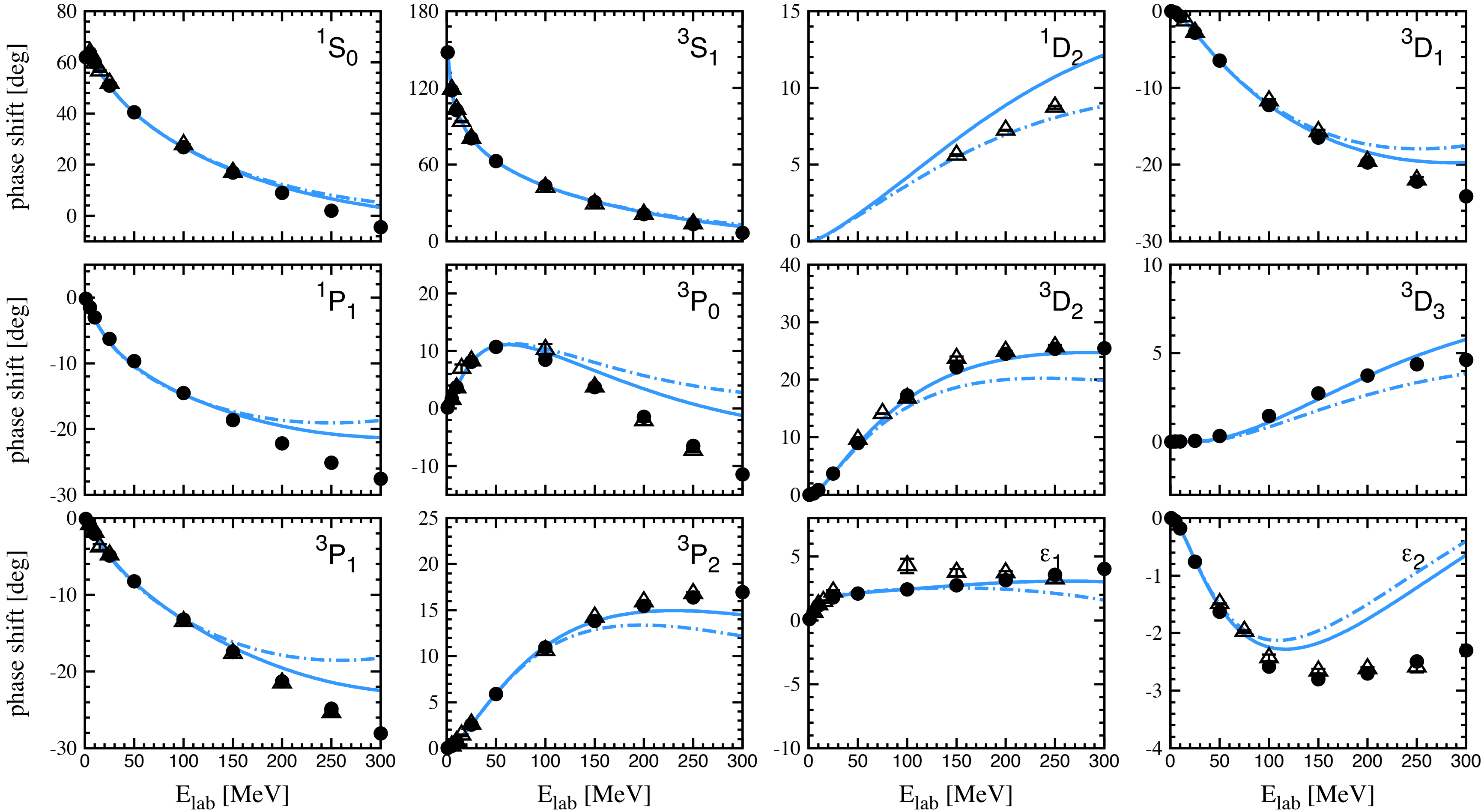

R ~ 0.9 fm [E$_{lab}$ ~ 410 MeV] **R ~ 1.0 fm [E$_{lab}$ ~ 330 MeV]**

Cutoff dependence of phase shifts

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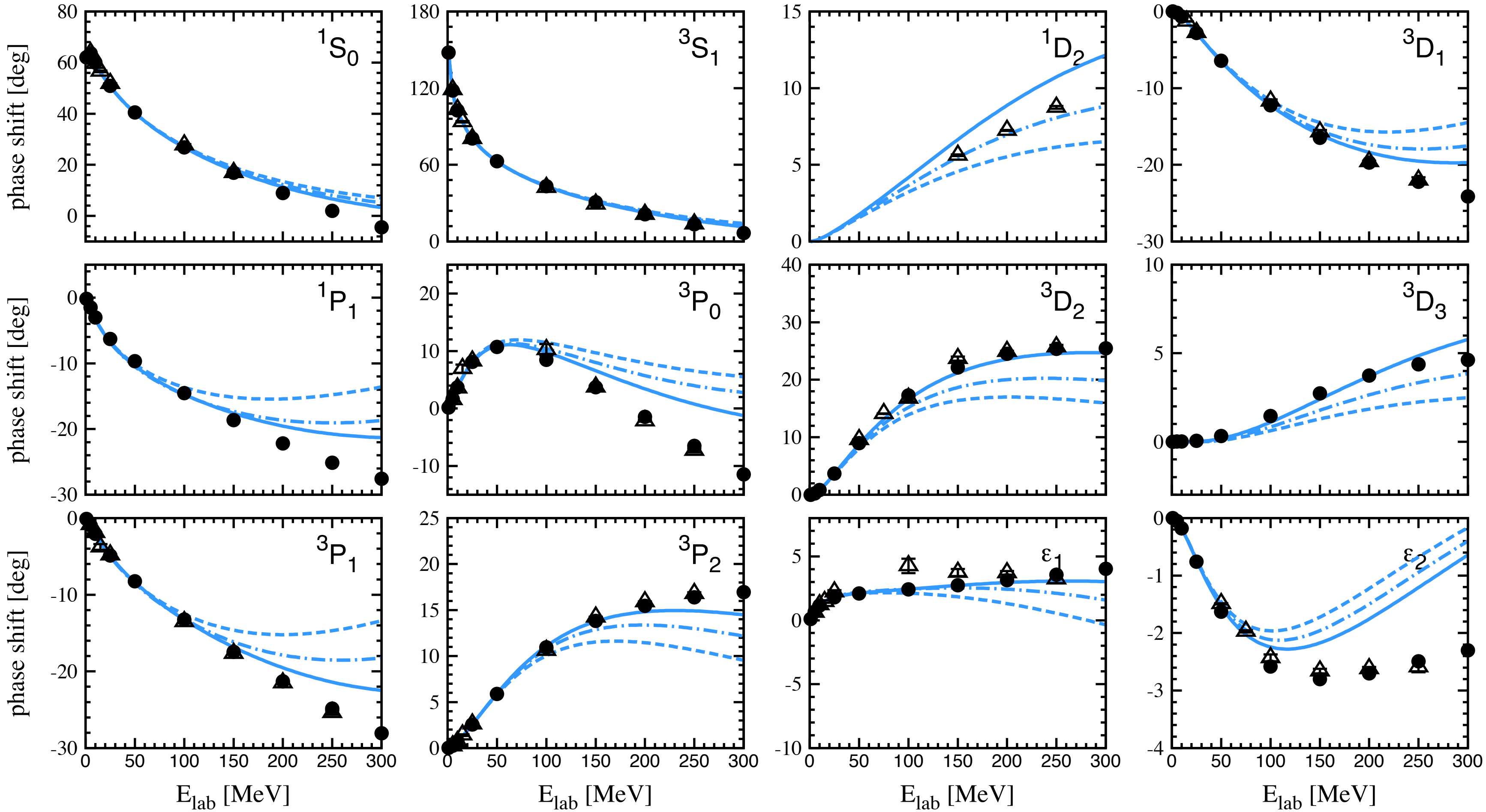

R ~ 0.9 fm [E_{lab} ~ 410 MeV] **R ~ 1.0 fm [E_{lab} ~ 330 MeV]** **R ~ 1.1 fm [E_{lab} ~ 270 MeV]**

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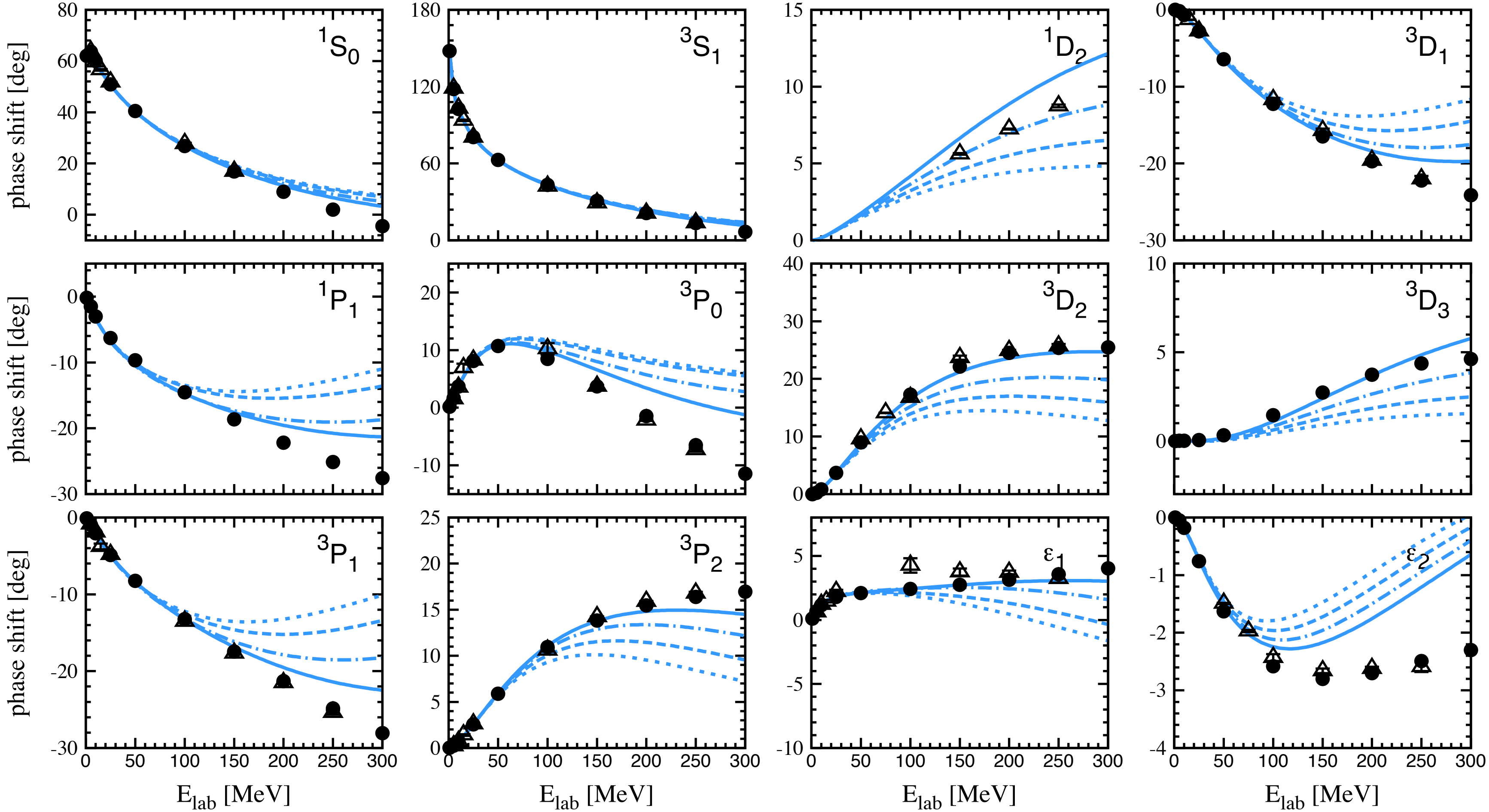

R ~ 0.9 fm [E_{lab} ~ 410 MeV] **R ~ 1.0 fm [E_{lab} ~ 330 MeV]** **R ~ 1.1 fm [E_{lab} ~ 270 MeV]** **R ~ 1.2 fm [E_{lab} ~ 230 MeV]**

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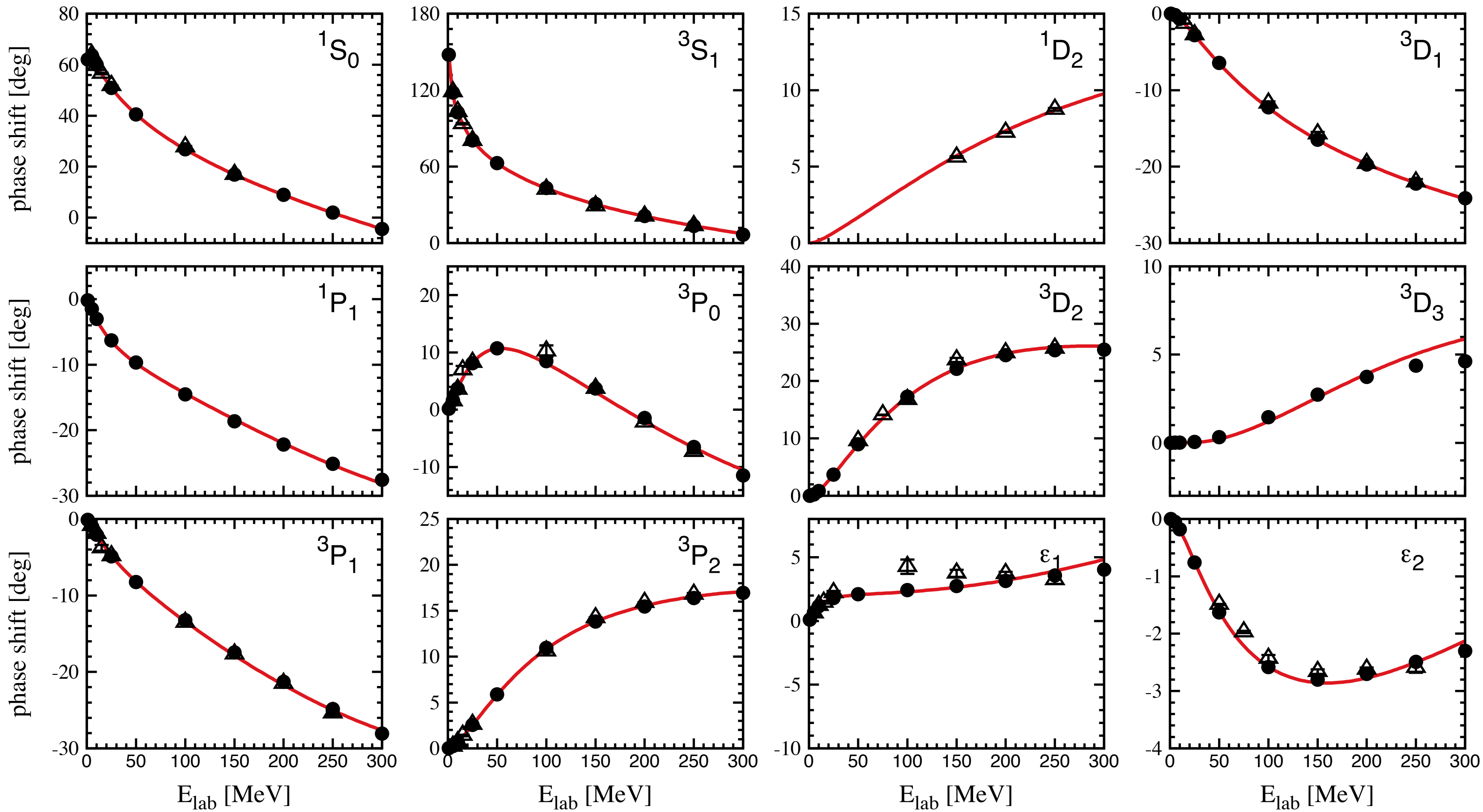

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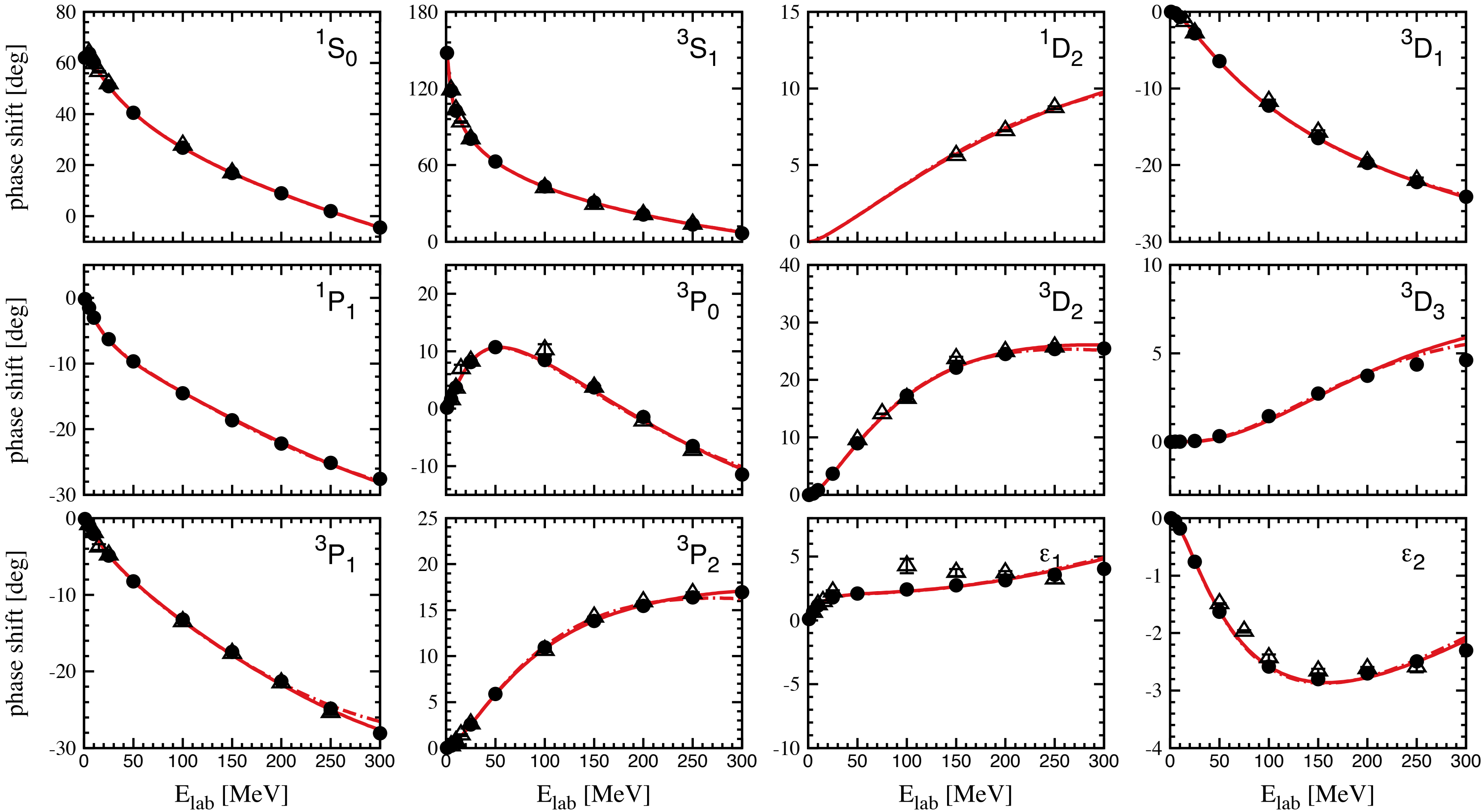

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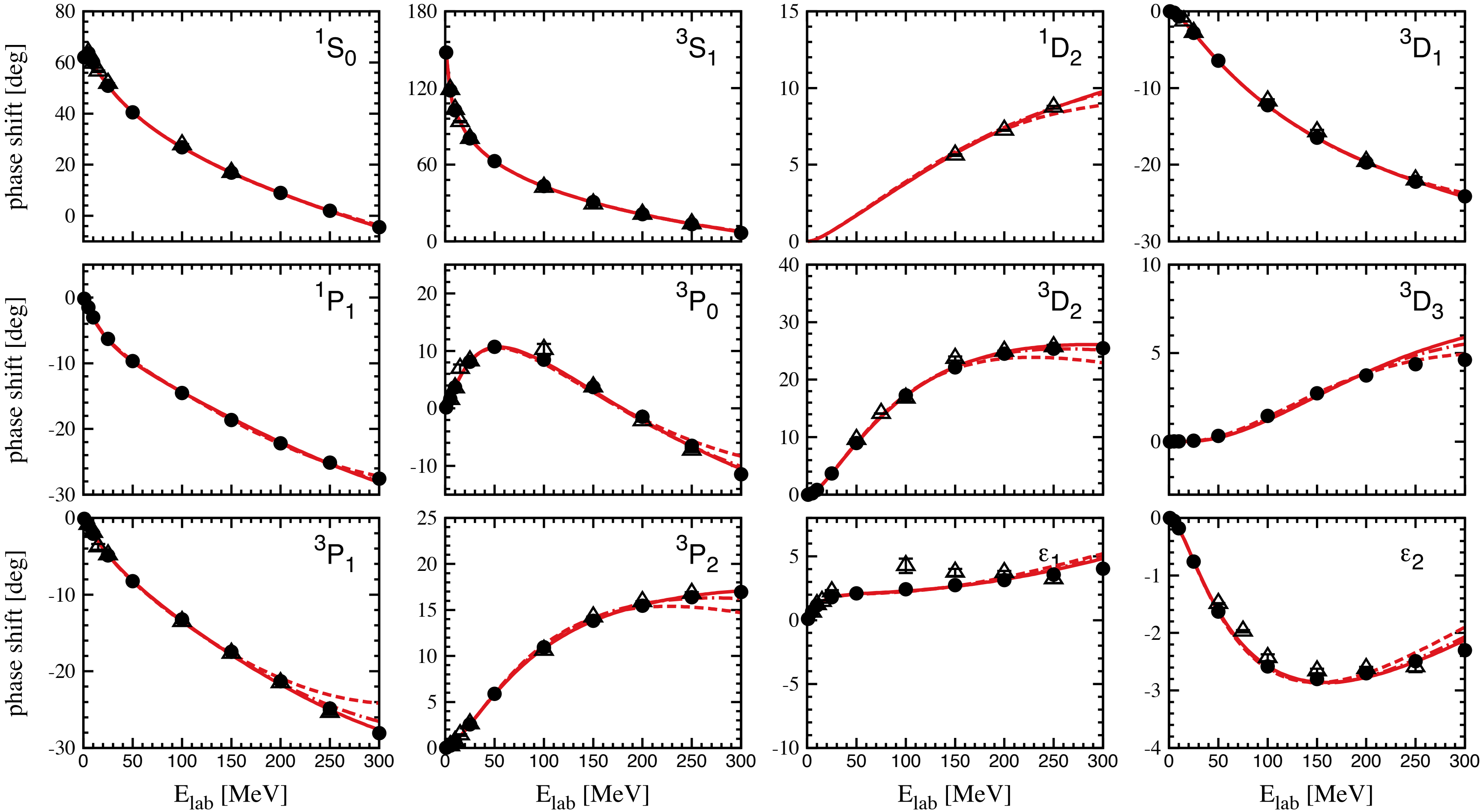
Cutoff dependence of phase shifts

Neutron-proton phase shifts



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Cutoff dependence of phase shifts

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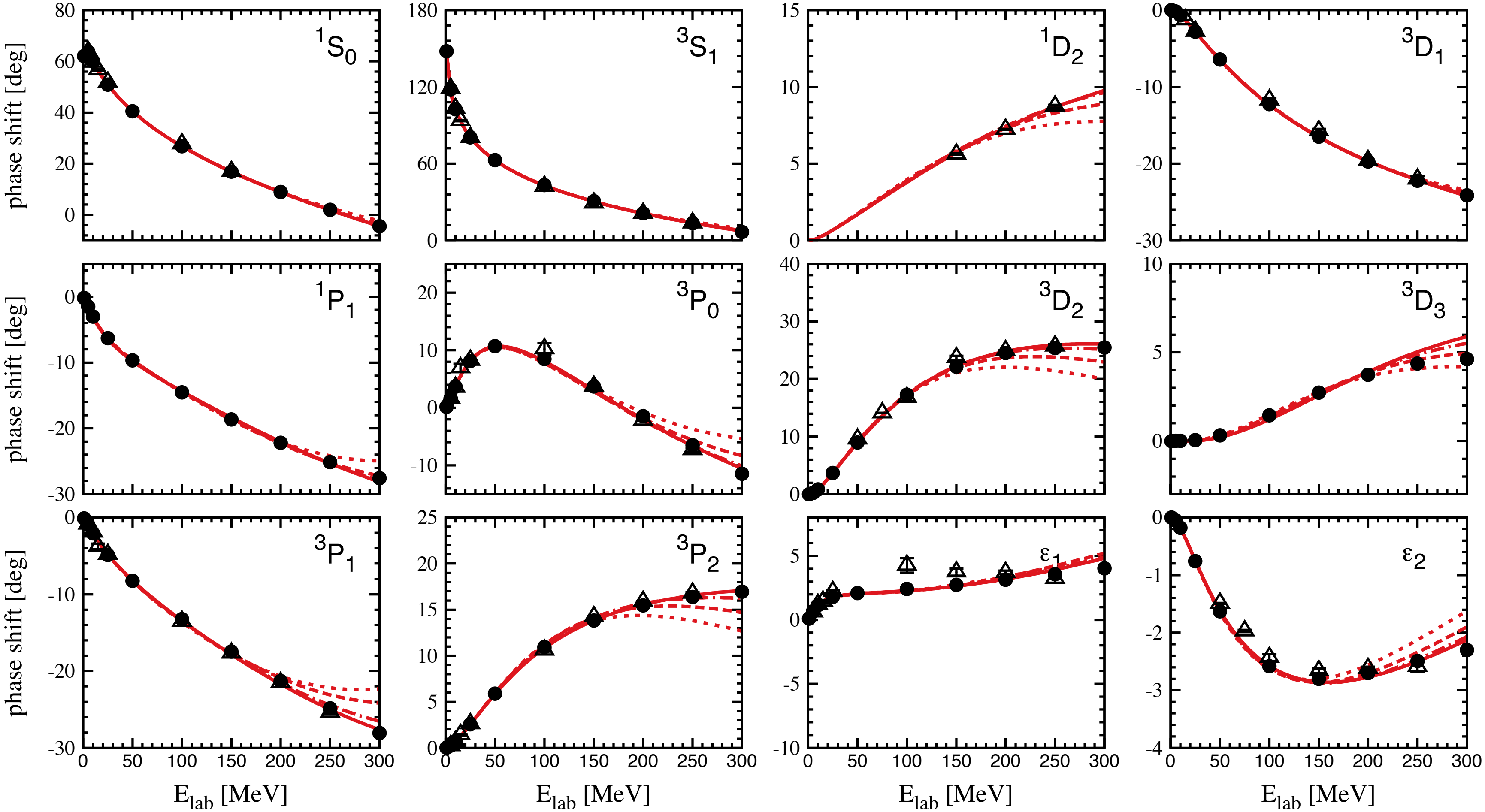

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Quantification of
Theoretical Uncertainties

Theoretical uncertainties

Sources of uncertainty:

- Uncertainty in NN PWA used as input to fix contact interactions (very small)
- Uncertainty in the knowledge of πN LECs
- **Uncertainty due to truncation of the chiral expansion at a given order**

  Often estimated by means of a **cutoff variation**. However…

  - underestimates the uncertainty at NLO and N^{3}LO
  - depends on the chosen range of cutoffs which in practice is rather restricted
  - softer cutoffs Λ overestimate the true uncertainty (expansion in Q/Λ)

$\longrightarrow$ **does not provide a reliable way to estimate theoretical uncertainty**

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$\longrightarrow$ **does not provide a reliable way to estimate theoretical uncertainty**

Theoretical uncertainty by estimating the size of higher-order contributions

(standard in ChPT)

Expansion parameter: $$Q = \frac{\text{momenta of pions and nucleons or } M_\pi \sim 140 \text{ MeV}}{\text{lowest hard scale [i.e. } \Lambda]}$$

Chiral expansion for the total cross section

- **Neutron-proton scattering**

E_{lab} = 25 MeV [p = 108 MeV]

R = 0.9 fm [Λ = 440 MeV]: $\sigma_{tot} = \overbrace{396}^{Q^0} \underbrace{\overbrace{- 15}^{Q^2}}_{\sim 40} \underbrace{\overbrace{- 1}^{Q^3}}_{\sim 12} \underbrace{\overbrace{+ 0}^{Q^4}}_{\sim 4} = 380$ mbarn

Q ~ 0.3 ⟶ expect: ~ 40 ~ 12 ~ 4

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$E_{lab} = 200$ MeV [p = 306 MeV]

R = 0.9 fm [Λ = 440 MeV]: $\sigma_{tot} = 35 \underbrace{+15}_{\sim 17} \underbrace{-7}_{\sim 12} \underbrace{+0}_{\sim 8} = 43$ mbarn

Q ~ 0.7 ⟶ expect: ~ 17 ~ 12 ~ 8

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Q ~ 1 ⟶ expect no convergence!

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Q ~ 1 ⟶ expect no convergence!

- **Neutron-deuteron scattering**

E_{lab} = 10 MeV [p = 69 MeV]

R = 0.9 fm [Λ = 440 MeV]: $\sigma_{tot} = 918 + \underbrace{85}_{\sim 80} + \underbrace{2}_{\sim 24} + \underbrace{11}_{\sim 7} = 1046$ mbarn

Q ~ 0.3 ⟶ expect: ~ 80 ~ 24 ~ 7

E_{lab} = 200 MeV [p = 306 MeV]

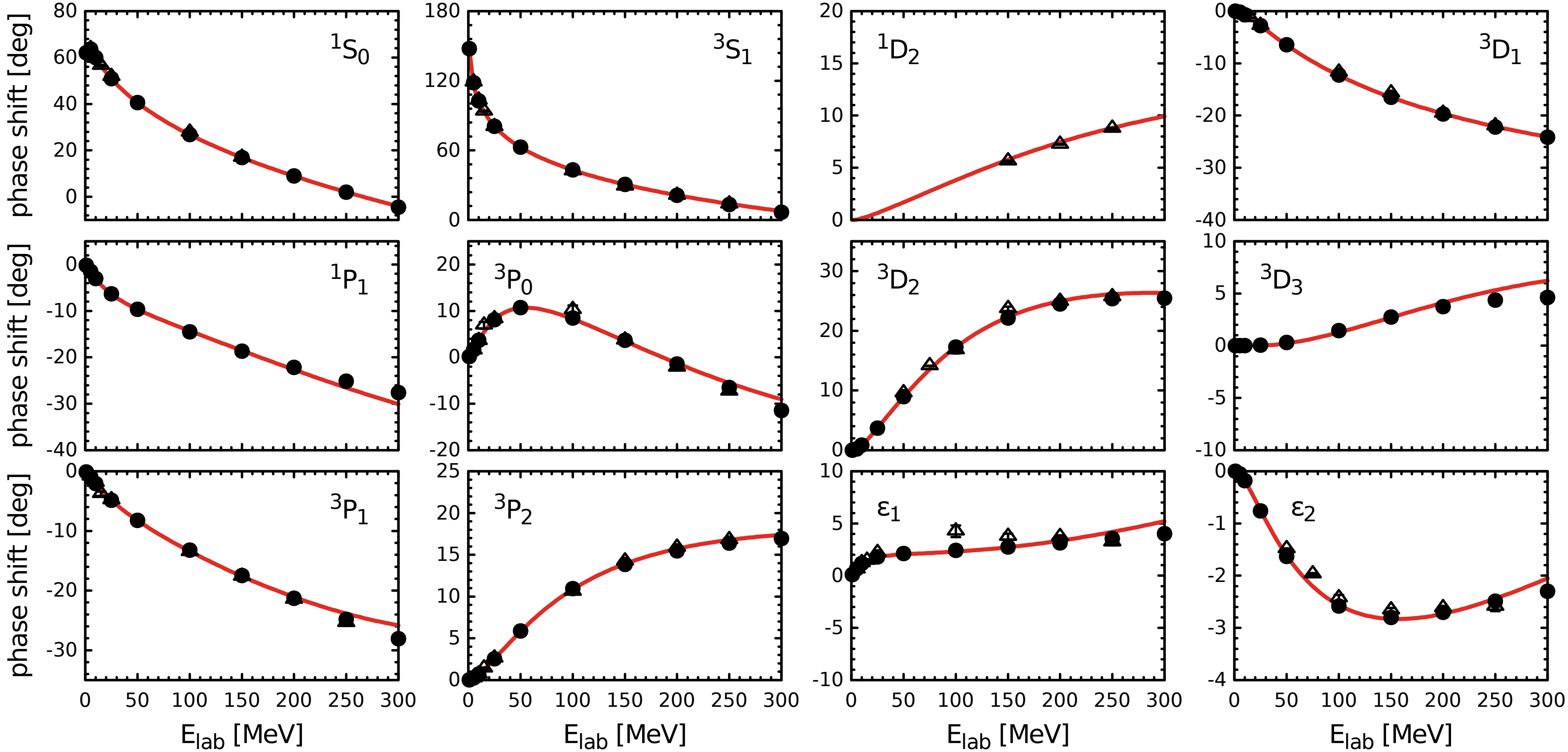
R = 0.9 fm [Λ = 440 MeV]: $\sigma_{tot} = 43 + \underbrace{11}_{\sim 21} + \underbrace{8}_{\sim 14} - \underbrace{1}_{\sim 10} = 61$ mbarn

Q ~ 0.7 ⟶ expect: ~ 21 ~ 14 ~ 10

R = 1.2 fm [Λ = 330 MeV]: $\sigma_{tot} = 20 + 10 + 4 + 20 = 54$ mbarn

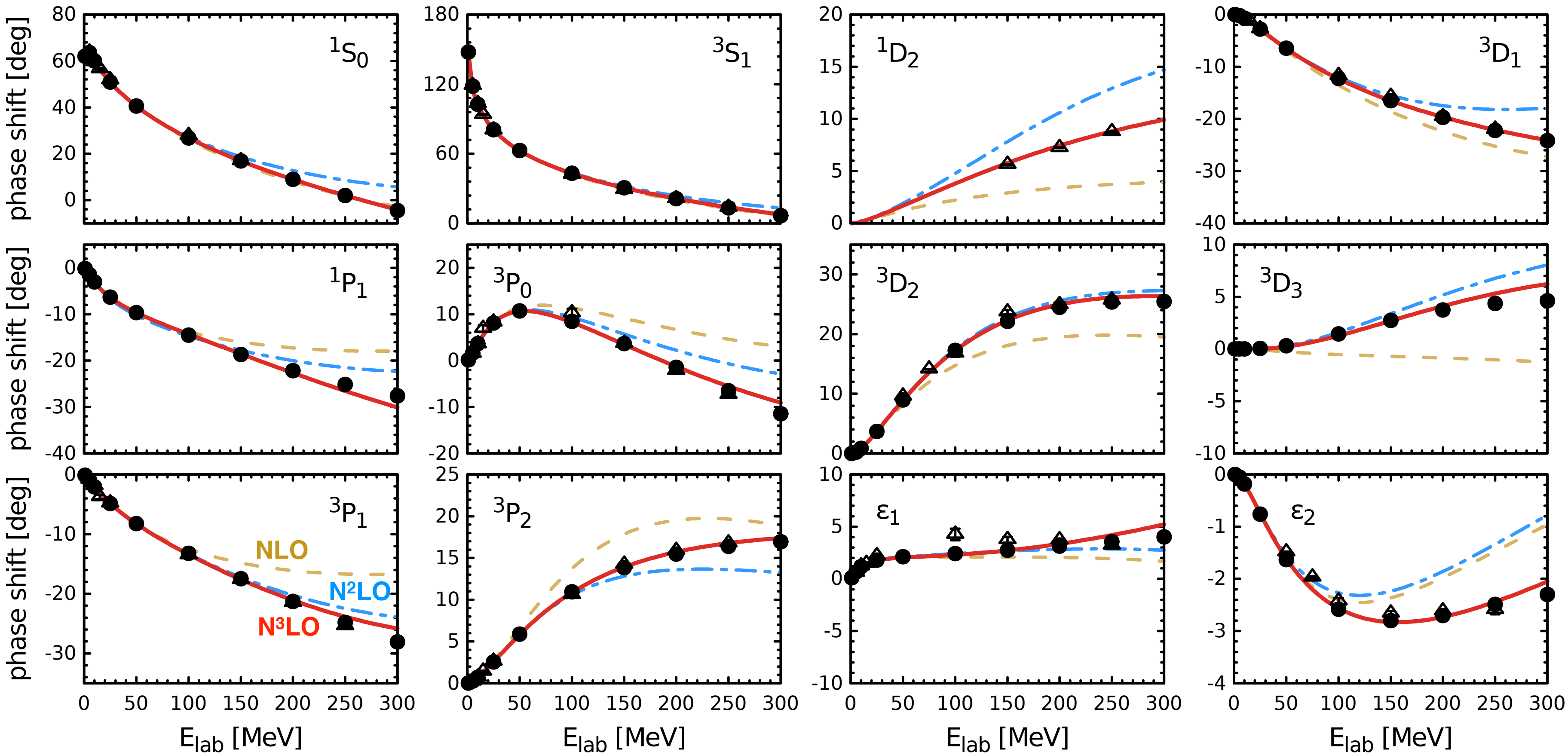
Q ~ 1 ⟶ expect no convergence!

Uncertainty in phase shifts



The description of phase shifts at N^{3}LO is nearly perfect. **But what is the expected theoretical uncertainty?**

Uncertainty in phase shifts



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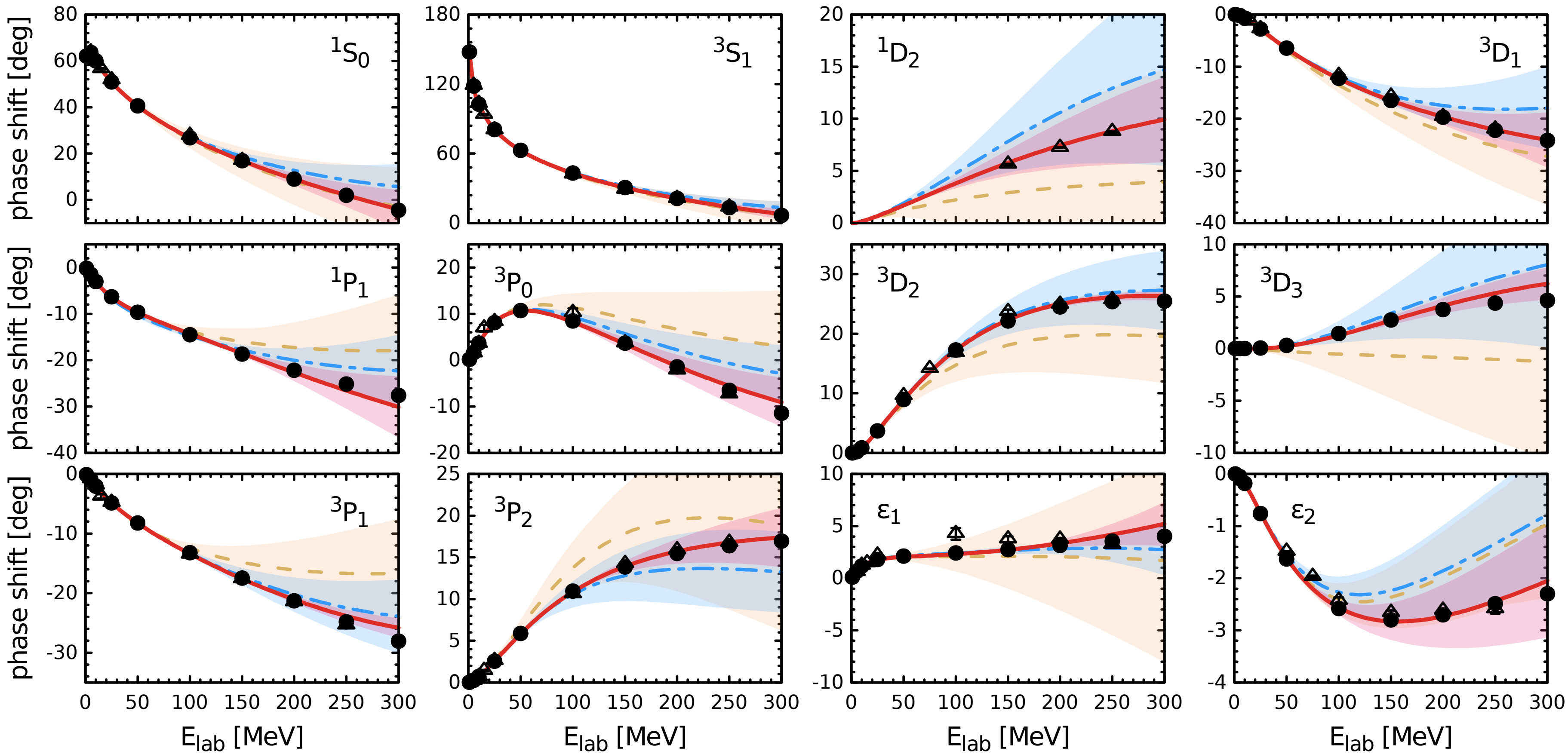
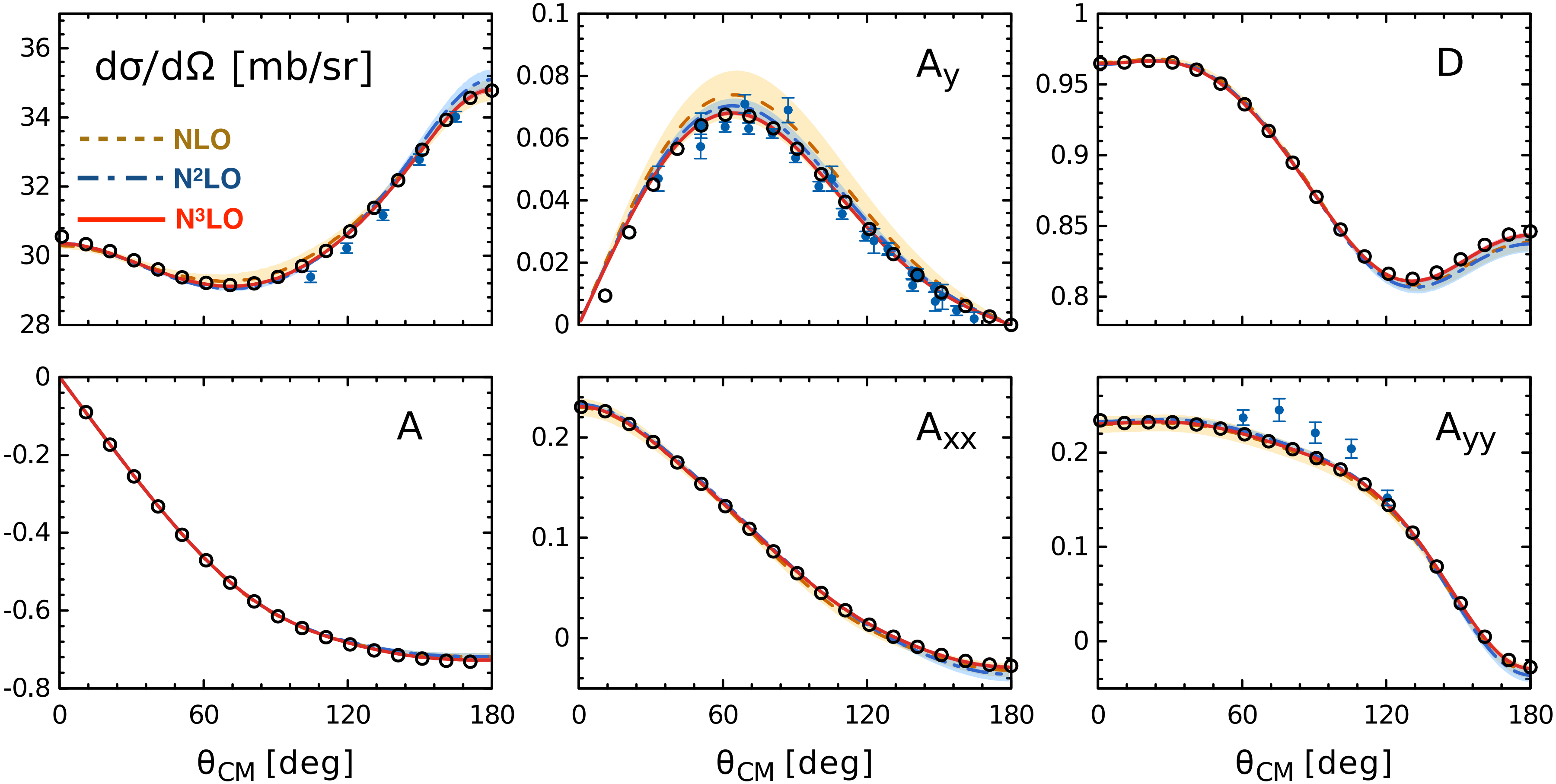


For an observable $X(p)$, we assign: $\Delta X^{(n)}(p) = \mathrm{Abs}\left(X^{(n)}(p) - X^{(n-1)}(p)\right) \times \mathrm{Max}\left(\frac{p}{\Lambda}, \frac{M_\pi}{\Lambda}\right)$ to estimate the size of corrections beyond the order Q^n.

(if higher-order corrections are available, we use them as a lower bound)

NN scattering observables at E_{lab}=25 MeV



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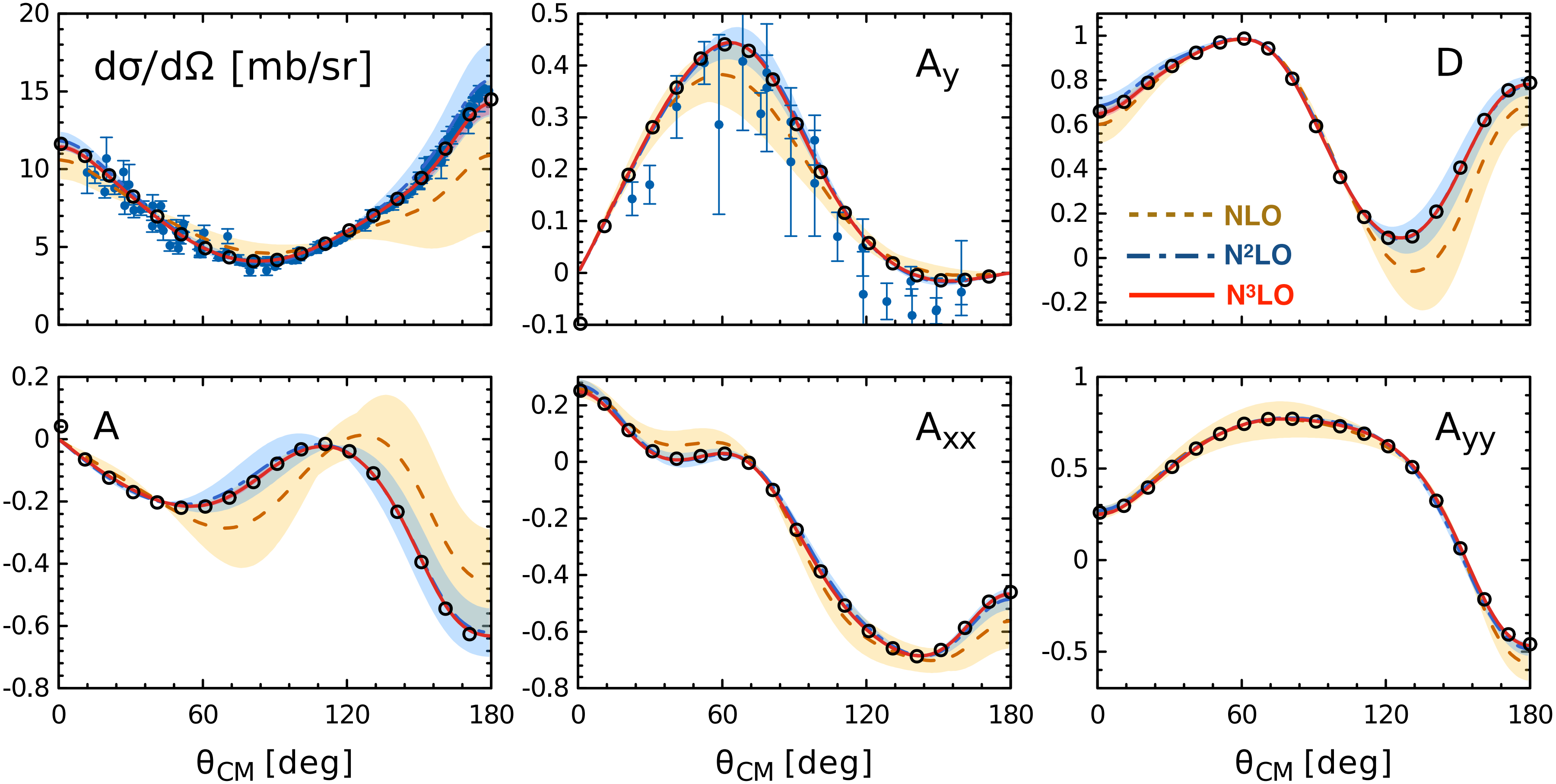
(if higher-order corrections are available, we use them as a lower bound)

NN scattering observables at E_{lab}=96 MeV

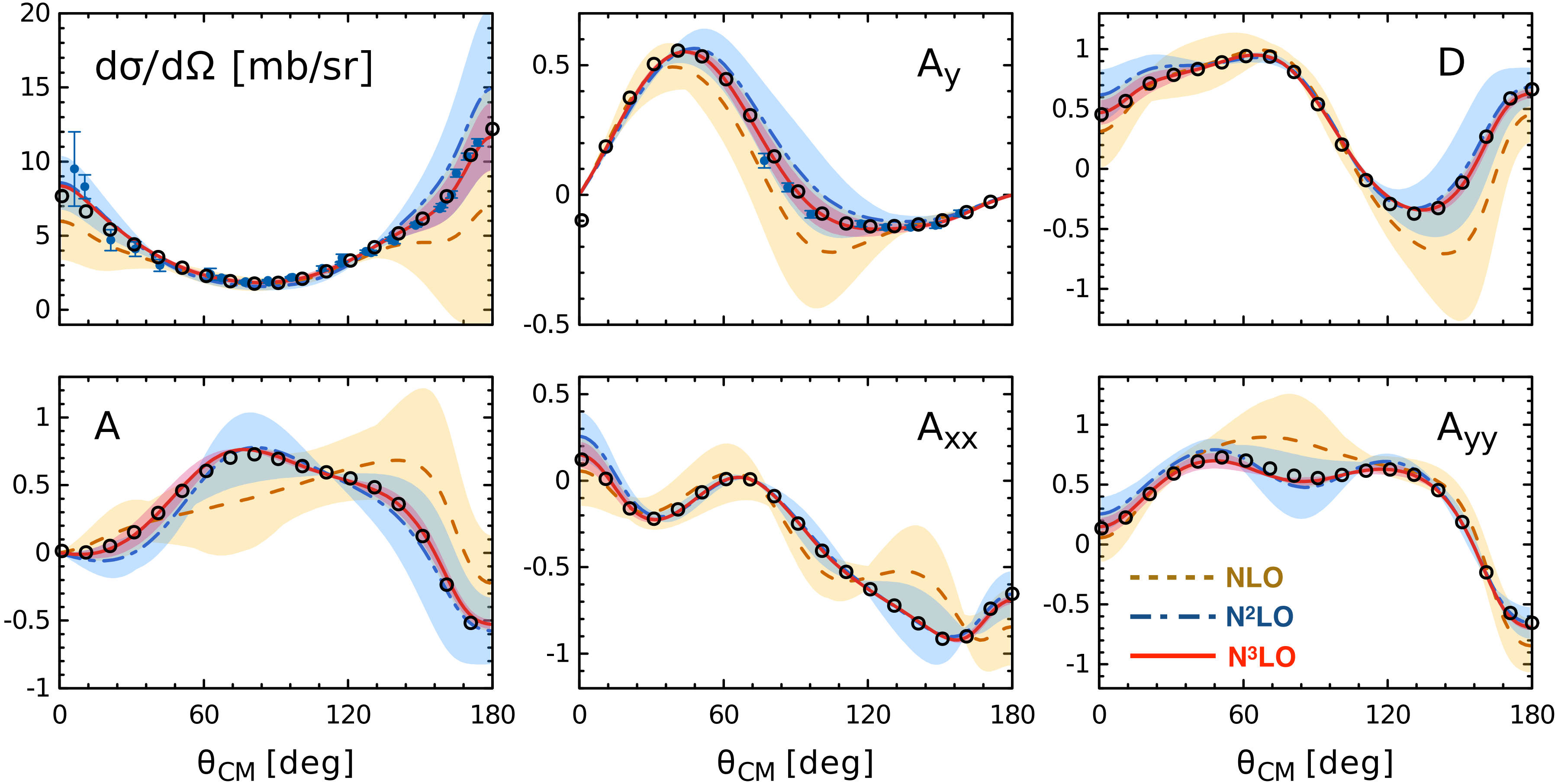


For an observable $X(p)$, we assign: $\Delta X^{(n)}(p) = \mathrm{Abs}\Big(X^{(n)}(p) - X^{(n-1)}(p)\Big) \times \mathrm{Max}\left(\frac{p}{\Lambda}, \frac{M_\pi}{\Lambda}\right)$

to estimate the size of corrections beyond the order Q^n.

(if higher-order corrections are available, we use them as a lower bound)

NN scattering observables at E_{lab}=200 MeV



For an observable $X(p)$, we assign: $\Delta X^{(n)}(p) = \mathrm{Abs}\big(X^{(n)}(p) - X^{(n-1)}(p)\big) \times \mathrm{Max}\left(\frac{p}{\Lambda}, \frac{M_\pi}{\Lambda}\right)$

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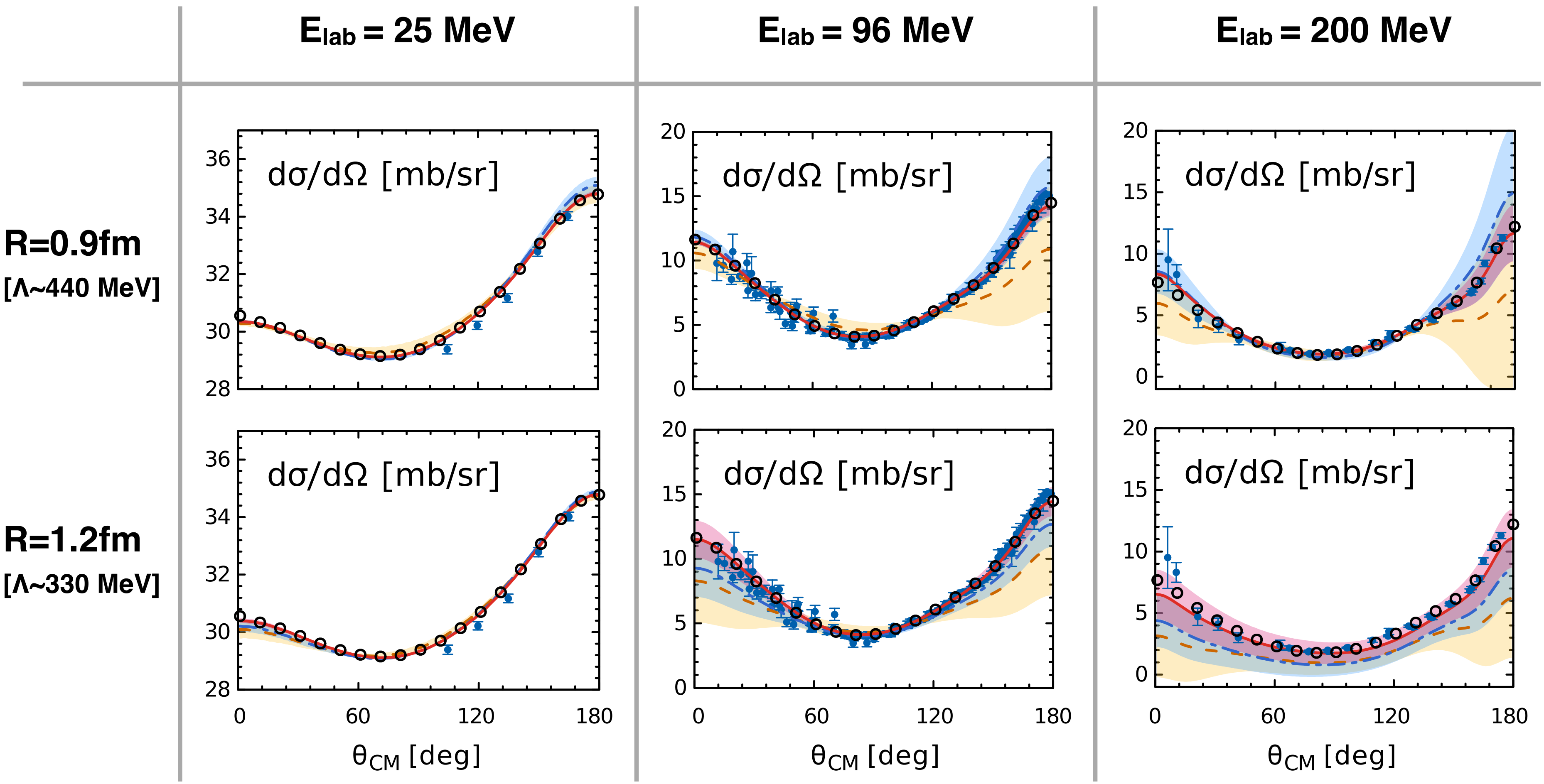
NN cross section: cutoff dependence



Deuteron properties

Cutoff 0.9 fm [Λ=440 MeV]:

	NLO	N^2LO	N^3LO	empirical
B_d (MeV)	2.20(3)	2.23(1)	2.220(3)	2.224575(9)
A_S ($fm^{-1/2}$)	0.877(8)	0.886(3)	0.8835(8)	0.8846(9)
η	0.0256(5)	0.0255(1)	0.0256(0)	0.0256(4)
r_d (fm)	1.968(7)	1.965(7)	1.972(2)	1.97535(85)
Q (fm^2)	0.274(10)	0.270(10)	0.280(3)	0.2859(3)

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Cutoff 1.2 fm [Λ=330 MeV]:

	NLO	N^2LO	N^3LO	empirical
B_d (MeV)	2.22(1)	2.23(1)	2.218(5)	2.224575(9)
A_S ($fm^{-1/2}$)	0.882(5)	0.887(4)	0.8829(13)	0.8846(9)
η	0.0260(13)	0.0259(3)	0.0256(1)	0.0256(4)
r_d (fm)	1.964(16)	1.963(16)	1.979(7)	1.97535(85)
Q (fm^2)	0.267(26)	0.265(26)	0.291(11)	0.2859(3)

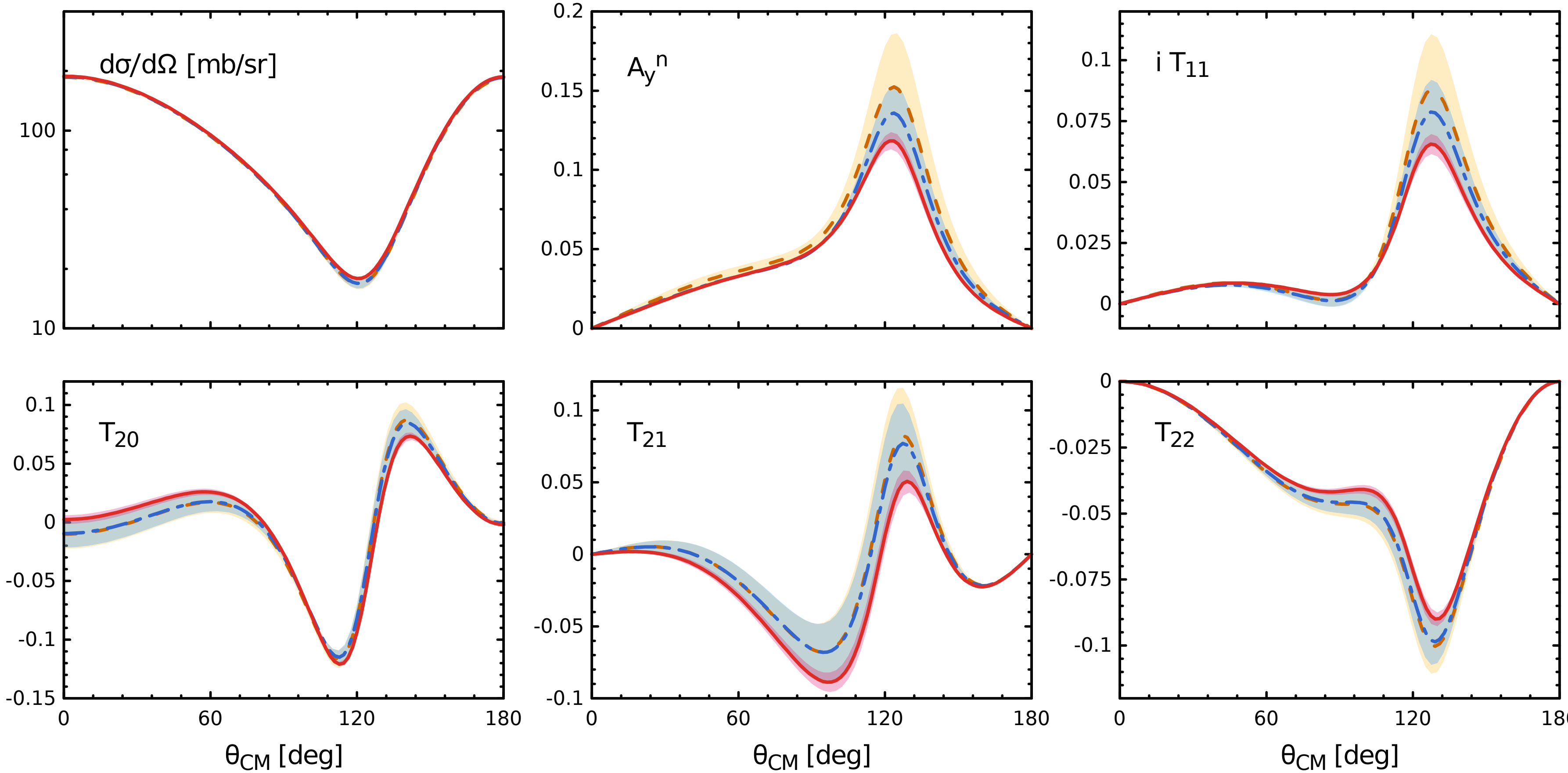
estimation of higher orders: NLO, N^2LO - a priori and a posteriori; N^3LO - a priori

Nd elastic scattering:
Where to search for 3NF effects?

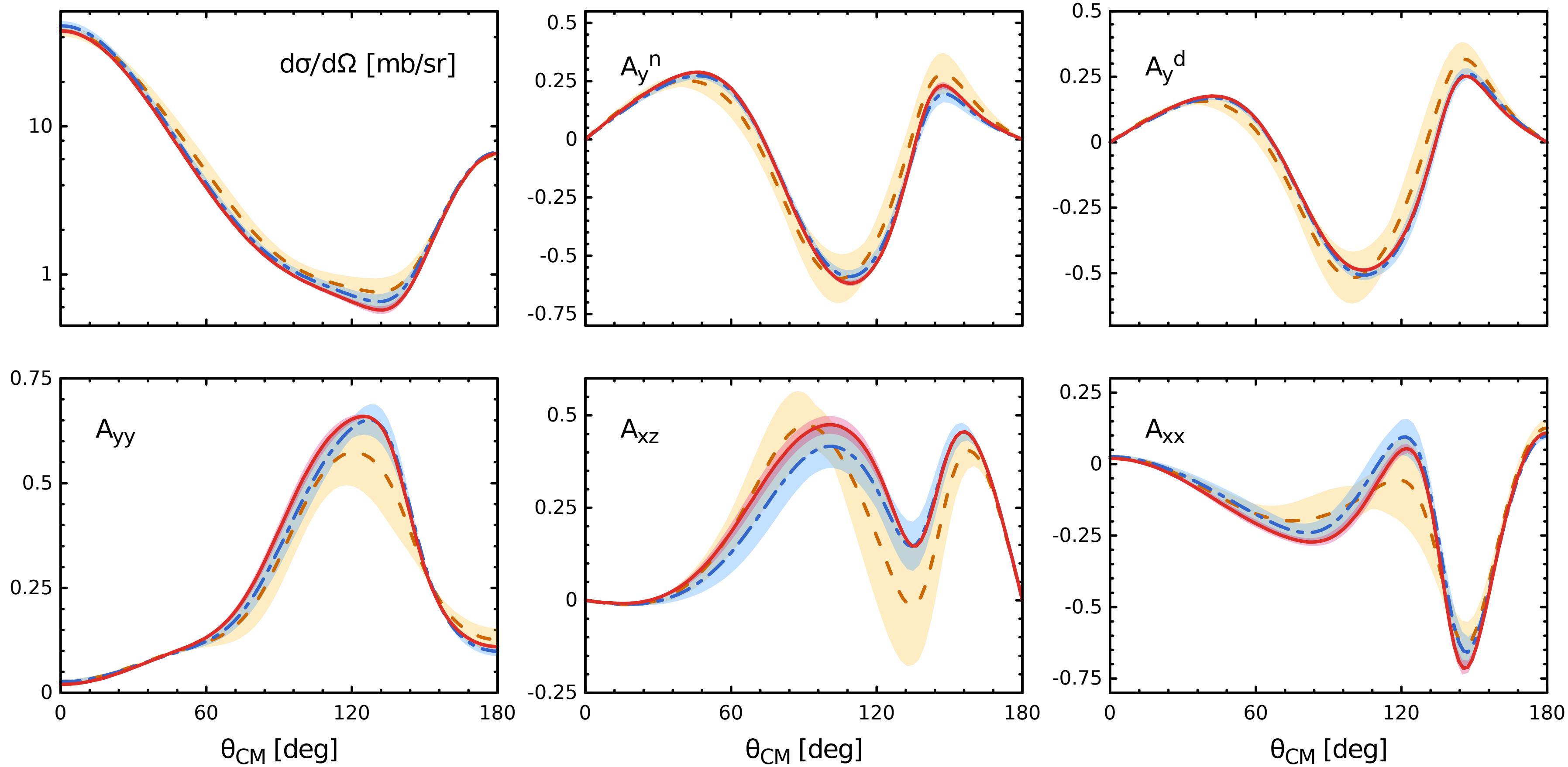
Calculate 3N scattering observables using NN potential at N^3LO and identify large deviations from the data (i.e. >> the uncertainty)

→ **clear indications for missing 3NFs** (start contributing at N^2LO)!

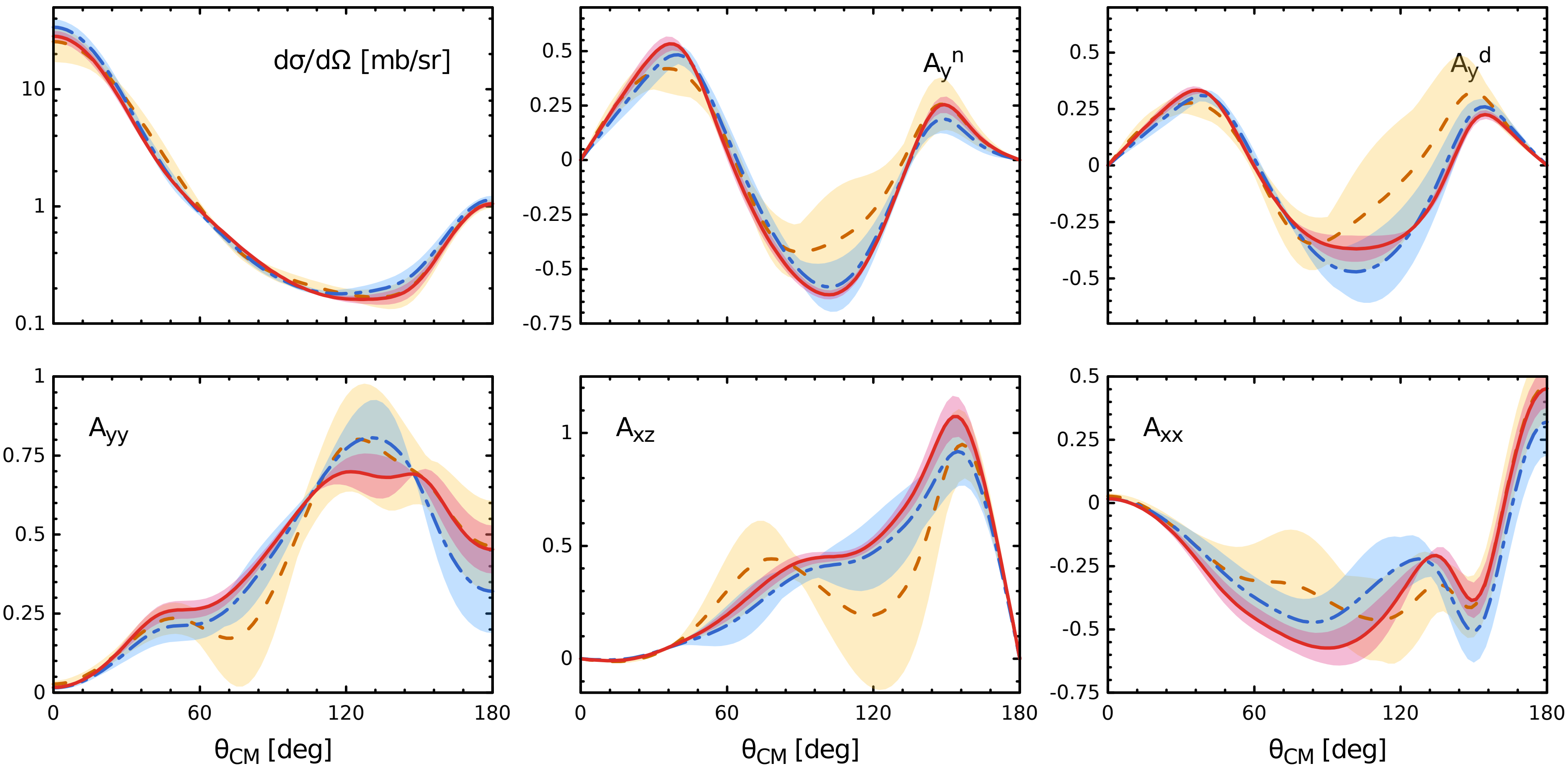
Elastic Nd scattering at E_{lab}=10 MeV: convergence



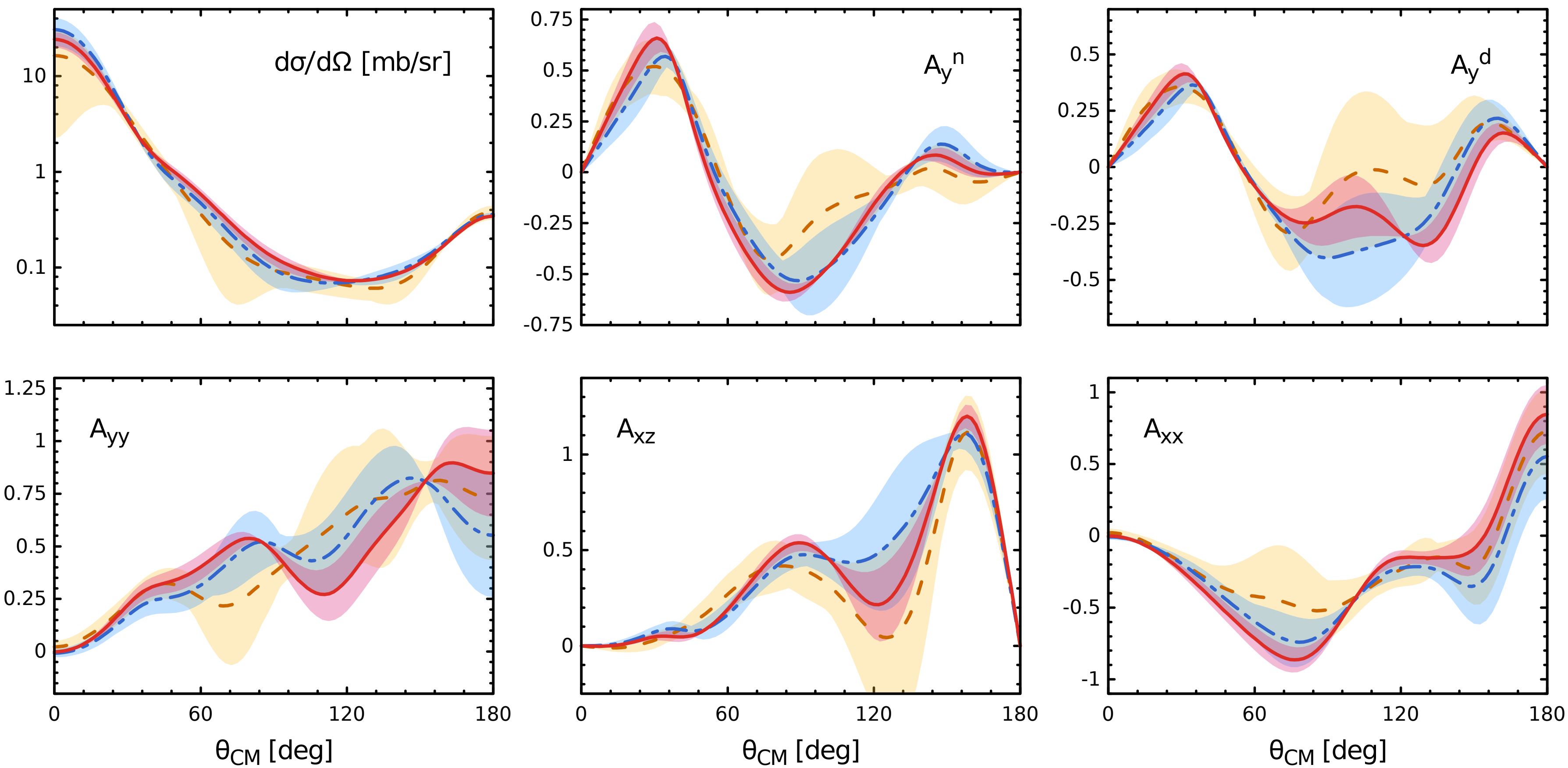
Elastic Nd scattering at E_{lab}=70 MeV: convergence



Elastic Nd scattering at E_{lab}=135 MeV: convergence

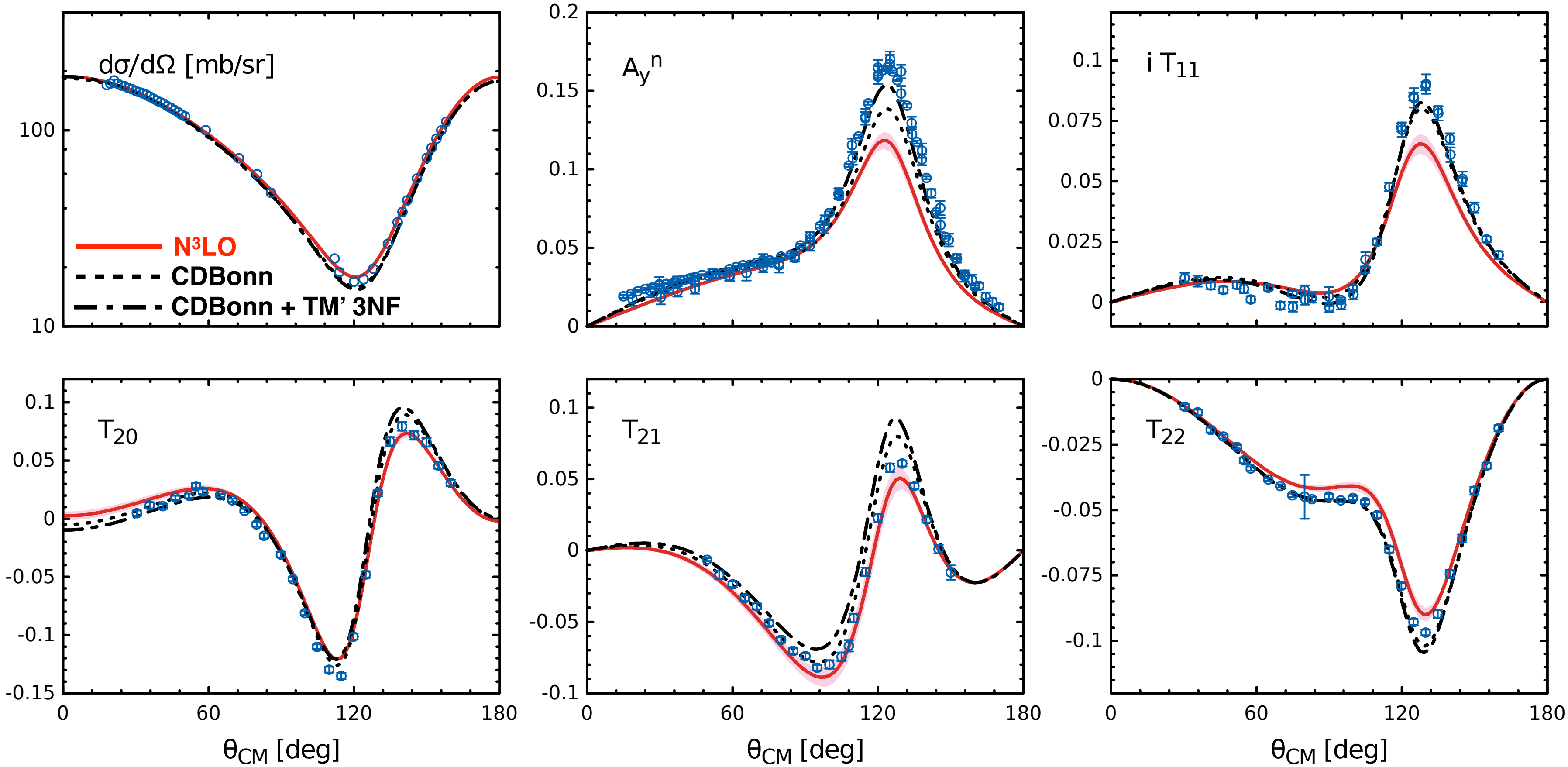


Elastic Nd scattering at E_{lab}=200 MeV: convergence



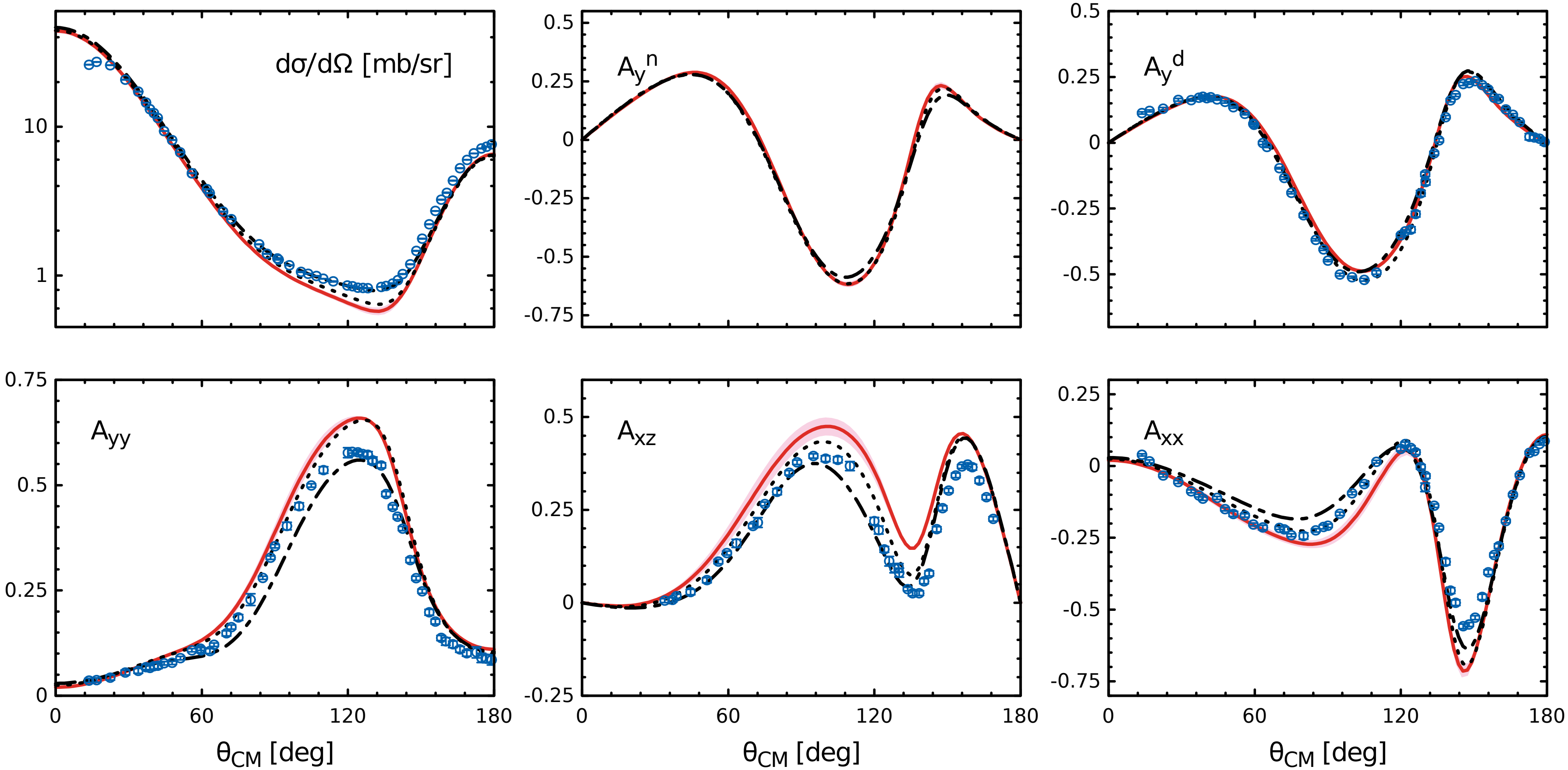
Notice: N³LO error bands are larger for Nd than for NN scattering. It is likely that the uncertainty at N³LO is overestimated (including 3NF is expected to bring N²LO and N³LO results closer to each other)

Elastic Nd scattering at E_{lab}=10 MeV



Notice: most of the data are Coulomb-corrected pd data

Elastic Nd scattering at E_{lab}=70 MeV



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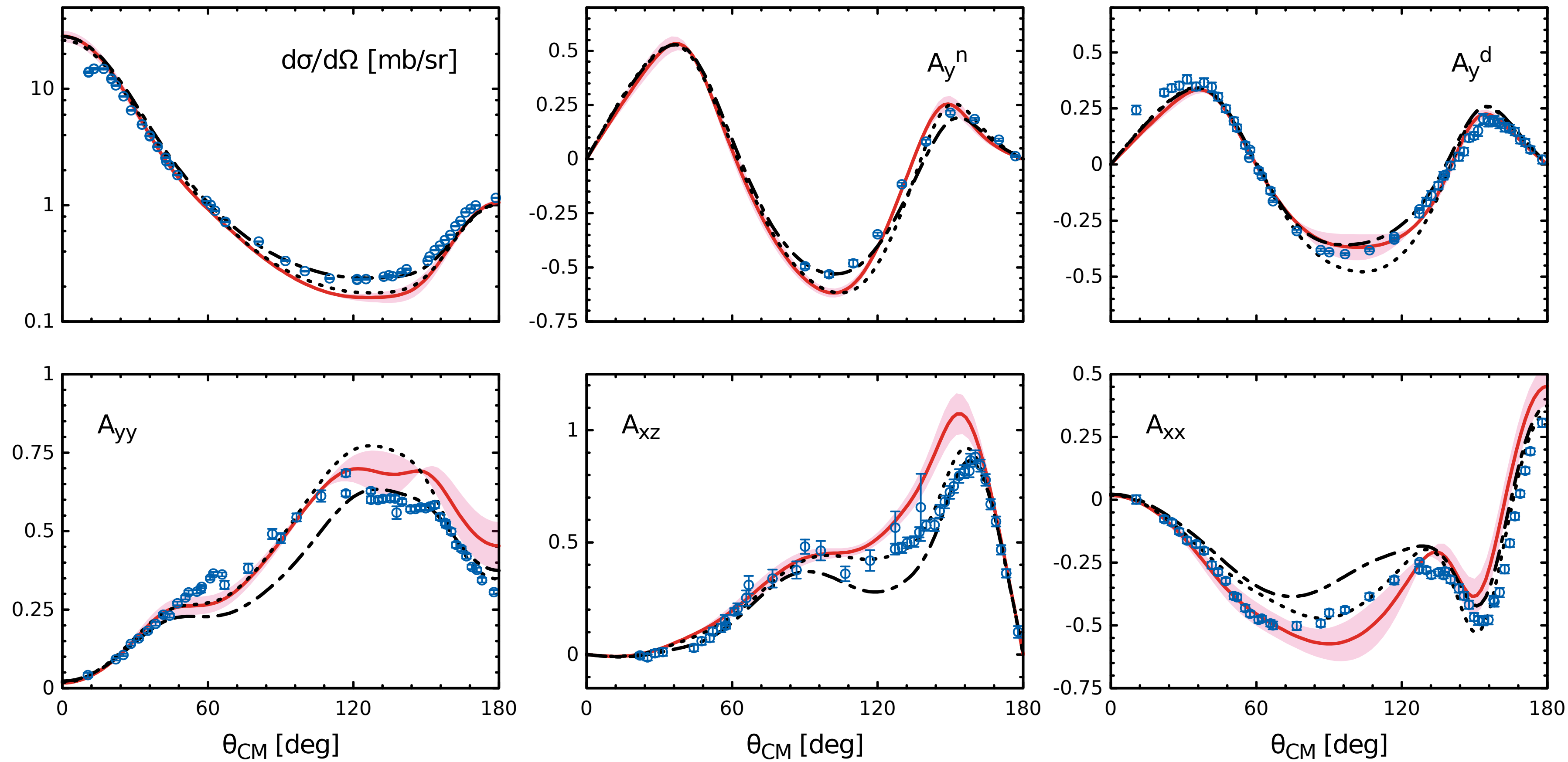
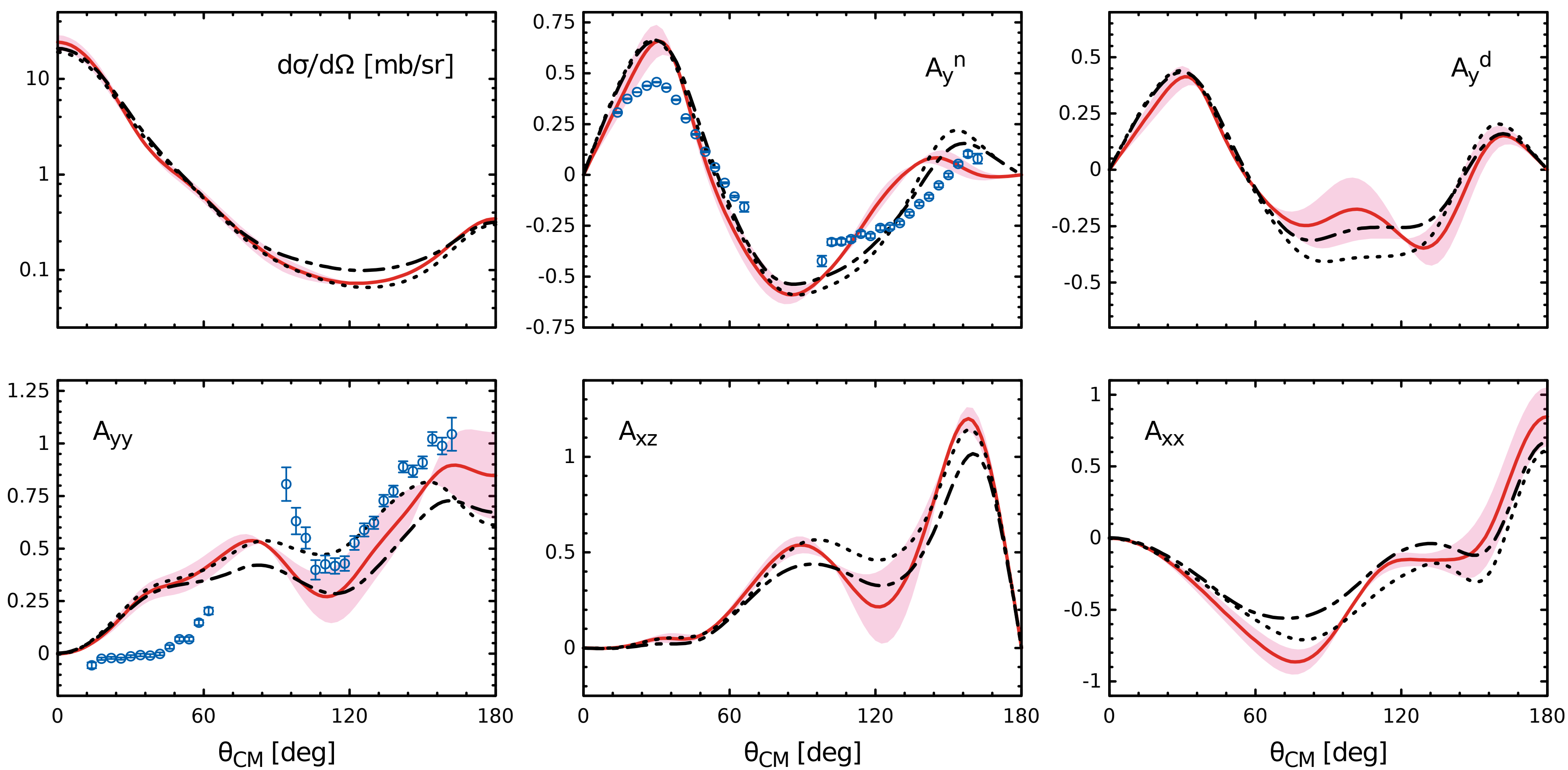
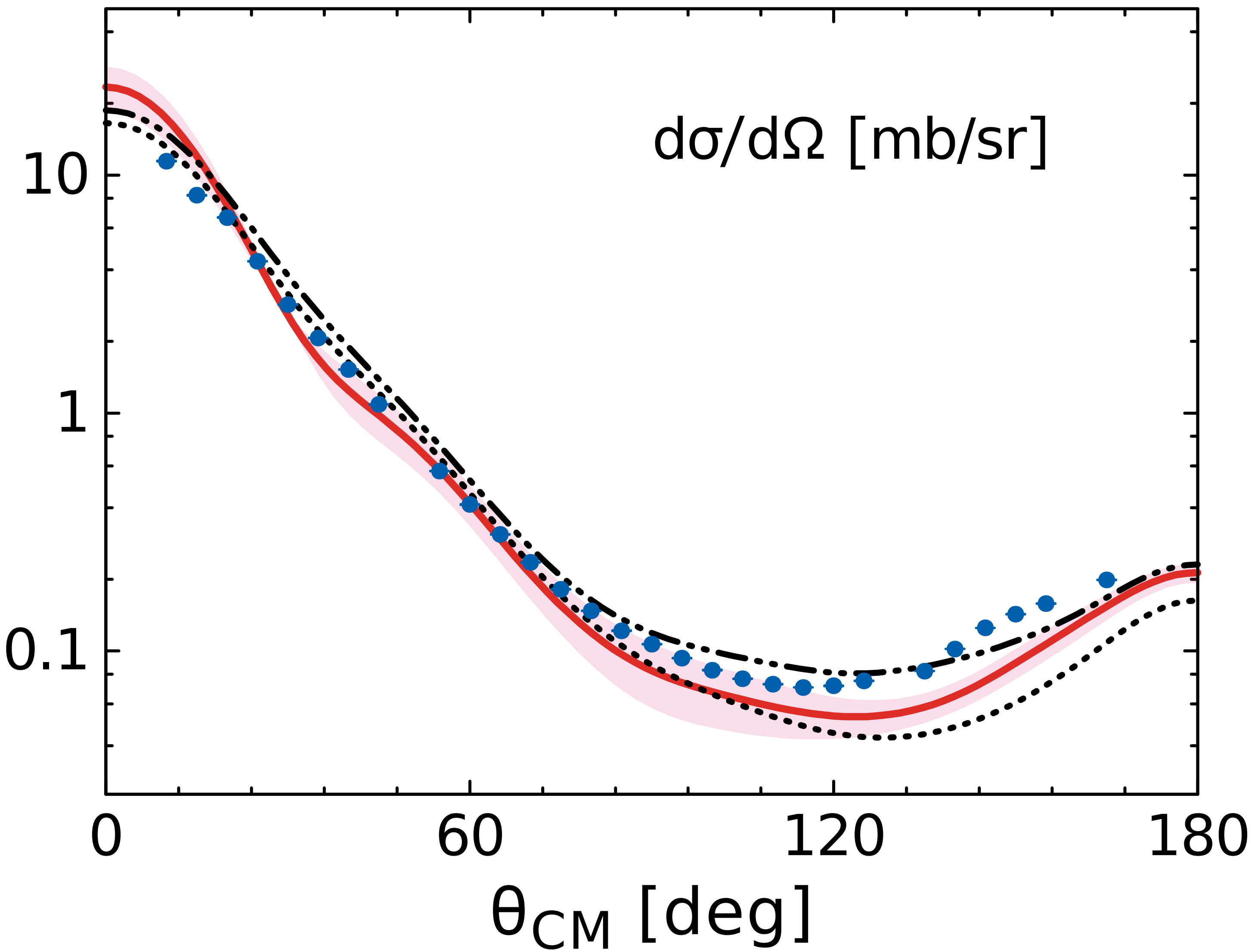


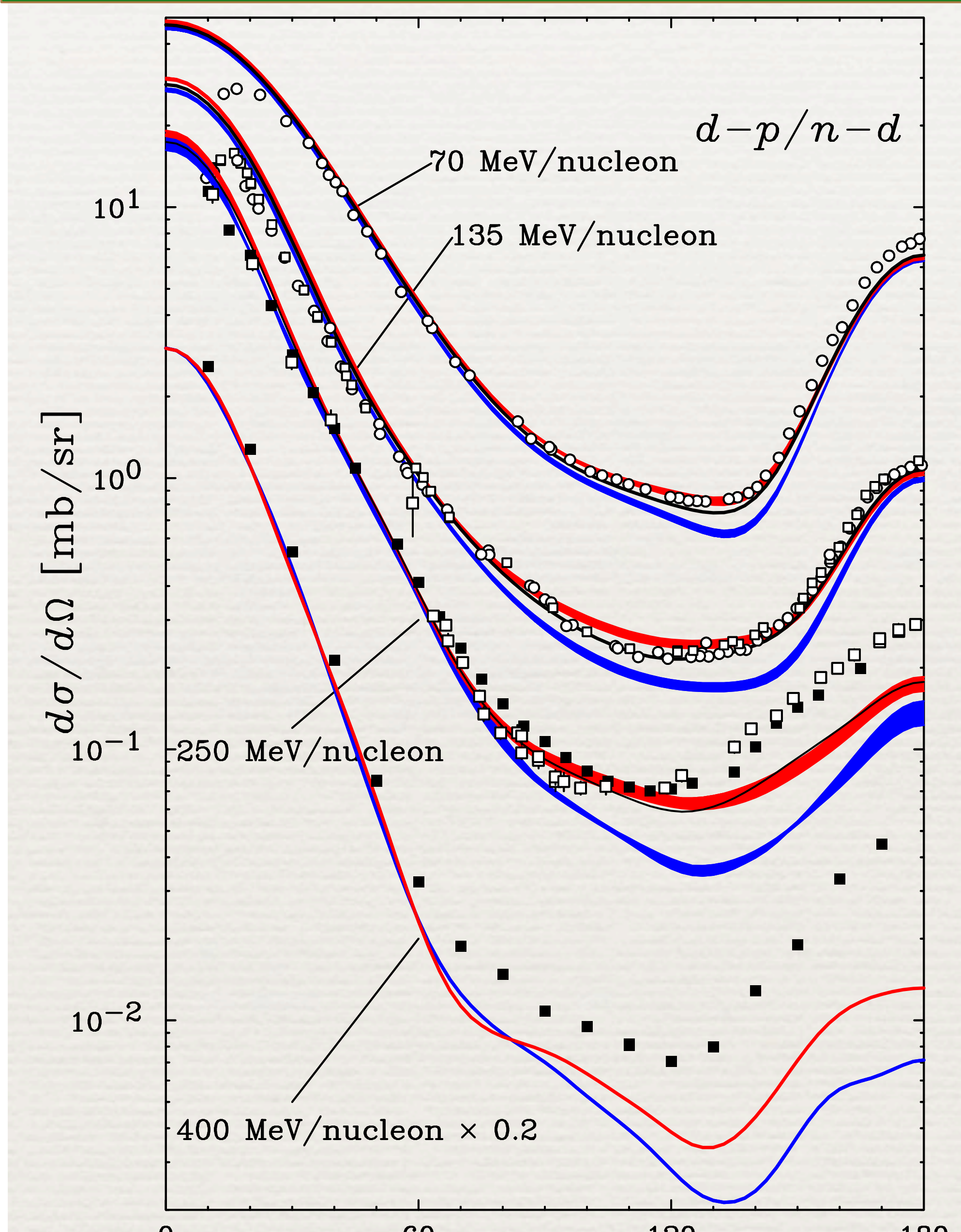
Elastic Nd scattering at E_{lab}=200 MeV

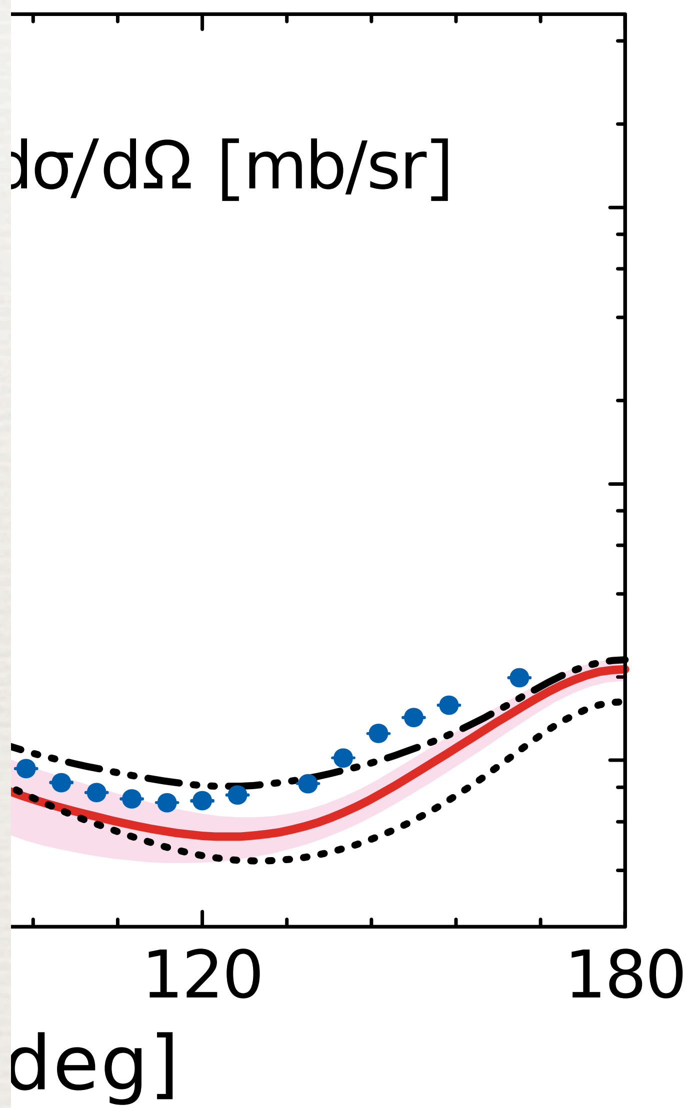


Elastic Nd scattering at E_{lab}=250 MeV



Elastic Nd scattering at E_{lab}=250 MeV





Summary

A new generation of chiral NN potentials up to N^3LO is being developed:

Better performance at higher energies, less sensitivity to cutoffs, no need for Spectral Function Regularization, no fine tuning for πN LECs.

Proposed a simple approach to estimate theor. uncertainty at a given order

Applicable for a single cutoff, more reliable/meaningful than cutoff variation, 2N and 3N scattering calculations show good convergence of the chiral expansion

Identified the promising cases to search for 3NF in elastic Nd scattering

Not much room for 3NF at low energy except for A_y, iT_{11} and smaller but visible deviations in T_{21}, T_{22}; large discrepancies with the data at intermediate energies of $E_{lab} \sim 70 \ldots 150$ MeV where results at N^3LO are still accurate;
cross sections at higher energies…

Ready for quantitative tests of the novel chiral 3NFs !