Electroweak Current in Chiral Effective Field Theory

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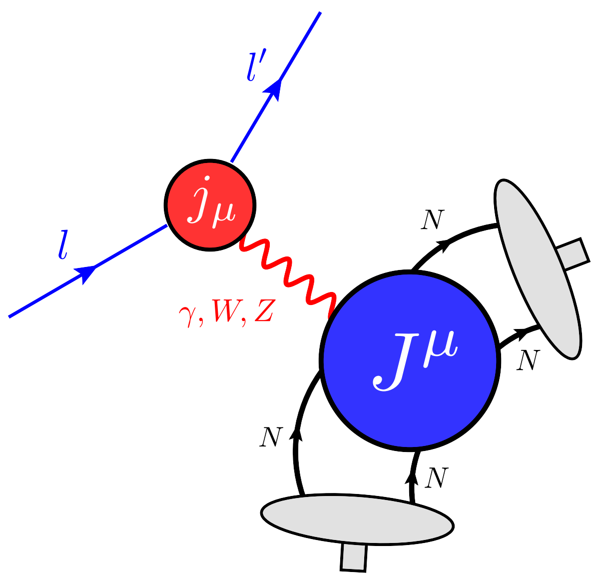
RUB

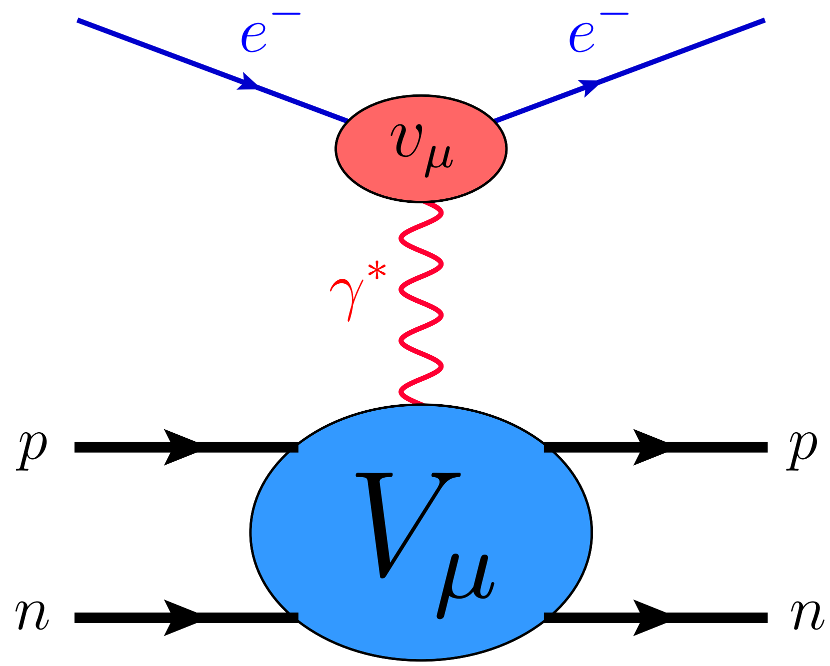
Outline

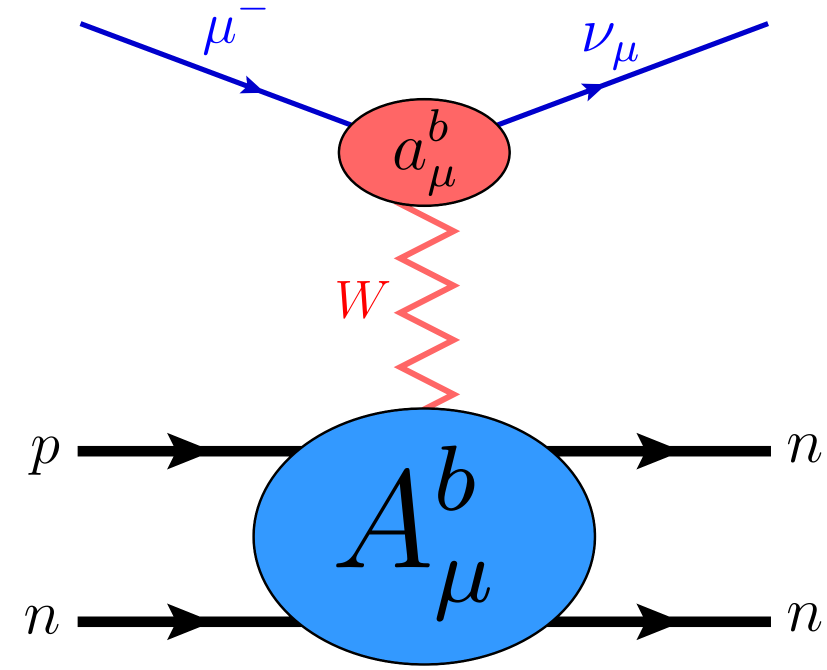
- Construction of nuclear currents in chiral EFT
- Symmetries for currents
- Nuclear currents up to N^3LO
- Symmetry preserving regularization
- Application to em deuteron form factor



Nuclear currents in chiral EFT

Electroweak probes on nucleons and nuclei can be described by current formalism





Chiral EFT Hamiltonian depends on external sources

$$H[a, v, s, p]$$

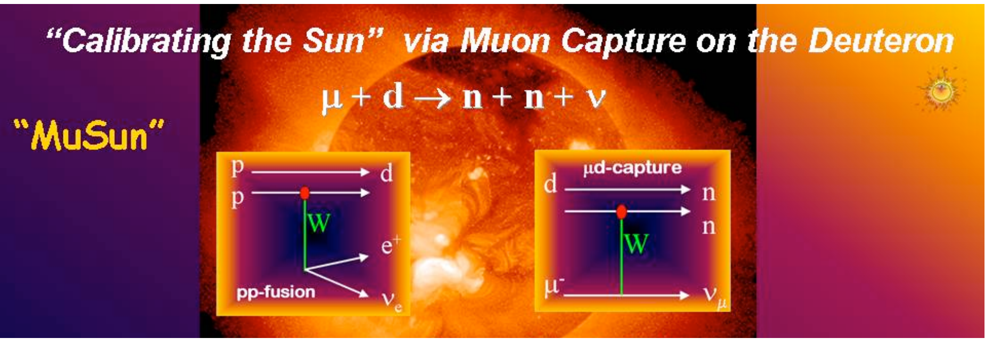
Axial-vector source

Pseudoscalar source

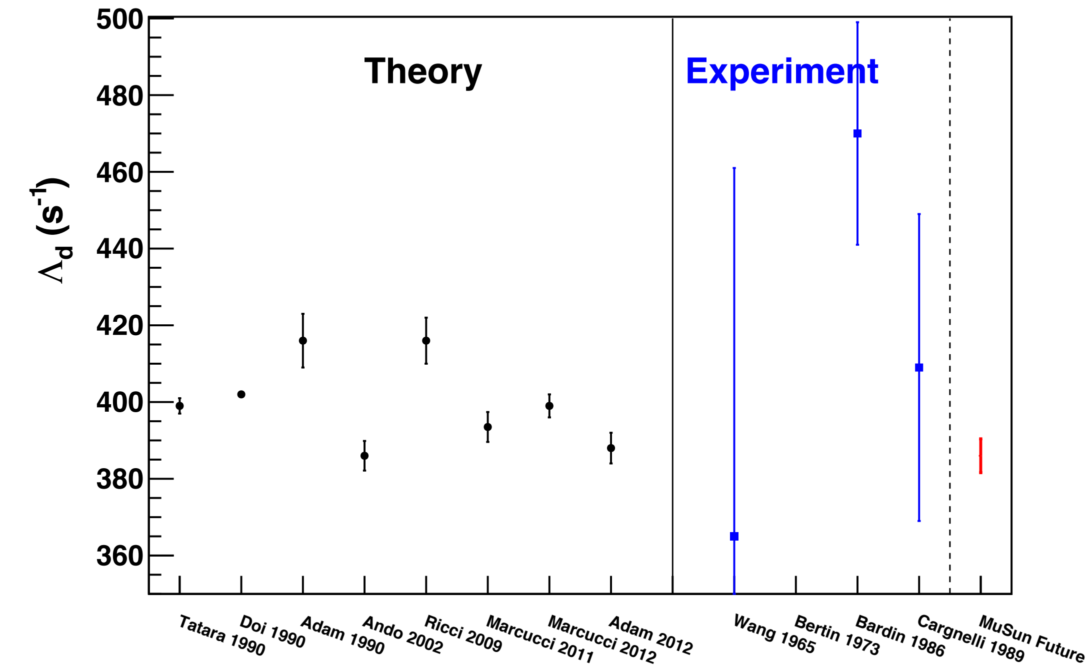
Vector source

Scalar source

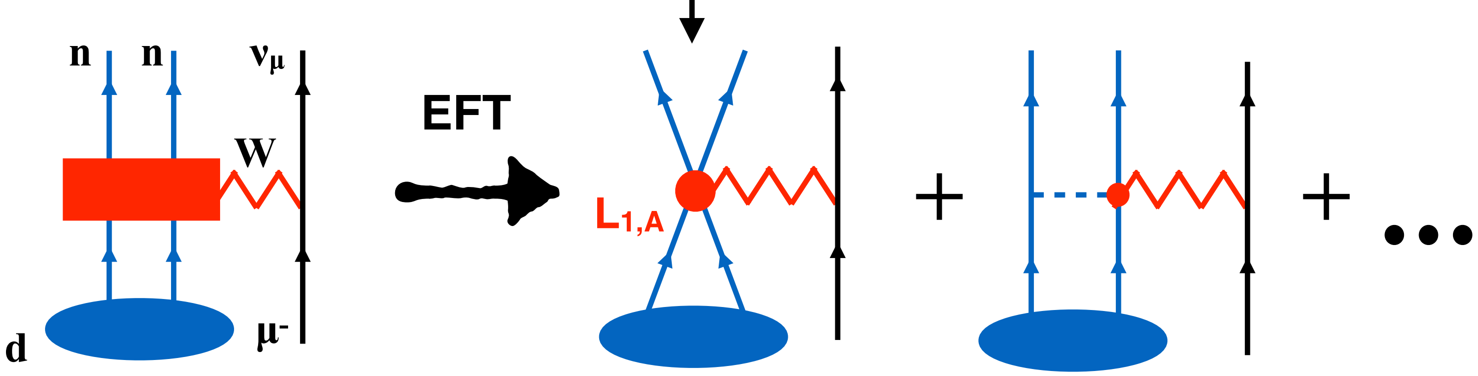
MuSun experiment at PSI



Main goal: measure the doublet capture rate Λ_d in $\mu^- + d \rightarrow \nu_\mu + n + n$ with the accuracy of ~ 1.5%



This will strongly constrain the short-range axial current



The resulting axial exchange current can be used to make precision calculations for

- triton half life, $fT_{1/2} = 1129.6 \pm 3.0$ s, and the muon capture rate on ^{3}He, $\Lambda_0 = 1496 \pm 4\ s^{-1}$ → precision tests of the theory
- weak reactions of astrophysical interest such as e.g. the pp chain of the solar burning:
- $L_{1,A}$ governs the leading 3NF

$$p + p \rightarrow d + e^+ + \nu_e$$
$$p + p + e^- \rightarrow d + \nu_e$$
$$p + {}^3He \rightarrow {}^4He + e^+ + \nu_e$$
$${}^7Be + e^- \rightarrow {}^7Li + \nu_e$$
$${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$$

Historical remarks

- Meson-exchange theory, Skyrme model, phenomenology, …
  Brown, Adam, Mosconi, Ricci, Truhlik, Nakamura, Sato, Ando, Kubodera, Riska, Sauer, Friar, Gari …

- First derivation within chiral EFT to leading 1-loop order using TOPT
  Park, Min, Rho Phys. Rept. 233 (1993) 341; NPA 596 (1996) 515;
  Park et al., Phys. Rev. C67 (2003) 055206
  – only for the threshold kinematics
  – pion-pole diagrams ignored
  – box-type diagrams neglected
  – renormalization incomplete

- Leading one-loop expressions using TOPT for general kinematics (still incomplete, e.g. no 1/m corrections)
  Pastore, Girlanda, Schiavilla, Goity, Viviani, Wiringa;
  PRC78 (2008) 064002; PRC80 (2009) 034004; PRC84 (2011) 024001 ← Vector current

  Baroni, Girlanda, Pastore, Schiavilla, Viviani;
  PRC93 (2016) 015501, Erratum: PRC 93 (2016) 049902 ← Axial vector current

Complete derivation to leading one-loop order using the method of UT

Kölling, Epelbaum, HK, Meißner;
PRC80 (2009) 045502; PRC84 (2011) 054008 ← Vector current

HK, Epelbaum, Meißner, Ann. Phys. 378 (2017) 317 ← Axial vector current

Diagonalization via Okubo

- Decomposition of the Fock space $\mathcal{H}$

Projector operators: $\eta + \lambda = 1$

$$\mathcal{H}_M \overset{\eta}{\leftarrow} \mathcal{H} = \mathcal{H}_M \oplus \mathcal{H}_R \overset{\lambda}{\rightarrow} \mathcal{H}_R$$

Model-space including only pure nucleon states

Remainder-space including states with at least one pion

$$H|\Psi\rangle = (H_0 + H_I)|\Psi\rangle = E|\Psi\rangle \quad \Longleftrightarrow \quad \begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} \begin{pmatrix} \eta|\Psi\rangle \\ \lambda|\Psi\rangle \end{pmatrix} = E \begin{pmatrix} \eta|\Psi\rangle \\ \lambda|\Psi\rangle \end{pmatrix}$$

- Block-diagonalization by applying unitary transformation

$$\tilde{H} = U^\dagger H U = \begin{pmatrix} \eta \tilde{H} \eta & 0 \\ 0 & \lambda H \lambda \end{pmatrix}$$

$$V_{\rm eff} = \eta(\tilde{H} - H_0)\eta$$

$V_{\rm eff}$ is E - indep. $\Rightarrow$ important for few-nucleon simulations

Possible parametrization by Okubo '54

$$U = \begin{pmatrix} \eta(1 + A^\dagger A)^{-1/2} & -A^\dagger(1 + AA^\dagger)^{-1/2} \\ A(1 + A^\dagger A)^{-1/2} & \lambda(1 + AA^\dagger)^{-1/2} \end{pmatrix}$$

With decoupling eq. $\lambda(H - [A, H] - AHA)\eta = 0$

Can be solved perturbatively within ChPT

Epelbaum, Glöckle, Meißner, ´98

Unitary transformations for currents

- Step 1: $\tilde{H} \to \tilde{H}[a,v,s,p] = U^\dagger H[a,v,s,p] U$

Okubo transf. or further strong unitary transf. are not enough to renormalize the currents

- Step 2: additional (time-dependent) unitary transformations

$$i\frac{\partial}{\partial t}\Psi = H\Psi \longrightarrow i\frac{\partial}{\partial t}U(t)U^\dagger(t)\Psi = U(t)i\frac{\partial}{\partial t}U^\dagger(t)\Psi + \left(i\frac{\partial}{\partial t}U(t)\right)U^\dagger(t)\Psi = HU(t)U^\dagger(t)\Psi$$

$$\Psi' = U^\dagger(t)\Psi \longrightarrow i\frac{\partial}{\partial t}\Psi' = \left[U^\dagger(t)HU(t) - U^\dagger(t)\left(i\frac{\partial}{\partial t}U(t)\right)\right]\Psi'$$

Explicit time-dependence through source terms

$$\tilde{H}[a,v,s,p] \to \underbrace{U^\dagger[a,v]\tilde{H}[a,v,s,p]U[a,v] + \left(i\frac{\partial}{\partial t}U^\dagger[a,v]\right)U[a,v]}_{=:\, H_{\text{eff}}[a,\dot{a},v,\dot{v}]}$$

$$A^b_\mu(\vec{x},t) := \frac{\delta}{\delta a^{\mu,b}(\vec{x},t)} H_{\text{eff}}[a,\dot{a},v,\dot{v}]\Big|_{a=v=0}$$

Due to time-derivatives ($\dot{a}$, $\dot{v}$) the currents depend on energy transfer if transformed into momentum space

Chiral symmetry constraints

Chiral symmetry transformations on the path integral level

Gasser, Leutwyler Ann. Phys. (1984) 142: $v_\mu = \frac{1}{2}(r_\mu + l_\mu)$ and $a_\mu = \frac{1}{2}(r_\mu - l_\mu)$

$$\langle 0_{\rm out}|0_{\rm in}\rangle_{a,v,s,p} = \exp\left(i\,Z[a,v,s,p]\right) = \exp\left(i\,Z[a',v',s',p']\right) = \langle 0_{\rm out}|0_{\rm in}\rangle_{a',v',s',p'}$$

$$\begin{aligned}
r_\mu &\to r'_\mu = R\,r_\mu R^\dagger + iR\,\partial_\mu R^\dagger\ ,\\
l_\mu &\to l'_\mu = L\,l_\mu L^\dagger + iL\,\partial_\mu L^\dagger\ ,\\
s+i\,p &\to s'+i\,p' = R(s+i\,p)L^\dagger\ ,\\
s-i\,p &\to s'-i\,p' = L(s-i\,p)R^\dagger\ .
\end{aligned}$$

Chiral $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ rotation does not change the generating functional $\longrightarrow$ Ward identities

Chiral symmetry transformations on the Hamiltonian level

- There exists a unitary transformation $U(R,L)$ such that from Schrödinger eq.

$i\frac{\partial}{\partial t}\Psi = H_{\rm eff}[a,v,s,p]\Psi$ takes the form $i\frac{\partial}{\partial t}U^\dagger(R,L)\Psi = H_{\rm eff}[a',v',s',p']U^\dagger(R,L)\Psi$

Transformed Hamiltonian is unitary equivalent to the untransformed one

$$H_{\rm eff}[a',\dot a',v',\dot v',s',p'] = U^\dagger(R,L)H_{\rm eff}[a,\dot a,v,\dot v,s,p]U(R,L) + \left(i\frac{\partial}{\partial t}U^\dagger(R,L)\right)U(R,L)$$

Continuity equation

Infinitesimally we have $R = 1 + \frac{i}{2}\boldsymbol{\tau}\cdot\boldsymbol{\epsilon}_R(x)$ and $L = 1 + \frac{i}{2}\boldsymbol{\tau}\cdot\boldsymbol{\epsilon}_L(x)$

Expressed in $\boldsymbol{\epsilon}_V = \frac{1}{2}(\boldsymbol{\epsilon}_R + \boldsymbol{\epsilon}_L)$ and $\boldsymbol{\epsilon}_A = \frac{1}{2}(\boldsymbol{\epsilon}_R - \boldsymbol{\epsilon}_L)$ we have

$$\begin{aligned} \boldsymbol{v}_\mu &\to \boldsymbol{v}'_\mu = \boldsymbol{v}_\mu + \boldsymbol{v}_\mu \times \boldsymbol{\epsilon}_V + \boldsymbol{a}_\mu \times \boldsymbol{\epsilon}_A + \partial_\mu \boldsymbol{\epsilon}_V \\ \boldsymbol{a}_\mu &\to \boldsymbol{a}'_\mu = \boldsymbol{a}_\mu + \boldsymbol{a}_\mu \times \boldsymbol{\epsilon}_V + \boldsymbol{v}_\mu \times \boldsymbol{\epsilon}_A + \partial_\mu \boldsymbol{\epsilon}_A \end{aligned} \quad \longrightarrow \quad \begin{aligned} \dot{v}_\mu &\to \dot{v}'_\mu = \partial_\mu \dot{\epsilon}_V + \dots \\ \dot{a}_\mu &\to \dot{a}'_\mu = \partial_\mu \dot{\epsilon}_A + \dots \end{aligned}$$

$$H_{\text{eff}}[a', \dot{a}', v', \dot{v}', s', p'] = U^\dagger(R,L) H_{\text{eff}}[a, \dot{a}, v, \dot{v}, s, p] U(R,L) + \left(i \frac{\partial}{\partial t} U^\dagger(R,L) \right) U(R,L)$$

- $H_{\text{eff}}[a', \dot{a}', v', \dot{v}', s', p']$ is a function of $\boldsymbol{\epsilon}_V, \dot{\boldsymbol{\epsilon}}_V, \ddot{\boldsymbol{\epsilon}}_V, \boldsymbol{\epsilon}_A, \dot{\boldsymbol{\epsilon}}_A, \ddot{\boldsymbol{\epsilon}}_A$

$$\longrightarrow \quad U = \exp\left(i \int d^3x \left[\boldsymbol{R}^v_0(\vec{x}) \cdot \boldsymbol{\epsilon}_V(\vec{x},t) + \boldsymbol{R}^v_1(\vec{x}) \cdot \dot{\boldsymbol{\epsilon}}_V(\vec{x},t) + \boldsymbol{R}^a_0(\vec{x}) \cdot \boldsymbol{\epsilon}_A(\vec{x},t) + \boldsymbol{R}^a_1(\vec{x}) \cdot \dot{\boldsymbol{\epsilon}}_A(\vec{x},t) \right] \right)$$

Expanding both sides in $\vec{\epsilon}_V, \vec{\epsilon}_A$, comparing the coefficients and transforming to momentum space we get the continuity equation

$$\mathcal{C}(\vec{k}, k_0) \quad = \quad \left[H_{\text{strong}}, \boldsymbol{A}_0(\vec{k}, k_0) \right] - \vec{k} \cdot \vec{\boldsymbol{A}}(\vec{k}, k_0) + i\, m_q \boldsymbol{P}(\vec{k}, k_0)$$

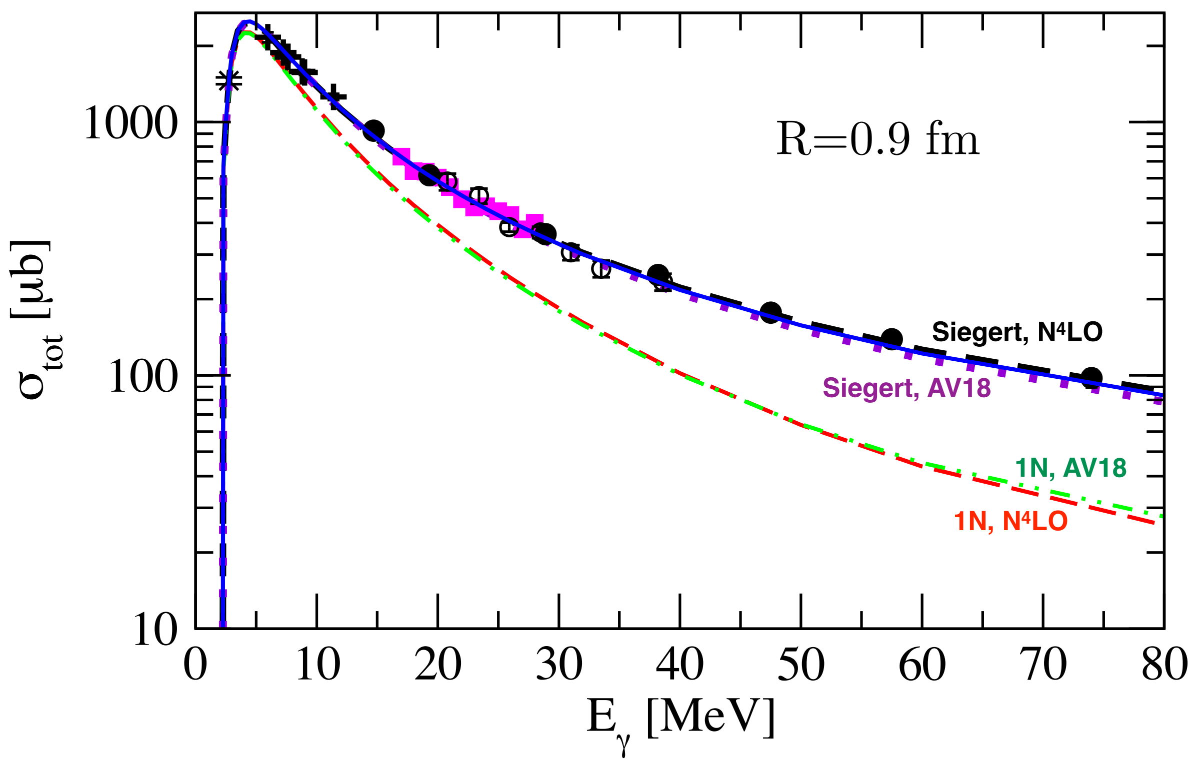
$$\mathcal{C}(\vec{k}, 0) + \underbrace{\left[H_{\text{strong}}, \frac{\partial}{\partial k_0} \mathcal{C}(\vec{k}, k_0) \right]}_{\text{new term}} = 0$$

Siegert theorem + N^{4}LO

Skibinski et al. PRC93 (2016) no. 6, 064002

Generate longitudinal component of NN current by continuity equation

$$\left[H_{\rm strong}, \boldsymbol{\rho}\right] = \vec{k} \cdot \vec{\boldsymbol{J}} \leftarrow$$ regularized longitudinal current (Siegert theorem)

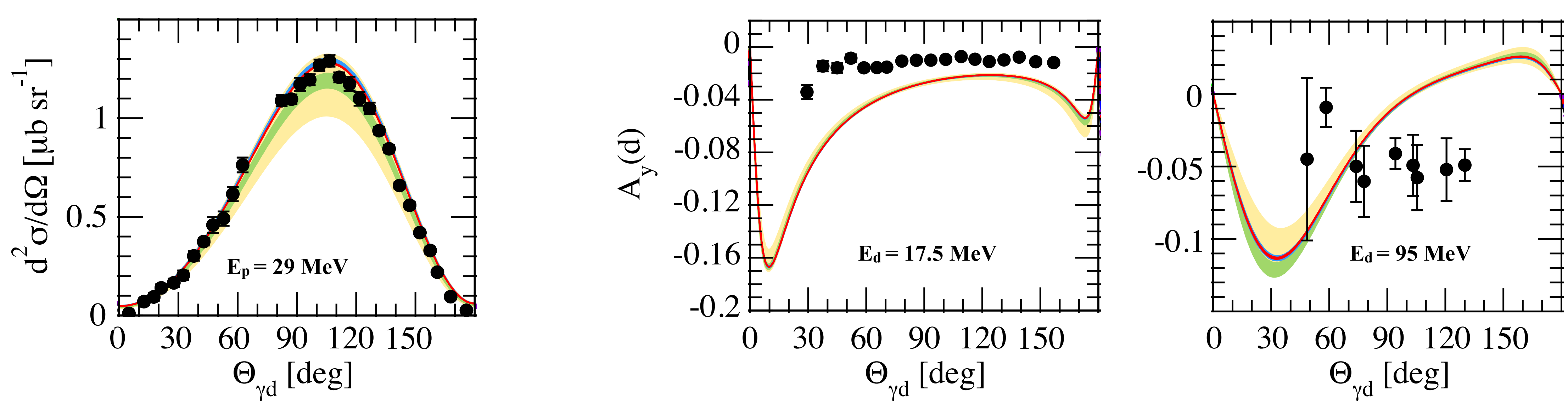


Deuteron photo-disintegration

$$\gamma + d \rightarrow p + n$$

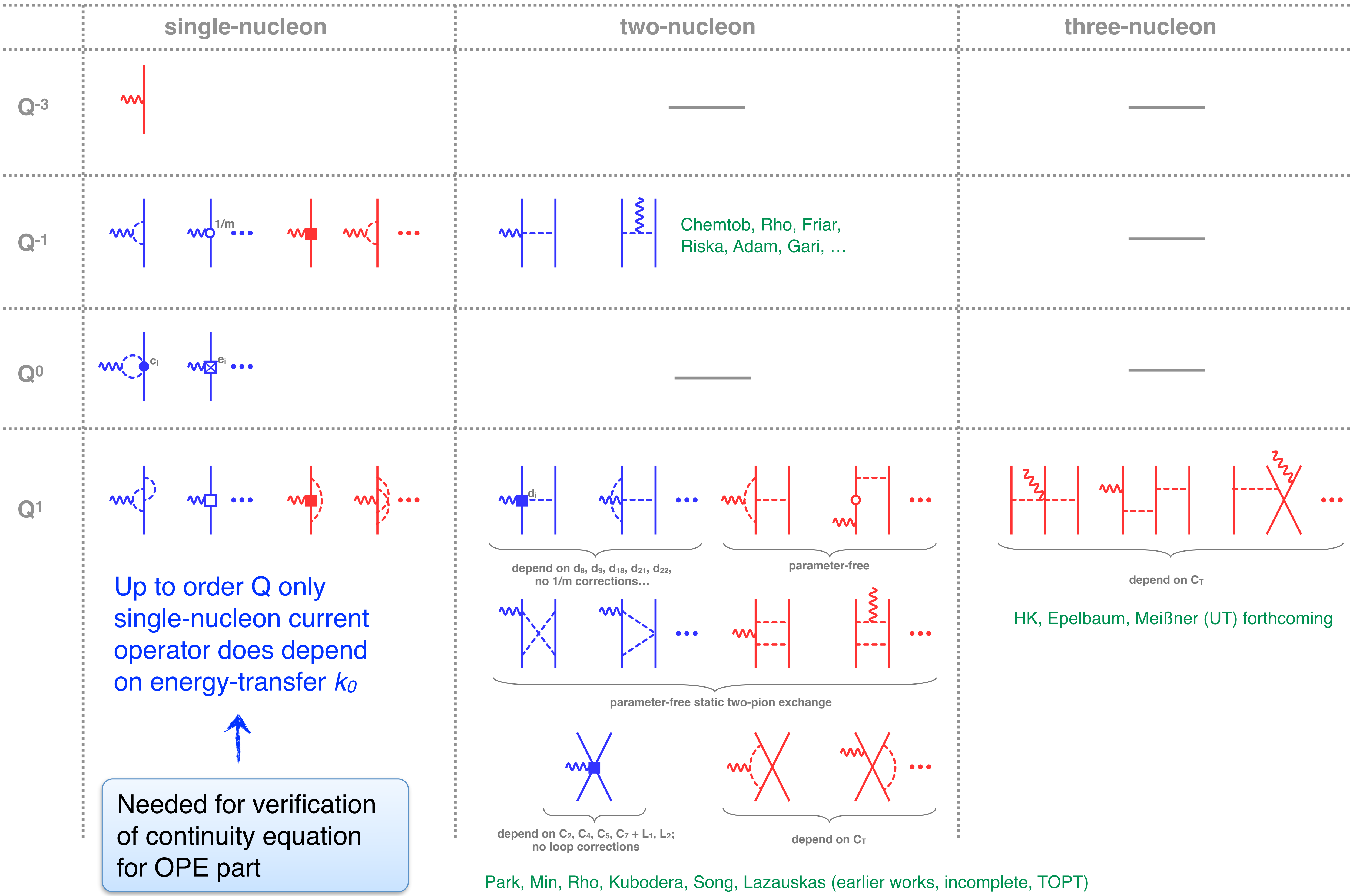
- consistent regularization via cont. eq.
- improvement by 1N+Siegert
- implementation of transverse part & exchange currents work in progress

Nucleon-deuteron radiative capture: $p(n) + d \rightarrow {}^3\mathrm{H}({}^3\mathrm{He}) + \gamma$



Vector currents in chiral EFT

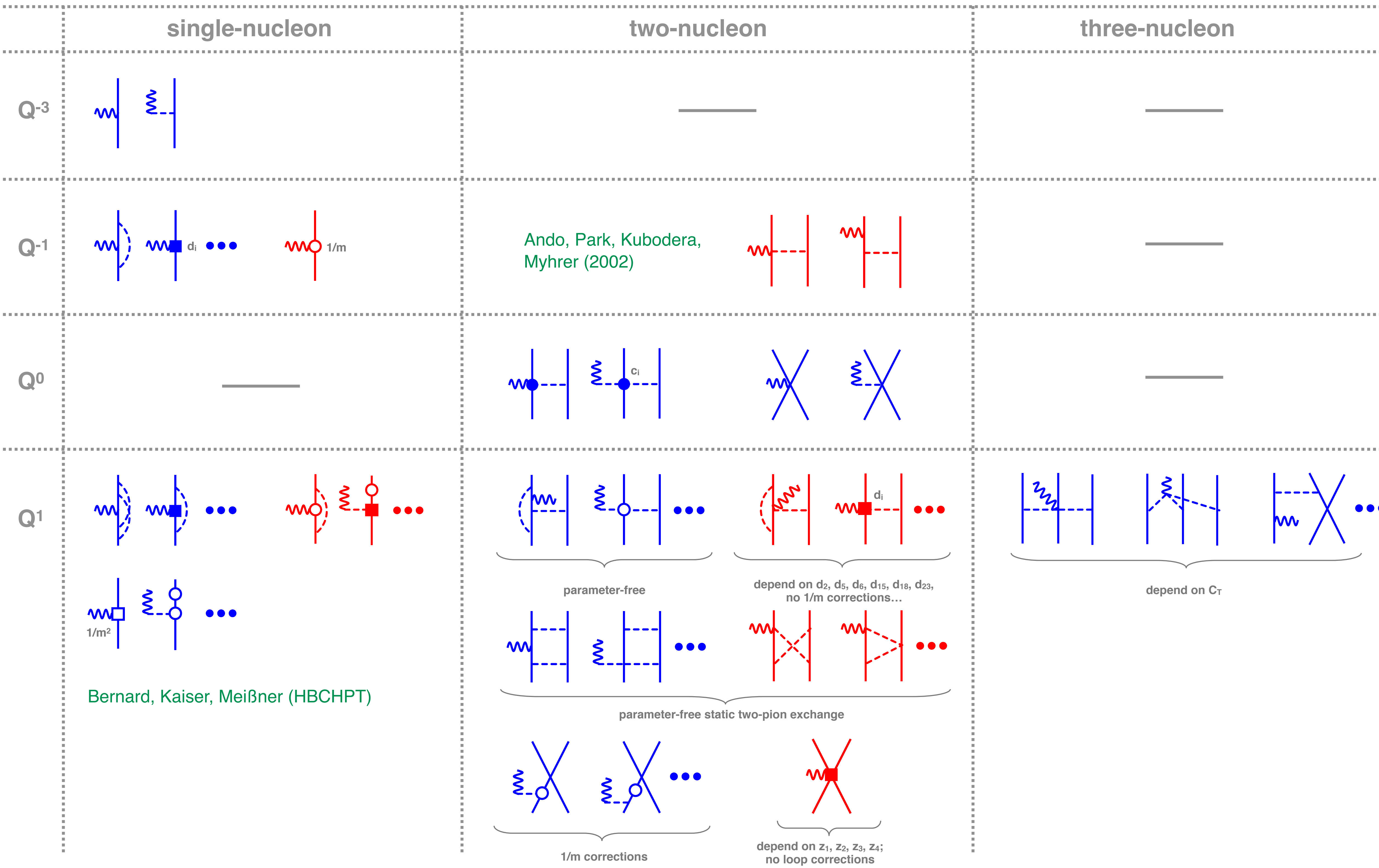
Chiral expansion of the electromagnetic **current** and **charge** operators



Park, Min, Rho, Kubodera, Song, Lazauskas (earlier works, incomplete, TOPT)
Pastore, Schiavilla et al. (TOPT), Kölling, Epelbaum, HK, Meißner (UT)

Axial vector operators in chiral EFT

Chiral expansion of the axial vector **current** and **charge** operators



Park, Min, Rho (earlier works, incomplete, TOPT)
Baroni et al. (TOPT), HK, Epelbaum, Meißner (UT)

Compare with Baroni et al.

Baroni et al. PRC94 (2016) no. 2, 024003; Erratum PRC95 (2017) no. 5, 059902;
PRC93 (2016) no. 1, 015501; Erratum PRC93 (2016) no. 4, 049902

At zero momentum transfer the result of Baroni et al. is

$$\mathbf{j}_{\pm}^{\rm N4LO}(\mathrm{OPE};\mathbf{k}) = \frac{g_A^5\, m_\pi}{256\,\pi\, f_\pi^4}\left[18\,\tau_{2,\pm}\,\mathbf{k} - (\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_\pm\,\boldsymbol{\sigma}_1\times\mathbf{k}\right]\boldsymbol{\sigma}_2\cdot\mathbf{k}\,\frac{1}{\omega_k^2} + (1\rightleftharpoons 2)\ , \tag{5}$$

$$\begin{aligned}\mathbf{j}_{\pm}^{\rm N4LO}(\mathrm{MPE};\mathbf{k}) =& \frac{g_A^3}{32\,\pi f_\pi^4}\,\tau_{2,\pm}\left[W_1(k)\,\boldsymbol{\sigma}_1 + W_2(k)\,\mathbf{k}\,\boldsymbol{\sigma}_1\cdot\mathbf{k} + Z_1(k)\left(2\,\mathbf{k}\,\boldsymbol{\sigma}_2\cdot\mathbf{k}\frac{1}{\omega_k^2} - \boldsymbol{\sigma}_2\right)\right]\\ &+\frac{g_A^5}{32\,\pi f_\pi^4}\,\tau_{1,\pm}\,W_3(k)\,(\boldsymbol{\sigma}_2\times\mathbf{k})\times\mathbf{k} - \frac{g_A^3}{32\,\pi f_\pi^4}\,(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_\pm\,Z_3(k)\,\boldsymbol{\sigma}_1\times\mathbf{k}\\ &\times\boldsymbol{\sigma}_2\cdot\mathbf{k}\,\frac{1}{\omega_k^2} + (1\rightleftharpoons 2)\ ,\end{aligned} \tag{6}$$

$$\begin{aligned}W_1(k) &= \frac{M_\pi}{2}\left(1+g_A^2\left(-9+\frac{4M_\pi^2}{k^2+4M_\pi^2}\right)\right) + \frac{1}{2}\left((1-5g_A^2)k^2 + 4(1-2g_A^2)M_\pi^2\right)A(k),\\ W_2(k) &= \frac{M_\pi}{2k^2(k^2+4M_\pi^2)}\left(\left(1+3g_A^2\right)k^2 + 4\left(1+2g_A^2\right)M_\pi^2\right) - \frac{1}{2k^2}\left((-1+g_A^2)k^2 + 4(1+2g_A^2)M_\pi^2\right)A(k)\\ W_3(k) &= \cancel{-\frac{1}{6M_\pi} - \frac{4}{3}A(k)} = -2A(k),\\ Z_1(k) &= 2M_\pi + 2(k^2+2M_\pi^2)A(k),\\ Z_3(k) &= \frac{M_\pi}{2} + \frac{1}{2}(k^2+4M_\pi^2)A(k).\end{aligned}$$

Baroni et al. PRC94 (2016) no. 2, 024003;
Erratum PRC95 (2017) no. 5, 059902

The current of Baroni et al. does ~~not~~ exist in the chiral limit!

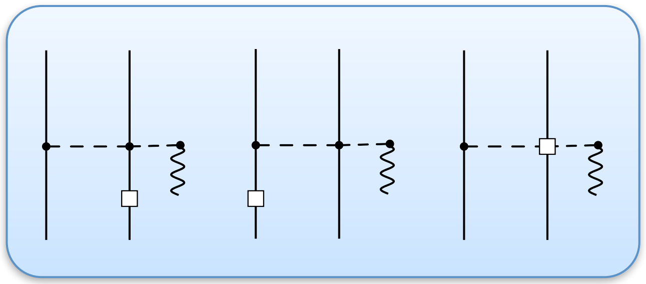
$$\begin{aligned}\vec{j}_a^{\rm \,N4LO}(\mathrm{MPE},\vec{q}_1\,) - \vec{A}_{\rm 2N:2\pi}^{a\,(Q)} - \vec{A}_{\rm 2N:1\pi}^{a\,(Q)} &= -\vec{q}_1\,\frac{g_A^5 A(q_1)(4M_\pi^2+q_1^2)\vec{q}_1\cdot\vec{\sigma}_2\tau_1^a}{32\pi F_\pi^4 q_1^2}\\ &+\ \text{rational function in } \vec{q}_1 + 1\leftrightarrow 2\end{aligned}$$

where $A(q) = \frac{1}{2q}\arctan\frac{q}{2M_\pi}$

Contributions of the difference to GT: *Baroni et al. ArXiv:180610245*

Call for Consistent Regularization

Violation of chiral symmetry due to different regularizations: Dim. reg. vs cutoff reg.



1/m - corrections to pion-pole OPE current proportional to g_A

$$\vec{A}^{a,(Q:\,g_A)}_{\rm 2N:\,1\pi,1/m} = i\,[\tau_1\times\tau_2]^a \frac{g_A}{8F_\pi^2 m}\frac{\vec{k}}{(k^2+M_\pi^2)(q_1^2+M_\pi^2)}\Big(\vec{k}_2\cdot(\vec{k}+\vec{q}_1)-\vec{k}_1\cdot\vec{q}_1+i\,\vec{k}\cdot(\vec{q}_1\times\vec{\sigma}_2)\Big)+1\leftrightarrow 2$$

Naive local cut-off regularization of the current and potential

$$\vec{A}^{a,(Q:\,g_A,\Lambda)}_{\rm 2N:\,1\pi,1/m} = \vec{A}^{a,(Q:\,g_A)}_{\rm 2N:\,1\pi,1/m}\exp\left(-\frac{q_1^2+M_\pi^2}{\Lambda^2}\right) \quad \& \quad V^{Q^0,\Lambda}_{1\pi} = -\frac{g_A^2}{4F_\pi^2}\tau_1\cdot\tau_2\frac{\vec{q}_1\cdot\vec{\sigma}_1\vec{q}_1\cdot\vec{\sigma}_2}{q_1^2+M_\pi^2}\exp\left(-\frac{q_1^2+M_\pi^2}{\Lambda^2}\right)$$

First iteration with OPE NN potential

$$\vec{A}^{a,(Q:\,g_A,\Lambda)}_{\rm 2N:\,1\pi,1/m}\frac{1}{E-H_0+i\epsilon}V^{O^0,\Lambda}_{1\pi}+V^{O^0,\Lambda}_{1\pi}\frac{1}{E-H_0+i\epsilon}\vec{A}^{a,(Q:\,g_A,\Lambda)}_{\rm 2N:\,1\pi,1/m} = \Lambda\frac{g_A^3}{32\sqrt{2}\pi^{3/2}F_\pi^4}\left([\tau_1]^a-[\tau_2]^a\right)\frac{\vec{k}}{k^2+M_\pi^2}\vec{q}_1\cdot\vec{\sigma}_1 + 1\leftrightarrow 2 + \dots$$

No such counter term in chiral Lagrangian

To be compensated by two-pion-exchange current $\vec{A}^{a\,(Q)}_{\rm 2N:\,2\pi}$ if calculated via cutoff regularization

In dim. reg. $\vec{A}^{a\,(Q)}_{\rm 2N:\,2\pi}$ is finite

Higher Derivative Regularization

Based on ideas: Slavnov, NPB31 (1971) 301;
Djukanovic et al. PRD72 (2005) 045002; Long and Mei PRC93 (2016) 044003

- Change leading order pion - Lagrangian (modify free part)

$$S_\pi^{(2)} = \int d^4x \frac{1}{2}\vec{\pi}(x)\left(-\partial^2 - M_\pi^2\right)\vec{\pi}(x) \to S_{\pi,\Lambda}^{(2)} = \int d^4x \frac{1}{2}\vec{\pi}(x)\left(-\partial^2 - M_\pi^2\right)\exp\left(\frac{\partial^2 + M_\pi^2}{\Lambda^2}\right)\vec{\pi}(x)$$

$$\frac{1}{q^2 + M_\pi^2} \to \frac{\exp\left(-\frac{q^2+M_\pi^2}{\Lambda^2}\right)}{q^2 + M_\pi^2}$$

$\mathcal{L}_{\pi,\Lambda}^{(2)}$ has to be invariant under $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \times \mathrm{U}(1)_\mathrm{V}$

- Every derivative should be covariant one
- Lagrangian $\mathcal{L}_{\pi,\Lambda}^{(2)}$ should be formulated in terms of $U(\vec{\pi}(x)) \in \mathrm{SU}(2)$

Gasser, Leutwyler ´84, ´85; Bernard, Kaiser, Meißner ´95

Building blocks $\chi = 2B(s + ip)$

$$\nabla_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu)U$$

Higher Derivative Lagrangian

- To construct a parity-conserving regulator it is convenient to work with building-blocks

$$u_\mu = i\, u^\dagger \nabla_\mu U u^\dagger, \quad D_\mu = \partial_\mu + \Gamma_\mu, \quad \Gamma_\mu = \frac{1}{2}\left[u^\dagger, \partial_\mu u\right] - \frac{i}{2} u^\dagger r_\mu u - \frac{i}{2} u\, l_\mu u^\dagger$$

$$\chi_\pm = u^\dagger \chi\, u^\dagger \pm u \chi^\dagger u, \quad \chi = 2B(s + i\, p), \quad u = \sqrt{U}, \quad \mathrm{ad}_A B = [A, B]$$

Possible ansatz for higher derivative pion Lagrangian

$$\mathcal{L}^{(2)}_{\pi,\Lambda} = \mathcal{L}^{(2)}_{\pi} + \frac{F^2}{4} \mathrm{Tr}\left[\mathrm{EOM} \frac{1 - \exp\left(\frac{\mathrm{ad}_{D_\mu} \mathrm{ad}_{D^\mu} + \frac{1}{2}\chi_+}{\Lambda^2}\right)}{\mathrm{ad}_{D_\mu} \mathrm{ad}_{D^\mu} + \frac{1}{2}\chi_+} \mathrm{EOM} \right]$$

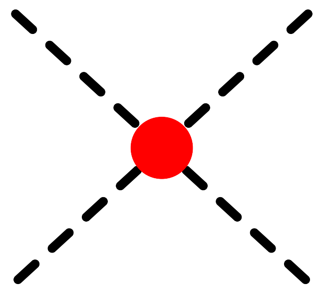
$$\mathcal{L}^{(2)}_{\pi} = \frac{F^2}{4} \mathrm{Tr}\left[u_\mu u^\mu + \chi_+\right] \qquad \mathrm{EOM} = -\left[D_\mu, u^\mu\right] + \frac{i}{2}\chi_- - \frac{i}{4}\mathrm{Tr}\left(\chi_-\right)$$

Expand $\mathcal{L}^{(2)}_{\pi,\Lambda}$ in D_0 → Lorentz-invariance only perturbatively

Use dimensional regularization on top of higher derivative one

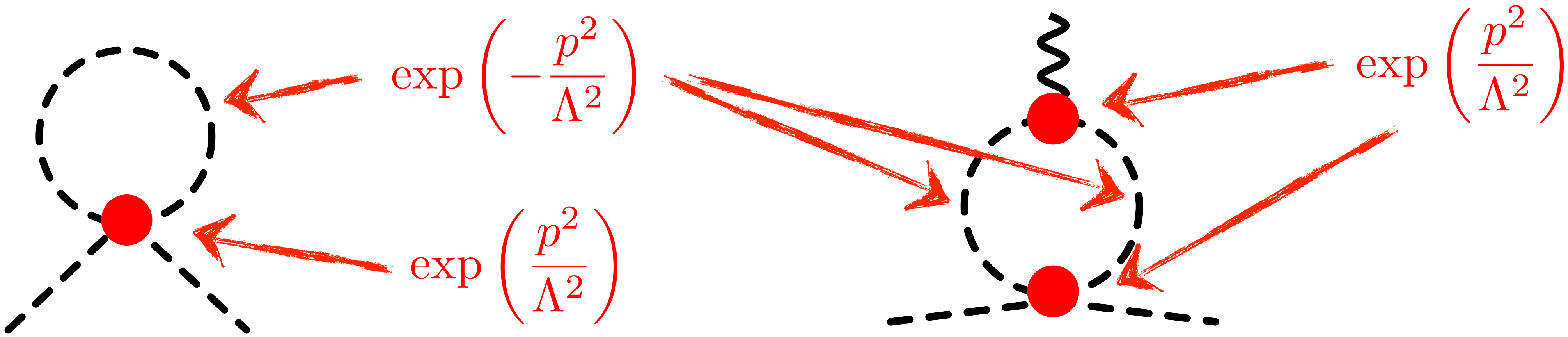
→ regularization of remaining divergencies in pion sector

Modified Vertices



- Enhanced by $\exp\left(\frac{p^2}{\Lambda^2}\right)$
- Every propagator is suppressed by $\exp\left(-\frac{p^2}{\Lambda^2}\right)$

Pionic sector becomes unregularized



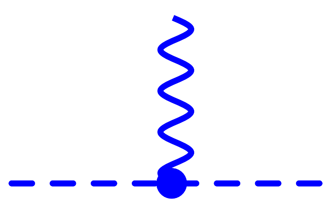
- Use dimensional on top of higher derivative regularization
- Dimensional regularization will not affect effective potential and Schrödinger or LS equations but will regularize pionic sector

Regularization of Vector Current

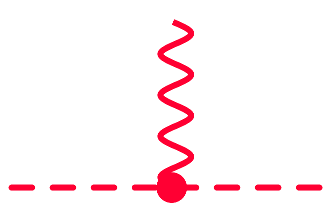
- Modify pion-propagators in a vector current

 $$= \frac{1}{q^2+M^2} \to \frac{\exp\left(-\frac{q^2+M^2}{\Lambda^2}\right)}{q^2+M^2} =$$

- Modify two-pion-photon vertex

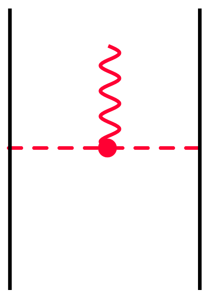
 $$= e\,\epsilon_\mu(q_2^\mu - q_1^\mu)\epsilon_{3,a_1,a_2}$$

Modified two-pion-photon vertex leads to exponential increase in momenta

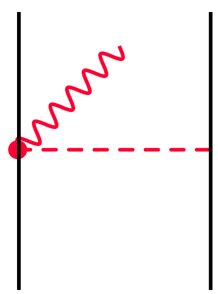
 $$= e\,\epsilon_\mu(q_2^\mu - q_1^\mu)\epsilon_{3,a_1,a_2} \times \frac{1}{q_1^2-q_2^2}\left[(q_1^2+M^2)\exp\left(\frac{q_1^2+M^2}{\Lambda^2}\right) - (q_2^2+M^2)\exp\left(\frac{q_2^2+M^2}{\Lambda^2}\right)\right]$$

Regularization of Vector Current

Regularization of pion-exchange vector current



$$= \frac{i\,e\,g_A^2}{4F^2}\vec{q}_1\cdot\vec{\sigma}_1\vec{q}_2\cdot\vec{\sigma}_2[\tau_1\times\tau_2]_3\frac{\vec{\epsilon}\cdot(\vec{q}_2-\vec{q}_1)}{q_1^2-q_2^2}\left[\frac{\exp\left(-\frac{q_2^2+M^2}{\Lambda^2}\right)}{q_2^2+M^2}-\frac{\exp\left(-\frac{q_1^2+M^2}{\Lambda^2}\right)}{q_1^2+M^2}\right]$$



$$= -\frac{i\,e\,g_A^2}{4F^2}\vec{\epsilon}\cdot\vec{\sigma}_1\vec{q}_2\cdot\vec{\sigma}_2[\tau_1\times\tau_2]_3\frac{\exp\left(-\frac{q_1^2+M^2}{\Lambda^2}\right)}{q_1^2+M^2}+(1\leftrightarrow 2)$$

Riska prescription: longitudinal part of the current can be derived from continuity equation

Riska, Prog. Part. Nucl. Phys. 11 (1984) 199

$$\left[H_{\mathrm{strong}},\boldsymbol{\rho}\right]=\vec{k}\cdot\vec{\boldsymbol{J}}$$

Higher orders $\longrightarrow$ work in progress

Application to Electromagnetic Charge

Electromagnetic charge operators in chiral EFT

- 1N charge operator is parametrized in terms of em form factors

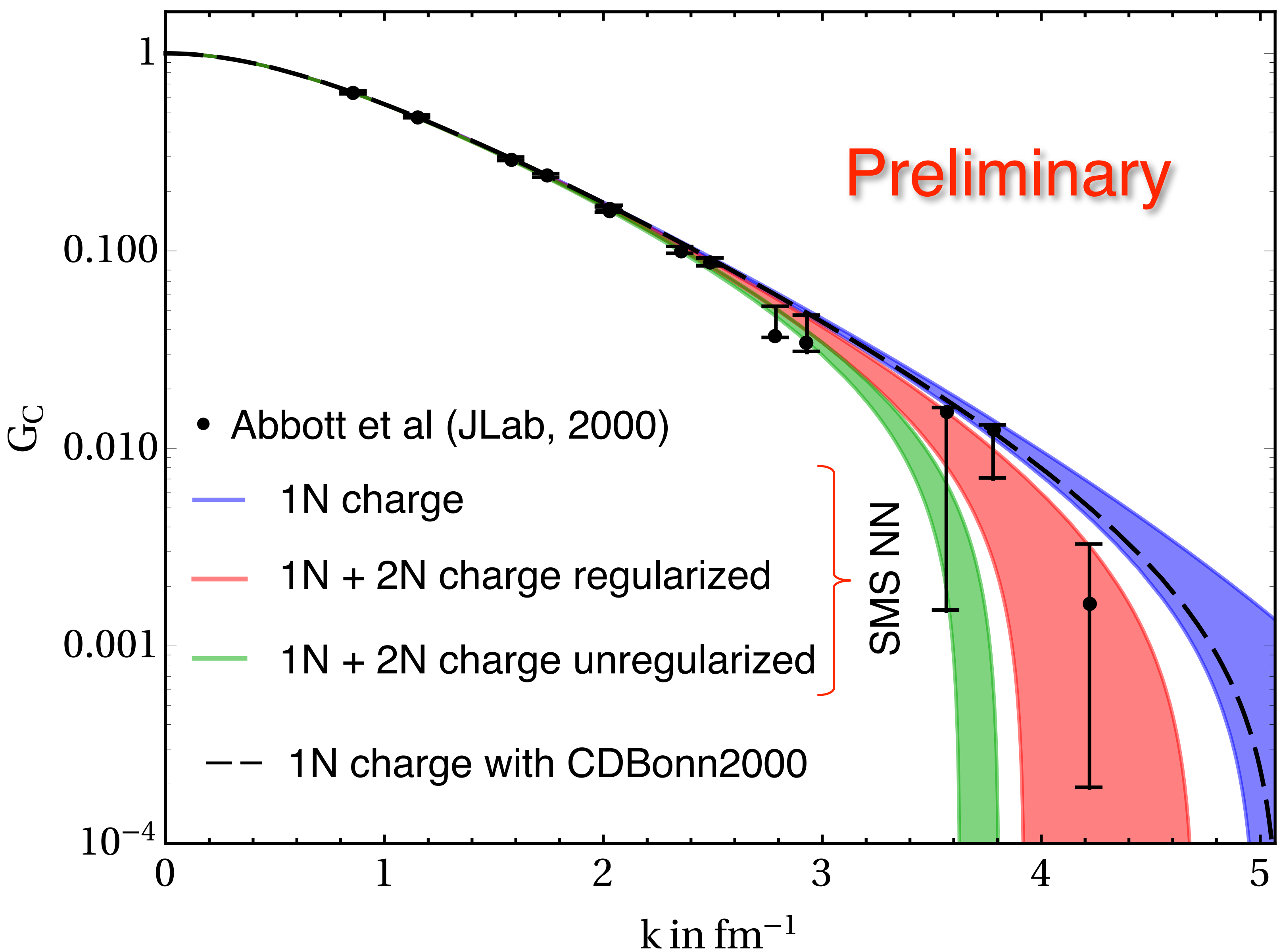
$$
\begin{aligned}
V^0_{\rm 1N:static} &= eG_E(Q^2),\\
V^0_{\rm 1N:1/m} &= \frac{i\,e}{2m^2}\boldsymbol{k}\cdot(\boldsymbol{k}_1\times\boldsymbol{\sigma})G_M(Q^2),\\
V^0_{\rm 1N:1/m^2} &= -\frac{e}{8m^2}\left[Q^2+2\,i\,\boldsymbol{k}\cdot(\boldsymbol{k}_1\times\boldsymbol{\sigma})\right]G_E(Q^2)
\end{aligned}
$$

- Static 2N charge operator does not contribute to deuteron form factors

$$
\begin{aligned}
V^{0,(Q)}_{\rm 2N:1\pi,1/m} &= \frac{eg_A^2}{16F_\pi^2 m_N}\frac{1}{q_2^2+M_\pi^2}\Big\{(1-2\bar\beta_9)\\
&\times\big([\boldsymbol{\tau}]_2^3+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\big)\,\boldsymbol{\sigma}_1\cdot\boldsymbol{k}\boldsymbol{\sigma}_2\cdot\boldsymbol{q}_2-i(1+2\bar\beta_9)\,[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^3\\
&\times\Big[\boldsymbol{\sigma}_1\cdot\boldsymbol{k}_1\boldsymbol{\sigma}_2\cdot\boldsymbol{q}_2-\boldsymbol{\sigma}_2\cdot\boldsymbol{k}_2\boldsymbol{\sigma}_1\cdot\boldsymbol{q}_2-2\,\frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{q}_1}{q_1^2+M_\pi^2}\boldsymbol{\sigma}_2\cdot\boldsymbol{q}_2\\
&\times\boldsymbol{q}_1\cdot\boldsymbol{k}_1\Big]\Big\}+\frac{eg_A^2}{16F_\pi^2 m_N}\frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{q}_2\,\boldsymbol{\sigma}_2\cdot\boldsymbol{q}_2}{(q_2^2+M_\pi^2)^2}\Big[(2\bar\beta_8-1)\\
&\times([\boldsymbol{\tau}_2]^3+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\boldsymbol{q}_2\cdot\boldsymbol{k}+i\,[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^3\,\big((2\bar\beta_8-1)\,\boldsymbol{q}_2\cdot\boldsymbol{k}_1\\
&-\,(2\bar\beta_8+1)\,\boldsymbol{q}_2\cdot\boldsymbol{k}_2\big)\Big]+1\leftrightarrow 2. \qquad (59)
\end{aligned}
$$

Apply higher derivative regularization to relativistic correction of the charge

Charge Form Factor of Deuteron



Semilocal momentum space (SMS) regularized NN force

Reinert, HK, Epelbaum '17

1N electromagnetic form factors

Belushkin, Hammer, Meißner '07

Cutoff variation 400 - 550 MeV

- Excellent description of the data for regularized charge even at higher momentum transfer k

Summary on Currents

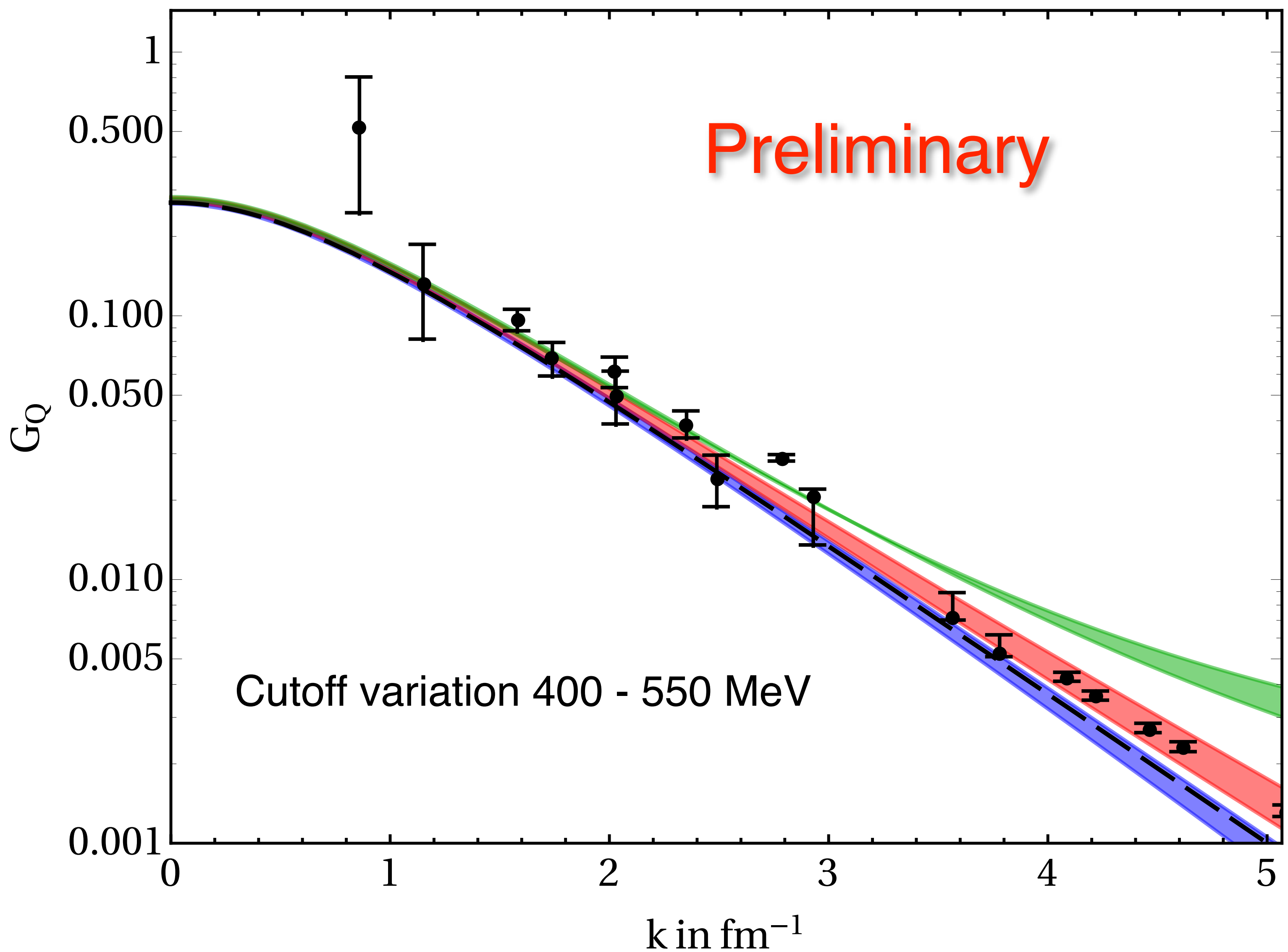
- Electroweak currents are analyzed up to order Q
- Modified continuity equation
- Differences in long range part between our results and Baroni et al.

- Violation of chiral symmetry at one loop level if different regularizations for currents and potentials are used
- Higher derivative regularization respects symmetries
- Excelent description of data for charge form factor at higher virtuality

Outlook

- Regularization and PWD of the currents
- Electroweak currents up to order Q^2

Quadrupole Form Factor of Deuteron



- Abbott et al (JLab, 2000)
- 1N charge
- 1N + 2N charge regularized
- 1N + 2N charge unregularized
- 1N charge with CDBonn2000

SMS NN

Semilocal momentum space (SMS) regularized NN force
Reinert, HK, Epelbaum '17

1N electromagnetic form factors
Belushkin, Hammer, Meißner '07

- Excellent description of the data for regularized charge even at higher momentum transfer k

order	single-nucleon	two-nucleon	three-nucleon
LO (Q^{-3})	$\vec{A}^a_{1\mathrm{N:\,static}}$,	—	—
NLO (Q^{-1})	$\vec{A}^a_{1\mathrm{N:\,static}}$,	—	—
$\mathrm{N^2LO}$ (Q^0)	—	$\vec{A}^a_{2\mathrm{N:}\,1\pi}$, ✓ $+\ \vec{A}^a_{2\mathrm{N:\,cont}}$, ✓	—
$\mathrm{N^3LO}$ (Q)	$\vec{A}^a_{1\mathrm{N:\,static}}$, $+\ \vec{A}^a_{1\mathrm{N:}\,1/m,\mathrm{UT}'}$, $+\ \vec{A}^a_{1\mathrm{N:}\,1/\mathrm{m}^2}$,	$\vec{A}^a_{2\mathrm{N:}\,1\pi}$, $+\ \vec{A}^a_{2\mathrm{N:}\,1\pi,\mathrm{UT}'}$, ✗ $+\ \vec{A}^a_{2\mathrm{N:}\,1\pi,\,1/m}$, ✗ $+\ \vec{A}^a_{2\mathrm{N:}\,2\pi}$, $+\ \vec{A}^a_{2\mathrm{N:\,cont,\,UT}'}$, ✗ $+\ \vec{A}^a_{2\mathrm{N:\,cont,}\,1/\mathrm{m}}$, ✗	$\vec{A}^a_{3\mathrm{N:}\,\pi}$, $+\ \vec{A}^a_{3\mathrm{N:\,cont}}$, ✗ Baroni et al. considered only irr. diagrams of 3N current

✗ terms not discussed by Baroni et al. ´16   ✓ terms on which we agree with Baroni et al. ´16

order	single-nucleon	two-nucleon	three-nucleon
LO (Q^{-3})	—	—	—
NLO (Q^{-1})	$A^{0,a}_{1\mathrm{N:\,UT}'}$, $+\ A^{0,a}_{1\mathrm{N:}\,1/m}$,	$A^{0,a}_{2\mathrm{N:}\,1\pi}$, ✓	—
$\mathrm{N^2LO}$ (Q^0)	—	—	—
$\mathrm{N^3LO}$ (Q)	$A^{0,a}_{1\mathrm{N:\,static,\,UT}'}$, $+\ A^{0,a}_{1\mathrm{N:}\,1/m}$,	$A^{0,a}_{2\mathrm{N:}\,1\pi}$, $+\ A^{0,a}_{2\mathrm{N:}\,2\pi}$, ✓ $+\ A^{0,a}_{2\mathrm{N:\,cont}}$, ✓	—

Pseudoscalar current

order	single-nucleon	two-nucleon	three-nucleon
LO (Q^{-4})	$P^a_{\rm 1N:\, static}$,	—	—
NLO (Q^{-2})	$P^a_{\rm 1N:\, static}$,	—	—
N^{2}LO (Q^{-1})	—	$P^a_{\rm 2N:\, 1\pi}$, $+ P^a_{\rm 2N:\, cont}$,	—
N^{3}LO (Q^{0})	$P^a_{\rm 1N:\, static}$, $+ P^a_{{\rm 1N:}\, 1/m, {\rm UT}'}$, $+ P^a_{\rm 1N:\, 1/m^2}$,	$P^a_{\rm 2N:\, 1\pi}$, $+ P^a_{{\rm 2N:}\, 1\pi, {\rm UT}'}$, $+ P^a_{{\rm 2N:}\, 1\pi,\, 1/m}$, $+ P^a_{\rm 2N:\, 2\pi}$, $+ P^a_{\rm 2N:\, cont,\, UT'}$, $+ P^a_{\rm 2N:\, cont,\, 1/m}$,	$P^a_{{\rm 3N:}\, \pi}$, $+ P^a_{\rm 3N:\, cont}$,

Continuity equations are verified for all currents

Unitary ambiguities

34 different unitary transformations are possible at the order Q

$$U_i(a) = \exp\left(S_i^{\rm ax} - h.c.\right)$$

$$S_1^{ax} = \alpha_1^{ax} \eta A_{2,0}^{(0)} \eta H_{2,1}^{(1)} \lambda^1 \frac{1}{E_\pi^3} H_{2,1}^{(1)} \eta,$$

$$S_2^{ax} = \alpha_2^{ax} \eta H_{2,1}^{(1)} \lambda^1 \frac{1}{E_\pi^2} A_{2,0}^{(0)} \lambda^1 \frac{1}{E_\pi} H_{2,1}^{(1)} \eta$$

$$\ldots$$

Vertices without axial source are denoted by $H_{n,p}^{(\kappa)}$

Vertices with one axial source are denoted by $A_{n,p}^{(\kappa)}$

n — number of nucleons

p — number of pions

a — number of axial sources

$\kappa = d + \frac{3}{2} n + p + a - 4 \longleftarrow$ inverse mass dimension

Large unitary ambiguity is related to appearance of the axial-vector-one-pion interaction $A_{0,1}^{(-1)}$ (30 out of 34 transformations depend on it)

Reasonable constraints come from

- Perturbative renormalizability of the current

$$l_i = l_i^r(\mu) + \gamma_i \lambda =: \frac{1}{16\pi^2} \bar{l}_i + \gamma_i \lambda + \frac{\gamma_i}{16\pi^2} \ln\left(\frac{M_\pi}{\mu}\right),$$

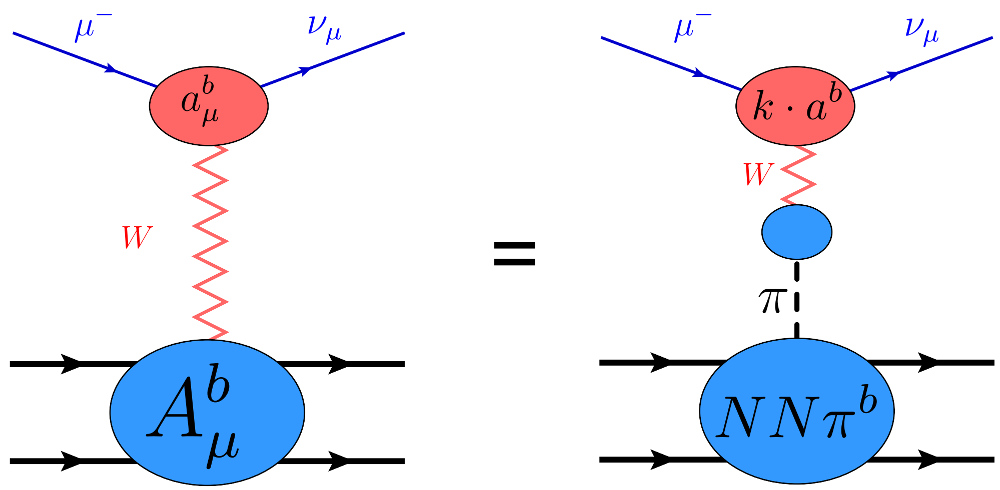
$$d_i = d_i^r(\mu) + \frac{\beta_i}{F^2} \lambda =: \bar{d}_i + \frac{\beta_i}{F^2} \lambda + \frac{\beta_i}{16\pi^2 F^2} \ln\left(\frac{M_\pi}{\mu}\right)$$

$$\gamma_3 = -\frac{1}{2},$$
$$\gamma_4 = 2,$$
$$\beta_2 = -2\beta_5 = \frac{1}{2}\beta_6 = -\frac{1}{12}(1 + 5g_A^2),$$
$$\beta_{15} = \beta_{18} = \beta_{22} = \beta_{23} = 0,$$
$$\beta_{16} = \frac{1}{2} g_A + g_A^3.$$

After renormalizing LECs l_i from $\mathcal{L}_\pi^{(4)}$ and d_i from $\mathcal{L}_{\pi N}^{(3)}$ and using well known β- and γ-functions (*Gasser et al. Eur. Phys. J. C26 (2002), 13*) we require the current to be finite

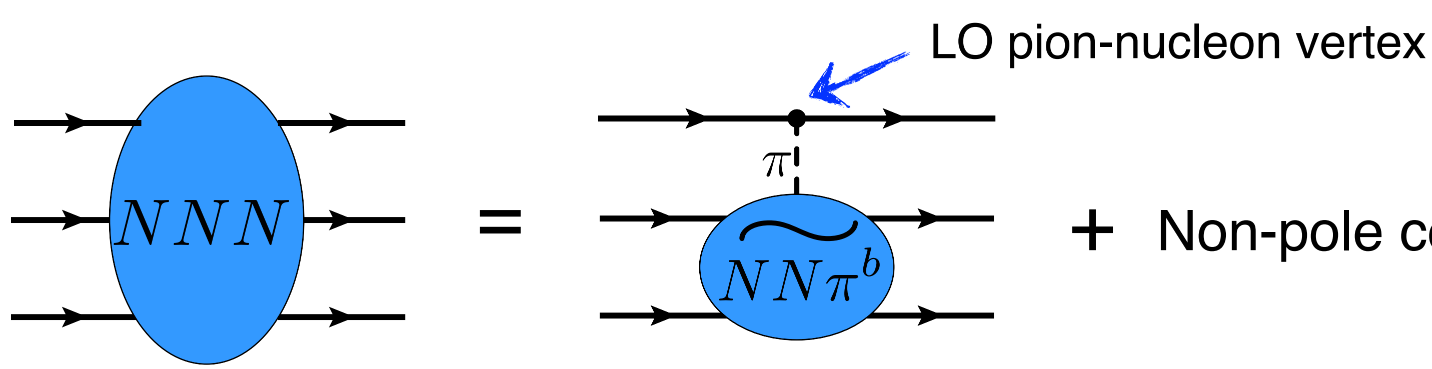
Matching to nuclear forces

Dominance of the pion production operator at the pion-pole (axial-vector current)



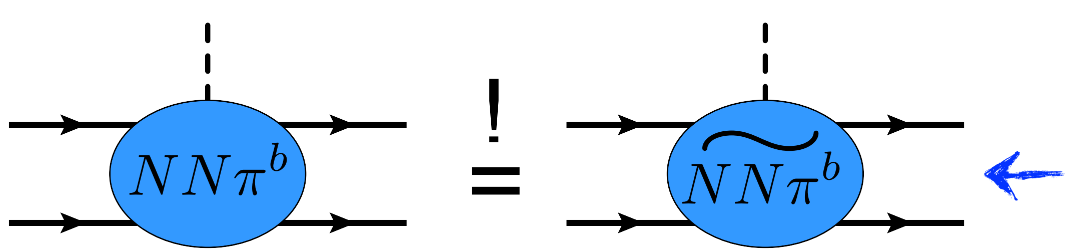
+ Non-pole contributions

Dominance of the pion production operator at the pion-pole (three-nucleon force)



+ Non-pole contributions

Consistent regularization of nuclear forces and currents calls for matching requirement between pion-production operators in different processes



Matching requirement is fulfilled only for particular choice of unitary phases

After renormalizability and matching requirement there are no further unitary ambiguities!

Uncertainty Estimate

Epelbaum, HK, Meißner '15

- Uncertainties in the experimental data
- Uncertainties in the estimation of πN LECs
- Uncertainties in the determination of contact interaction LECs
- Uncertainties of the fits due to the choice of E_{max}
- Systematic uncertainty due to truncation of the chiral expansion at a given order

Estimate the uncertainty via expected size of higher-order corrections

For a N^{4}LO prediction of an observable $X^{\mathrm{N^4LO}}$ we get an uncertainty

$$\begin{aligned}\Delta X^{\mathrm{N^4LO}}(p) \;=\; \max\Big(& Q \times |X^{\mathrm{N^3LO}}(p) - X^{\mathrm{N^4LO}}(p)|, Q^2 \times |X^{\mathrm{N^2LO}}(p) - X^{\mathrm{N^3LO}}(p)|, \\ & Q^3 \times |X^{\mathrm{NLO}}(p) - X^{\mathrm{N^2LO}}(p)|, Q^4 \times |X^{\mathrm{LO}}(p) - X^{\mathrm{NLO}}(p)|, Q^6 \times |X^{\mathrm{LO}}(p)|\Big)\end{aligned}$$

with chiral expansion parameter $Q = \max\left(\frac{p}{\Lambda_b}, \frac{M_\pi}{\Lambda_b}\right)$

For σ_{tot} errors → 68% degree-of-belief intervals(Bayesian analysis): *Furnstahl et al. '15*

Pion-Nucleon Scattering

- Effective chiral Lagrangian:

low-energy constants

$$\mathcal{L}_\pi = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \dots$$

$$\mathcal{L}_{\pi N} = \underbrace{\bar{N}\left(i\gamma^\mu D_\mu[\pi] - m + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu[\pi]\right)N}_{\mathcal{L}_{\pi N}^{(1)}} + \underbrace{\sum_i \boldsymbol{c}_i \bar{N}\hat{O}_i^{(2)}[\pi]N}_{\mathcal{L}_{\pi N}^{(2)}} + \underbrace{\sum_i \boldsymbol{d}_i \bar{N}\hat{O}_i^{(3)}[\pi]N}_{\mathcal{L}_{\pi N}^{(3)}} + \underbrace{\sum_i \boldsymbol{e}_i \bar{N}\hat{O}_i^{(4)}[\pi]N}_{\mathcal{L}_{\pi N}^{(4)}} + \dots$$

- Pion-nucleon scattering is calculated up to Q^4 in heavy-baryon ChPT

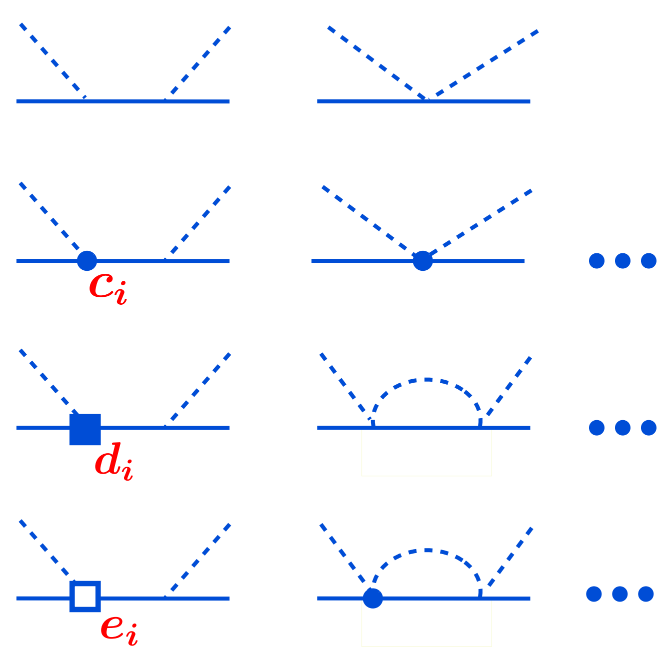
Fettes, Meißner '00; HK, Gasparyan, Epelbaum '12

Order Q:

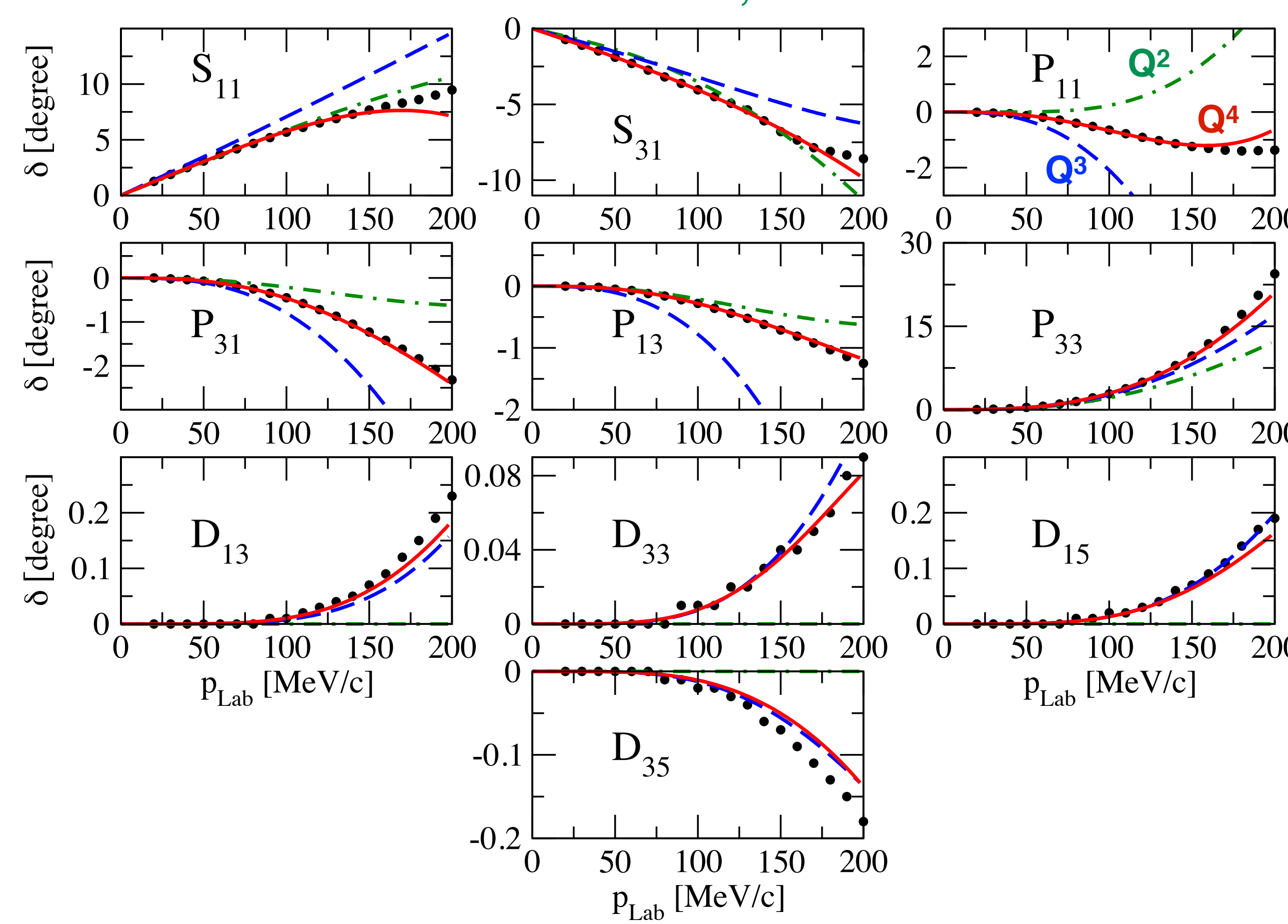
Order Q^2:

Order Q^3:

Order Q^4:

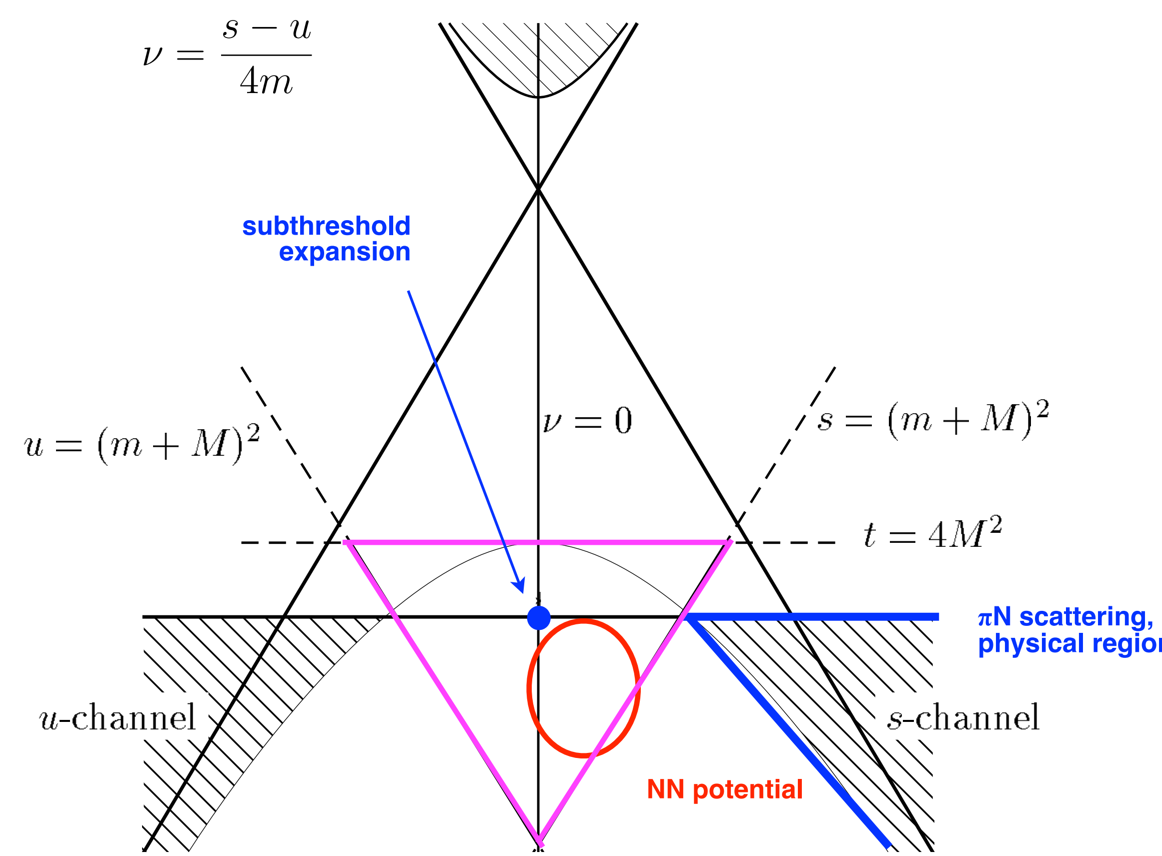


Karlsruhe-Helsinki PWA, Koch et al. '86



Dispersive analysis of πN scattering

- Roy-Steiner equations for πN scattering
  Hoferichter et al., Phys. Rept. 625 (16) 1



Partial Wave Decomposition of
Hyperbolic dispersion relations
πN → πN & ππ → $\bar{\text{N}}$N channels

Input:
S- and P-waves above $s_\mathrm{m} = (1.38\,\mathrm{GeV})^2$
Higher partial waves for all s
Inelasticities for $s < s_m$ and scattering lengths

Output:
S- and P-waves with error bands, σ-term,
Subthreshold coefficients $\bar{X} = \sum_{m,n} x_{mn}\, \nu^{2m+k} t^n, \qquad X = \{A^\pm,\, B^\pm\}$

- c_i, d_i, e_i are fixed from subthreshold coefficients (within Mandelstam triangle where one expects best convergence of chiral expansion)

- Subthreshold point is closer to kinematical region of NN force than the physical region of πN scattering

NN Data Used in the Fits

Reinert, HK, Epelbaum '17

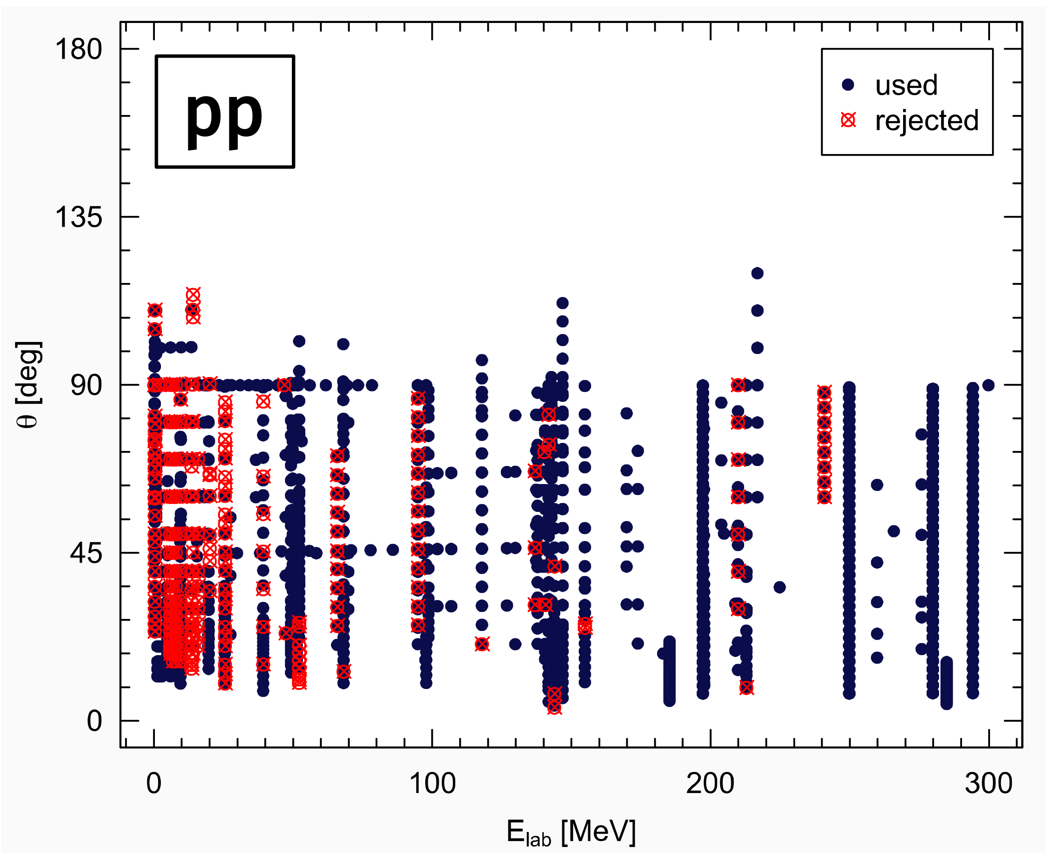
- From 1950 on around 3000 proton-proton + 5000 neutron-proton scattering data below 350 MeV have been measured

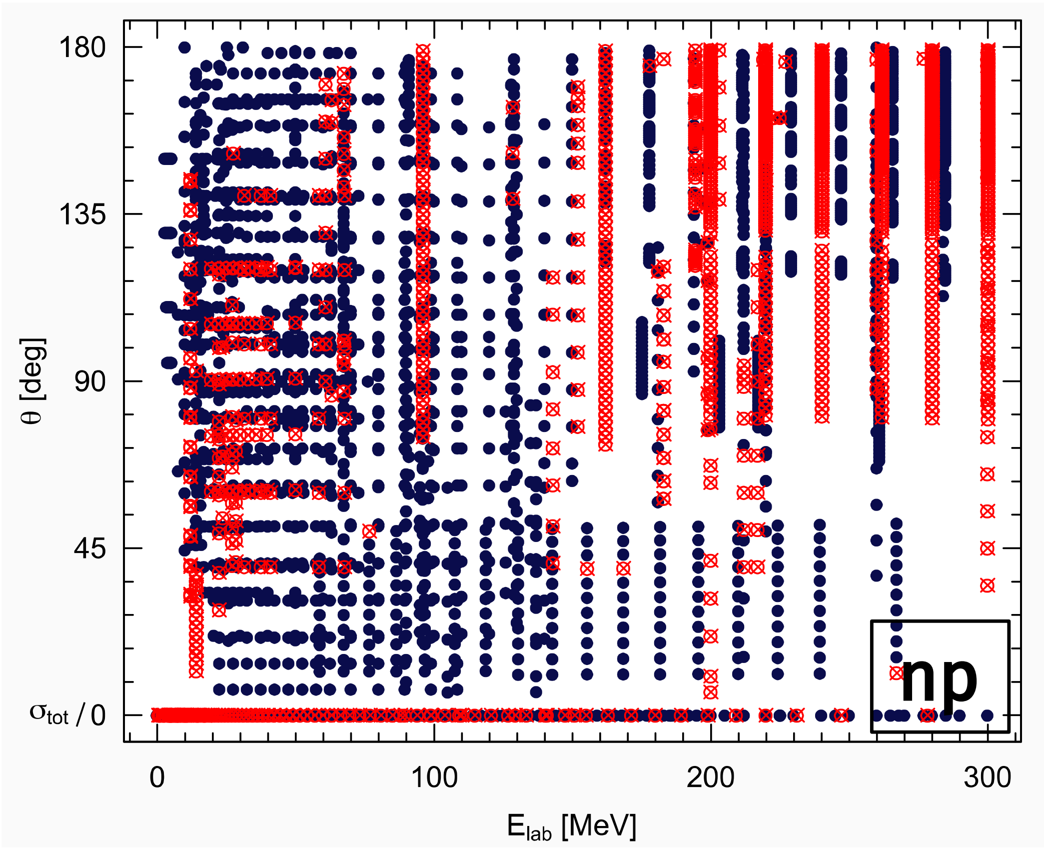
- Not all of these data are compatible. Rejections are required to get a reasonable fit

- Granada 2013 base used: *Navarro Perez et al. '13* rejection by 3σ-criterion

  ⇨ 31% of np + 11% of pp data have been rejected

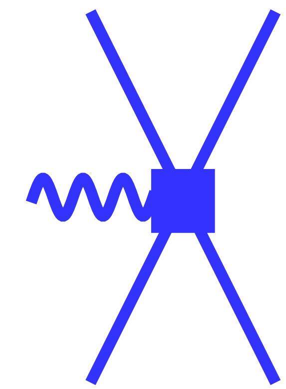
Resulting data base consists of 2697 np + 2158 pp data for E_{lab}=0-300 MeV





Call for Consistent Regularization

$$
\begin{aligned}
\boldsymbol{V}^{(Q)}_{\text{cont: tree}} = {} & e\,\frac{i}{16}\,[\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2]^3 \Big[(C_2 + 3C_4 + C_7)\ \boldsymbol{q}_1 \\
& - (-C_2 + C_4 + C_7)\ (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\ \boldsymbol{q}_1 \\
& + C_7\ (\boldsymbol{\sigma}_2 \cdot \boldsymbol{q}_1\, \boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{q}_1\, \boldsymbol{\sigma}_2) \Big] \\
& - e\,\frac{C_5\, i}{16}\,[\boldsymbol{\tau}_1]^3\ [(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \times \boldsymbol{q}_1] \\
& + ieL_1\,[\boldsymbol{\tau}_1]^3\ [(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \times \boldsymbol{k}] \\
& + ieL_2\ [(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \times \boldsymbol{q}_1]\ .
\end{aligned}
$$



- Short-range component of a vector current which is not longitudinal
  - They can not be cured by Riska prescription: $\left[H_{\text{strong}}, \boldsymbol{\rho}\right] = \vec{k} \cdot \vec{\boldsymbol{J}}$
- Strong dependence on the cut-off is to be compensated by cut-off dependence of C_i LECs from NN at NLO

works out only if current and force are consistently regularized

Call for consistent symmetry-preserving regularization of forces and currents!